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A stochastic model of asset pricing function with additive effects series for capital market prices

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Abstract

In this paper, a combination of two stochastic systems namely: Stochastic Differential Equation (SDE) and Stochastic Delay Differential Equation (SDDE) respectively were considered. A closed form analytical solutions were presented in details which determined asset values through additive effects series. The validity of the analytical solution were obtained and verified using initial stock prices; which were explicitly analysed using Kolmogorov-Smirnov and results showed that the two asset values do not come from a common distribution. Finally, the impressions of other key stock quantities were discussed and graphical results presented using MATLAB.

Keywords: asset value; Kolmogorov –Smirnov (KS); additive effects; SDE and SDDE.

1 Introduction

The investment of assets in businesses creates a room where the returns or profits would be used for the management of the daily cost of the running businesses in financial investments. So, an investor might potentially require more capital investment so as to bring about the improvement of the investment proceedings. Accordingly, financial resources are invested in essence to obtain more asset investments for the purpose of expansion and growth of the business. This create a room for improved unit productivities, creation of innovativeness for new products or bring in values to the investments and to look for modern technology that improves on productivity and minimizes cost and replace outdated assets. If capital assets are skipped, stock exchange will definitely be confronted with the challenges of getting off the ground. There has been progressive growth of stock market activities and performances in the financial market as investors earn incomes to have a living.

At the same time, the stock market is typically avowed for its high returns. Returns on investment for some capital market are approaches for aptly evaluating returns on an investment. Investors adopt these measures to evaluate or assess the significance of expenditure [1]. In the capital market, the fluctuations of stock prices are caused by changes of their values over time. As a result, it is absolutely essential to understand properly the type of physical quantities to model so that realistic results of the stock variables could be obtained; [2] looked at the stochastic model of price changes on the floor of stock market. Thus far, the equilibrium price and the market growth

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rate of assets were found out. [3] studied the stochastic analysis of stock market expected returns and growth-rates. The precise conditions for obtaining the drifts, volatilities, Growth-Rates of four different assets were looked at. [4] studied on the stochastic model of the changes or variations of stock market prices. Conditions for finding out the equilibrium price, adequate circumstances for dynamic stability and convergence to equilibrium of the growth rate of the value function of shares. Furthermore, [5] studied the unstable nature of stock market forces using proposed differential equation model. [6] looked at stochastic analysis of the behaviour of stock prices. Results show that the proposed model if efficient for the prediction of stock prices. [7] worked on the solution of differential equations and stochastic differential equations of time-varying investment returns; where precise conditions were obtained which governs stock return rates through multiplicative effect and multiplicative inverse trends series. [8] considered the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in this study, the drift and volatility coefficients for the stochastic differential equations were determined. [9] worked on the problem of stock price fluctuations using stochastic differential equations, principal component analysis and KS goodness of fit test were studied. Details of the analytical solutions were given; the computational and graphical results were presented and discussed respectively. [10] considered the stochastic analysis of Markov Chain in Finite state. The transition Matrix replicated the use of 3-State transition probability matrix which enables them to present precise condition of obtaining expected mean rate of returns of each stock.

This paper combined two stochastic systems such as: Stochastic Differential Equation (SDE) and Stochastic Delay Differential Equation (SDDE) respectively which gave précised conditions of determining asset values through additive effects series. The validity of the analytical solution was verified using initial stock prices. Hence, this paper compliments the works of [11] and [12] respectively by incorporating time dependent delay parameter to realistically assess asset pricing function with additive effects series for capital market price changes which have not been seen elsewhere.

The arrangement of this paper is set as follows: Section 2.1 presents the mathematical framework, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2 Mathematical framework

At this stage we present few intricate definitions touching this dynamic area of study, hence we have the following:

Definition 1: Probability space is a triple $(\Omega, \mathrm{F}, \tilde{A})$ where Ω represents a set of sample space, F represents a collection of subsets of Ω, while $\wp$ is the probability measure defined on each event $A \in \mathrm{F}$. The collection F is a σ -algebra or σ -field such as $\Omega \in \mathrm{F}$ and F is closed under the arbitrary unions and finite intersections. Hence it is called probability measure when the following condition holds:

(i) $$P(A) \geq 0 \text{ for all } A \subset \Omega \quad (1)$$

(ii) $$P(\Omega) = 1 \quad (2)$$

(iii) $$A, B \subset \Omega,\ A \cap B = \phi \text{ then } P(A \cup B) = P(A) + P(B) \quad (3)$$

Definition 2: Normal Distribution: A normal distribution function is a peculiar distribution in probability theory and is usually used for modeling asset returns. A normal distribution is used in the Black-Scholes Partial differential equation to value European options. A normal distribution depends on two parameters.

Mean, $\mu \in \mathbb{R}$, is the expectation of a random variable normal distribution.
Variance , $\sigma^2 > 0$, deals with the magnitude of the spread from the mean.

In Black-Scholes formula, normal distributions are used. The cumulative distribution, usually denoted as $\phi(X)$, is the probability that X will be equal to or less than X ,expressed as $F_x(x) = P(X \le x)$. A standard normal cumulative distribution function is defined as.

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt \,. \tag{4}$$

A normal distribution is a symmetric distribution, which means that it touches around a vertical axis of symmetry. Obviously, there is a connection between any given points with same distance to the vertical axis. This relationship is defined in equation (5):

$$\phi(x) = 1 - \phi(-x)\,. \tag{5}$$

Definition 3: A σ -algebra is a set $\mathcal{F}$ of subsets of Ω with the following axioms:

(i) $\phi, \Omega \in \mathcal{F}$. (6)

(ii) If $A \in \mathcal{F}, then\ A^c \in \mathcal{F}$. (7)

(iii) If $A_1, A_2, ..., \in \mathcal{F}, then \bigcup_{k=1}^{\infty} A_k, \bigcap_{k=1}^{\infty} A_k \in \mathcal{F}$. (8)

Clearly $A^c := \Omega - A$ is the complement of A.

Definition 4: If $\mathcal{F}$ is a σ -algebra in Ω, then Ω is called a measurable space and the members of $\mathcal{F}$ are called the measurable sets in Ω.

Definition 5: Let $(\Omega, \mathcal{M})$ be a measurable space .A map $\mu : \mathcal{M} \to \overline{\mathbb{R}} = [0,\infty) \cup \{\infty\}$ is called a measure provided that

(i) $\mu(\phi) = 0$ (9)

(ii) $\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n)$ (10)

Definition 6: Stochastic process: It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

Definition 7: A stochastic process whose finite dimensional probability distributions are all Gaussian.(Normal distribution),
where X_k are independent and identically distributed random variables.

Therefore μ is not known completely which is subject to environmental effects.

Therefore the stock dynamics governing this process can be written as :

$$dS(t) = \mu S(t)\,\mathrm{dt} + \sigma S(t)\,\mathrm{dZ(t)}, \tag{11}$$

where σ is the volatility, dZ is the Brownian motion which is random term, the stochastic term added to (12) gives (3.15) which makes it stochastic differential equation.

Definition 8: A Stochastic Differential Equation is a differential equation with stochastic term.

Therefore assume that $(\Omega, \mathrm{F}, \wp)$ is a probability space with filteration $\{f_t\}_t \geq 0$ and $W(t) = (W_1(t), W_2(t), ..., W_m(t))^T, t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g *as* follows

$$dX(t) = f(t, X(t))dt + g(t, X(t))dZ(t),\ 0 \leq t \leq T, \tag{12}$$

$$X(0) = x_0, \tag{13}$$

where $T > 0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f : [0, \mathrm{T}] \times \mathbb{R}^n$ *and* $g : [0, T] \times \mathbb{R}^n \to \times \mathbb{R}^{n \times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S))dS + \int_0^t g(S, X(S))dZ(S) \tag{14}$$

where dX, dZ are known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$ satisfying (11).

Theorem 1.1: let $T > 0$, be a given final time and assume that the coefficient functions $f : [0, \mathrm{T}] \times \mathbb{R}^n \to \mathbb{R}^n$ *and* $g : [0, T] \times \mathbb{R}^n \to \times \mathbb{R}^{n \times n}$ are continuous. Moreover, $\exists$ finite constant numbers λ *and* β such that $\forall t \in [0, \mathrm{T}]$ and for all $x, y \in \mathbb{R}^n$, the drift and diffusion term satisfy

$$\| f(t, x) - f(t, y) \| + \| \mathrm{g}(t, x) - g(t, y) \| \leq \lambda \| x - y \|, \tag{15}$$

$$\| f(t, x) \| + \| \mathrm{g}(t, x) \| - g(t, \mathrm{x}) \| \leq \beta(1 + \| x \|). \tag{16}$$

Suppose also that x_0 is any $\mathbb{R}^n$ -valued random variable such that $E(\| x_0 \|^2) < \infty$. then the above SDE has a unique solution X in the interval $[0, T]$. Moreover, it satisfies $E\left(\sup_{0 \leq t \leq T} \| X(t) \|^2\right) < \infty$. the proof of the theorem 1.1 is seen in [17].

Theorem 3.2:(Ito's lemma). Let $f(S, t)$ be a twice continuous differential function on $[0, \infty) \times \mathrm{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0 ,$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (dS_t)^2 + \text{higer order terms } (h.\mathrm{o}, \mathrm{t}) ,$$

So, ignoring h.o.t and substituting for dS_t we obtain

$$dF_t = \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}\left(a_t dt + bdz(t)\right)^2 \tag{17}$$

$$= \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2 dt, \tag{18}$$

$$= \left(\frac{\partial F}{\partial S_t}a_t + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2\right)dt + \frac{\partial F}{\partial S_t}b_t dz(t) \tag{19}$$

More so, given the variable $S(\mathrm{t})$ denotes stock price, then following GBM implies (3.15) and hence, the function $F\left(S,t\right)$,Ito's lemma gives:

$$dF = \left(\mu S\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 F}{\partial S^2}\right)dt + \sigma S\frac{\partial F}{\partial S}dz(t) \tag{20}$$

2.1 Mathematical formulation

On the verge of assessing asset value and other financial market quantities there must be a delay τ in such assessments in one way or the other. Therefore, incorporating delay term in a model gives more realistic and better model. Therefore delay term is added in the stochastic differential equation which gives :

$$dX_i\left(t\right) = \left(t-\tau\right)X_i(t)dt + \sigma X_1(t)dZ(t) \tag{21}$$

where μ is an expected rate of returns on stock, σ is the volatility of the stock , dt is the relative change in the price during the period of time and Z is a Wiener process .Following the method of [7] and [11]on rate of returns gives as:

$$\begin{aligned} &R_t := \left(\lambda_1 + \lambda_2\right)^2, \ldots, \\ &\text{where } t = 1, 2.. \end{aligned} \tag{22}$$

Using (22) in (21) gives the following system of equations which is SDDE and SDE respectively:

$$dX_2(t) = \left(t-\tau\right)\left(\theta_1+\theta_2\right)^2 \mathrm{X}_2(t)dt + \sigma X_2(t)dZ^{(2)}(\mathrm{t}) \tag{23}$$

$$dX_4(\mathrm{t}) = \left(\theta_1+\theta_2\right)^2 \mathrm{X}_4(t)dt + \sigma X_4(t)dZ^{(4)}(\mathrm{t}), \tag{24}$$

where $\mathrm{X}_2(\mathrm{t}), and\ S_4(\mathrm{t})$ are underlying stocks with the following initial conditions:

$$\left.\begin{aligned} \mathrm{X}_2(0) = X_0 \\ \mathrm{X}_4(0) = X_0 \end{aligned}\right\} \tag{25}$$

where, $X_2(t), X_4(t),$ are asset prices, The expression dZ , which contains the randomness that is certainly a characteristic of asset prices is called a Wiener process or Brownian motion. $\lambda_1\ and\ \lambda_2$

represents rate of returns of first and second investments respectively, $\lambda_1\lambda_2$ is multiplicative effects, $\lambda_1+\lambda_2$ is additive effects.

2.2 Method of solution

The model (23) and (24) is made up of a system of stochastic delay differential equations whose solutions are not trivial. We implement the methods of Ito's lemma in solving for $\mathrm{X}_2(t), and\ X_4(t)$ To grab this problem we note that we can forecast the future worth of the asset with sureness.
From (23) Let $f\left(X_2,t\right)=\ln X_2$ so differentiating partially gives

$$\frac{\partial f}{\partial X_2}=\frac{1}{X_2},\ \frac{\partial^2 f}{\partial X_2^2}=-\frac{-1}{X_2^2},\ \frac{\partial f}{\partial t}=0 \tag{26}$$

According to Ito's gives

$$df\left(X_2,t\right)=\sigma X_2\frac{\partial f}{\partial X_2}dZ(t)+\left((\theta_1+\theta_2)^2X_2(t)\frac{\partial f}{\partial X_2}+\frac{1}{2}\sigma^2X_2^2\frac{\partial^2 f}{\partial X_2^2}+\frac{\partial f}{\partial t}\right)dt\ . \tag{27}$$

Substituting (26) and (27) into (23) gives

$$=\sigma \cancel{X_2}\frac{1}{\cancel{X_2}}dZ(t)+\left(\left(t-\tau\right)(\theta_1+\theta_2)^2\cancel{X_2}(t)\frac{1}{\cancel{X_2}}+\frac{1}{2}\sigma^2\cancel{X_2}^2(-\frac{1}{X_2^2})+0\right)dt$$
$$=\sigma dZ(t)+\left(\left(t-\tau\right)(\theta_1\theta_2)^2-\frac{1}{2}\sigma^2\right)dt\,.$$

Integrating both sides , talking upper and lower limits gives

$$\int_0^t d\ln X_2=\int_0^t df\left(X_2,u\right)=\left(t-\tau\right)\int\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)du+\int_0^t\sigma dZ(t)$$
$$\ln X_2-\ln X_0=\left(t-\tau\right)\left((\theta_1+\theta_2)^2u-\frac{1}{2}\sigma^2u\right)|_0^t+\left(\sigma Zu\right)|_0^t \tag{28}$$
$$\ln\left(\frac{X_2}{X_0}\right)=\left(t-\tau\right)\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)t+\sigma Z(t)$$

Taking the ln of the both sides

$$X_2(t)=X_0\exp\left(t-\tau\right)\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)t+\sigma Z(t) \tag{29}$$

From (24), let $f\left(X_4,t\right)=\ln X_4$ so differentiating partially gives

$$\frac{\partial f}{\partial X_4}=\frac{1}{X_4},\ \frac{\partial^2 f}{\partial X_4^2}=-\frac{-1}{X_4^2},\ \frac{\partial f}{\partial t}=0 \tag{30}$$

According to Ito's gives

$$df\left(X_4,t\right)=\sigma X_4\frac{\partial f}{\partial X_4}dZ(t)+\left((\theta_1+\theta_2)^2X_4(t)\frac{\partial f}{\partial X_4}+\frac{1}{2}\sigma^2X_4^2\frac{\partial^2 f}{\partial X_4^2}+\frac{\partial f}{\partial t}\right)dt \tag{31}$$

Substituting (30) and (31) into (24) gives

$$=\sigma\cancel{X_4}\frac{1}{\cancel{X_4}}dZ(t)+\left((\theta_1+\theta_2)^2\cancel{X_4}(t)\frac{1}{\cancel{X_4}}+\frac{1}{2}\sigma^2\cancel{X_4}^2(-\frac{1}{X_4^2})+0\right)dt$$

$$=\sigma dZ(t)+\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)dt\,.$$

Integrating both sides , talking upper and lower limits gives

$$\int_0^t d\ln X_4=\int_0^t df\left(X_4,u\right)=\int\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)du+\int_0^t\sigma dZ(t)$$

$$\ln X_4-\ln X_0=\left((\theta_1+\theta_2)^2u-\frac{1}{2}\sigma^2u\right)|_0^t+\left(\sigma Zu\right)|_0^t \tag{32}$$

$$\ln\left(\frac{X_4}{X_0}\right)=\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)t+\sigma Z(t)$$

Taking the ln of the both sides

$$X_4(t)=X_0\exp\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)t+\sigma Z(t)\,. \tag{33}$$

Table 1: The impact of time delay in the assessment of Asset values with additive effects through the solution below: $X_2\left(t\right)=X_0\exp\left\{\left(t-\tau\right)\left(\left(\theta_1+\theta_2\right)^2-\frac{1}{2}\sigma^2\right)t+\sigma dz\left(t\right)\right\}$ t = 2, $dz=1$

τ	S_0	σ	$(\theta_1+\theta_2)$	$X_2(t)$	$(\theta_1+\theta_2)\,X_2(t)$	
	5.0(	0.2	1.0000	194.3067	2.0000	7056346.005
0.2!	6.0(	0.2	1.0000	233.1681	2.0000	8467615.206
	5.7(	0.2	1.0000	221.5097	2.0000	8044234.446
	5.2(	0.2	1.0000	165.4483	2.0000	3297445.466
0.3!	4.6(	0.2	1.0000	146.3581	2.0000	2916790.99
	4.0(	0.2	1.0000	127.2679	2.0000	2536496.513
	3.7(	0.2	1.0000	96.3833	2.0000	1054242.249
0.4!	3.9(	0.2	1.0000	101.5932	2.0000	1111228.317
	3.5(	0.2	1.0000	91.1734	2.0000	997256.1817
	3.3:	0.2	1.0000	70.5942	2.0000	423770.8709
0.5!	3.7!	0.2	1.0000	79.9783	2.0000	480102.9504
	4.0(	0.2	1.0000	85.3102	2.0000	512109.8138

Table 1 shows the various levels of asset value increase as a result of delay parameter. This remark is realistic because as you delay in selling- off your landed properties, Estates, companies etc over time; will eventually yield more monies compared to when it is sold for today. That is why investors buy these assets and allow it for some time to appreciate in value before they are sold.

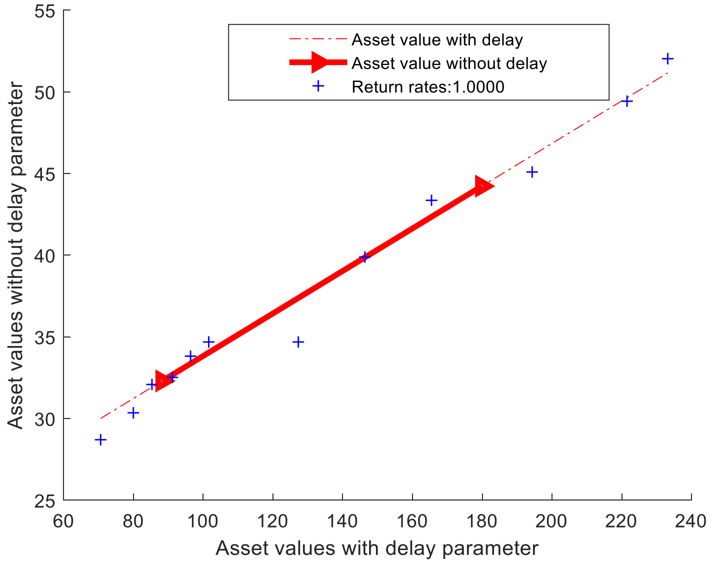

Figure 1: Graphical representation of two asset values (with and without delay) parameter which follows additive effects with:1.0000.

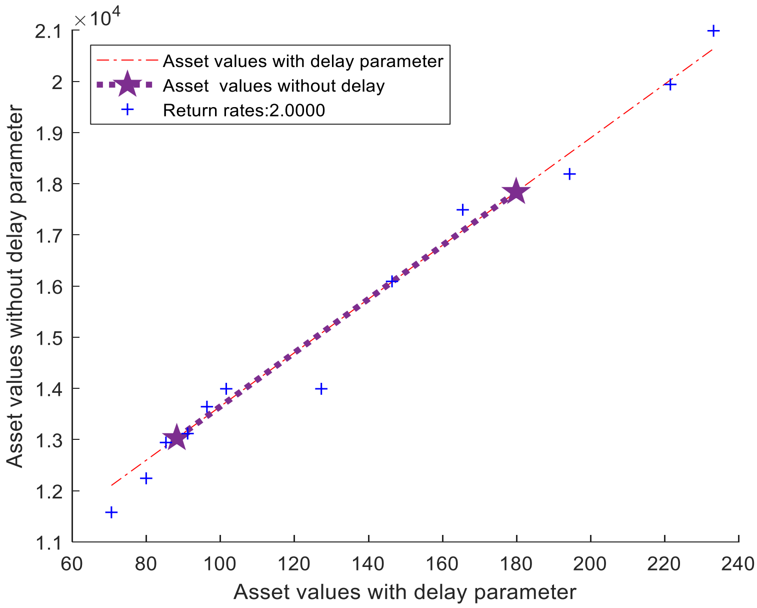

Figure 2: Graphical representation of two asset values (with and without delay) parameter which follows additive effects with:1.0000.

Figures 1 and 2 are quantile-quantile (q-q) of two asset values under considerations; it shows that two asset values do not come from a population with a common distribution. These results agree with our findings in Tables 1 and respectively using Kolmogorov –Smirnov (KS).

Table 2: The assessment of Asset values with additive effects through the solution below: $X_4(t) = X_0 \exp\left\{\left((\theta_1+\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma dz(t)\right\}$, $dz = 1$ and $t = 2$

S_0	σ	$(\theta_1+\theta_2)$	$X_4(t)$	$(\theta_1+\theta_2)$	$X_4(t)$
5.00	0.2	1.0000	43.3557	2.0000	17490.9330
6.00	0.2	1.0000	52.0268	2.0000	20989.1196
5.70	0.2	1.0000	49.4255	2.0000	19939.6636
5.20	0.2	1.0000	45.0899	2.0000	18190.5703
4.60	0.2	1.0000	39.8872	2.0000	16091.6584
4.00	0.2	1.0000	34.6846	2.0000	13992.7464
3.70	0.2	1.0000	32.0832	2.0000	12943.2904
3.90	0.2	1.0000	33.8174	2.0000	13642.9278
3.50	0.2	1.0000	30.3490	2.0000	12243.6531
3.31	0.2	1.0000	28.7015	2.0000	11578.9977
3.75	0.2	1.0000	32.5168	2.0000	13118.1998
4.00	0.2	1.0000	34.6846	2.0000	13992.7464

Table 2 displays asset values earned without delay. There is no much profit here because goods and services are sold as they arrived without minding the future outcome; hence asset value is a function of time and money. This remark informs investors or traders on how effectively they can grow more wealth in their investments.

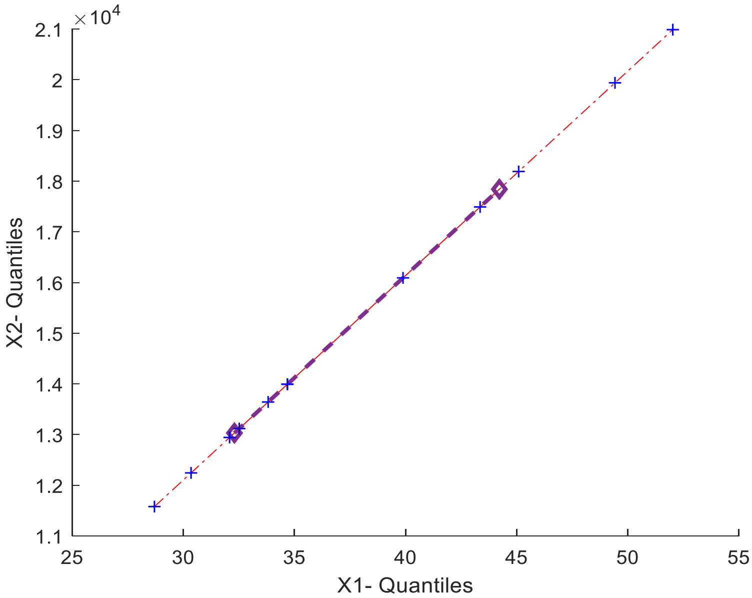

Figure 3: Graphical representation of two asset values (with and without delay) parameter which follows additive effects .

H_0 : The asset value functions $X_2(t)$ and $X_4(t)$ comes from a common distribution
H_1 : They are not from a common distribution

Table 3: Goodness of fit test for assessment of two asset values with and without delay when its return rates is: 1.0000.

$(\theta_1\theta_2)$	$X_4(t)$	$X_2(t)$	Mean	STD	KSStat	P-value	Decision
1.0000	43.3557	194.3067	118.8312	106.7385	1.0000	0.2890	Accept
1.0000	52.0268	233.1681	142.5975	128.0862	1.0000	0.2890	Accept
1.0000	49.4255	221.5097	135.4676	121.6819	1.0000	0.2890	Accept
1.0000	45.0899	165.4483	105.2691	85.1062	1.0000	0.2890	Accept
1.0000	39.8872	146.3581	93.1227	75.2863	1.0000	0.2890	Accept
1.0000	34.6846	127.2679	80.9762	65.4663	1.0000	0.2890	Accept
1.0000	32.0832	96.3833	64.2332	45.4670	1.0000	0.2890	Accept
1.0000	33.8174	101.5932	67.7053	47.9247	1.0000	0.2890	Accept
1.0000	30.3490	91.1734	60.7612	43.0093	1.0000	0.2890	Accept
1.0000	28.7015	70.5942	49.6478	29.6226	1.0000	0.2890	Accept
1.0000	32.5168	79.9783	56.2476	33.5603	1.0000	0.2890	Accept
1.0000	34.6846	85.3102	59.9974	35.7977	1.0000	0.2890	Accept

Following Tables 3 and 4 the alternative hypothesis of KS was accepted because it is the hypothesis we are will to accept if we reject the null hypothesis hence the p-values are not significant. The level of significance $\alpha = 0.01$ shows that the two asset values are not from a common distribution which has financial meaning as it affects asset returns; with these traders or investors are properly guided in terms of decision making. Clearly, we can boldly say that there is significant dissimilarity among the two asset values $\left(X_2(t)\ , X_4\ (t)\right)$ throughout the trading days. In another dimension, the mean rates which can be seen in columns 5 of Tables 3 and 4 respectively defines the average levels of asset values when additive rate of return is 1.0000 and 2.0000. The standard deviation measures the degree of each level of changes in respect to asset values in financial markets.

Table 4: Goodness of fit test for assessment of two asset values with and without delay when it return rates is: 2.0000 .

$(\theta_1\theta_2)$	$X_4(t)$	$X_2(t)$	Mean	STD	KSStat	P-value	Decision
2.0000	17490.9330	7056346.005	3.3369e+06	4.9772e+06	1.0000	0.2890	Accept
2.0000	20989.1196	8467615.206	4.2443e+06	5.9727e+06	1.0000	0.2890	Accept
2.0000	19939.6636	8044234.446	4.0321e+06	5.6740e+06	1.0000	0.2890	Accept
2.0000	18190.5703	3297445.466	1.6578e+06	2.3188e+06	1.0000	0.2890	Accept
2.0000	16091.6584	2916790.99	1.1664e+06	2.0511e+06	1.0000	0.2890	Accept
2.0000	13992.7464	2536496.513	1.2752e+06	1.7837e+06	1.0000	0.2890	Accept
2.0000	12943.2904	1054242.249	5.3359e+05	7.3631e+05	1.0000	0.2890	Accept
2.0000	13642.9278	1111228.317	5.6244e+05	7.7611e+05	1.0000	0.2890	Accept
2.0000	12243.6531	997256.1817	5.0475e+05	6.9651e+05	1.0000	0.2890	Accept
2.0000	11578.9977	423770.8709	2.1767e+05	2.9146e+05	1.0000	0.2890	Accept
2.0000	13118.1998	480102.9504	2.4661e+05	3.3021e+05	1.0000	0.2890	Accept
2.0000	13992.7464	512109.8138	2.6305e+05	3.5222e+05	1.0000	0.2890	Accept

3 Conclusion

Generally, an assessment of asset values and its return rates are very vital in every capital investment. The non-existence of assessment of asset values may have a hard time crumbling which does not benefit any trader or investor in a time varying investments. Therefore , this paper studied two stochastic systems which results showed as follows: (i) : increase in delay parameter dominantly increases the value of asset (ii) the solution of SDDE has more impact over solution of SDE in terms of asset value estimates, (iii) higher initial stock price has substantial effects on the value of asset over time (iv) a higher asset value also have a higher level of asset changes according to standard deviation rates(v) the two asset values under-consideration do not come from a common distribution . More so, comparing the existence and uniqueness of the models will be an interesting study to explore.

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