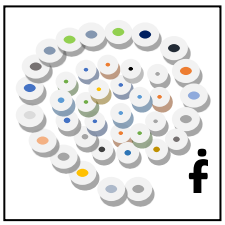

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Lassa fever disease: a sensitivity analysis of its treatment control model

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Abstract

A treatment model with seven mutually exclusive compartments related to Lassa fever is presented and the dynamics is completely determined by the effective reproduction number which was obtained using the next-generation approach. A sensitivity analysis of the treatment deterministic model is performed. This is done in order to determine the relative importance of the model parameters to the disease transmission. Stability and sensitivity based on such methods as Lyaponuv functional, Comparison theorem etc. Numerical simulation analysis (using MATLAB and MAPLE) were carried out to support the theoretical analysis. The result of the sensitivity analysis shows that the most sensitive parameter is the treatment rate, followed by human recovery rate, then person-to-person contact. This suggests that control strategies should target effective drugs for treatment and education to reduce person to person contact.

Keywords: Lassa fever; mathematical modeling; sensitivity; treatment control

1 Introduction

Lassa fever (LF) is a serious and potentially deadly viral disease caused by Lassa virus (LASV) that is prevalent in West Africa including Benin, Sierra Leone, Guinea, Liberia, Mali, and Nigeria because LASV reservoir and vector (Mastomys natalensis) is abundantly seen in these areas [3]. LASV has 2-21 day or more than that incubation period, thus its increases the risk of its transmission from endemic to non-endemic regions [17]. One of the major concerns with LASV is its high case fatality rate, additionally, the virus can easily spread from human-human contact, which makes it even more dangerous. Another major concern with LASV is the lack of effective vaccines and therapeutics for treating the disease. This is further compounded by the fact that the virus can also spread through aerosol, making it even more difficult to contain [2].

Due to the severe nature of LASV and its potential to spread easily, it is classified as a Biosafety Level 4 agent. This means that it is considered to be one of the most dangerous pathogens known to mankind. Despite this, the overall impact of LASV on the increased threat of viral diseases in West Africa has not been fully understood [1]. However, it is known that LASV is a significant cause of yearly morbidity and mortality in many of Africa's poverty-stricken communities such as during 2022 in Nigeria around 189 deaths reported out of 1067 confirmed cases of LASV. Annually, around 1–3 million infections occur of LF with around 5000 deaths [7].

This Lassa fever epidemic, with its increased frequency indicates that our current knowledge of its dynamics and public health guidelines to control the disease is not adequate. [5] carried out a study

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by the use of mathematical model which is formulated based on the natural history of Lassa fever. They also carried out another study on the mathematics of Lassa fever treatment model [5] which is more comprehensive than previous mathematical model studies. However, they did not consider the sensitivity of the model, and this prompted the present study. Hence, we shall consider the Lassa fever epidemic under such sub-headings as transmission, signs and symptoms, method of diagnosis, etc. We shall also consider vaccine and treatments, natural host, prevention and control, and such other information that may be very necessary for proper understanding of our mathematical model and the subsequent results and analysis. Hence, the present study will present a mathematical treatment model on the spread of the Lassa fever disease with current features and the sensitivity analysis on the transmission dynamics.

In this study a compartmental mathematical treatment model is presented to investigate the sensitivity analysis of Lassa fever transmission dynamic model with treatment control. Sensitivity analysis in mathematical biology tells us how important each parameter is to disease transmission and is used to assess how sensitive a model is to variations in the value of the parameters of the model and to changes in the structure of the model [12]. It refers to technique used to determine how independent variable values will impact a particular dependent variable under a given set of assumptions. It is used to determine parameters that have high impact on the basic reproduction number so that it is directly targeted by intervention strategies. It is necessary to determine how sensitive the threshold quantity basic reproductive number with respect to its parameters. This analysis reveals how crucial each of the parameter is to the disease transmission.

Sensitivity analysis has been used for various parameterization tasks of models of biological systems, such as finding essential parameters for research prioritization [9], identifying insignificant parameters or model reduction or parameters clustering (Peter et al in [12]). Classically, sensitivity of the model is determined by the partial derivatives of the outcome with respect to its parameters. Sensitivity analysis methods based on such quantities are called local as the derivative is taken at a fixed point in the state space of model parameters – at the parameter face value. Moreover, these methods belong to the class of one-factor-at-a-time (OAT) methods, because the net effect of a parameter on the property of the outcome is taken while assuming that all other factors are fixed [12].

In recent times, sensitivity analysis of Lassa fever models has received attention of researchers in mathematical epidemiology and has become a subject of intense study [5, 8, 10, 11, 12, 13]. A good number of mathematical models has been developed to assess the sensitivity of each factor driving the disease in order to determine the major factors to be targeted by appropriate interventions in order to eradicate Lassa fever. The present work is aimed at conducting sensitivity analysis on a new Lassa fever treatment model due to [6], for effective management of Lassa fever spread. We reproduce their Lassa fever treatment control model here for ease of reference:

$$\begin{cases} \frac{dS_h}{dt} = \Lambda_h + \phi(1-\nu)I_h + \psi R_h - \beta_1\sigma I_r S_h - \beta_2\varepsilon I_h S_h - \eta(1-e^{-rt})S_h - \mu_h S_h \\ \frac{dE_h}{dt} = \beta_1\sigma I_r S_h + \beta_2\varepsilon I_h S_h + \eta(1-e^{-rt})S_h - (\kappa+\mu_h)E_h \\ \frac{dI_h}{dt} = \kappa E_h - (\phi+\theta+\delta+\mu_h)I_h \\ \frac{dT_{h1}}{dt} = \alpha\theta I_h - \mu_h T_{h1} \\ \frac{dT_{h2}}{dt} = (1-\alpha)\theta I_h - (\delta+\mu_h)T_{h2} \\ \frac{dR_h}{dt} = \phi\nu I_h + \varpi T_{h1} - (\psi+\mu_h)R_h \\ \frac{dS_r}{dt} = \Lambda_r - \beta_3\vartheta I_r S_r + \varphi R_r - \mu_r S_r \\ \frac{dI_r}{dt} = \beta_3\vartheta I_r S_r - (\omega+\mu_r)I_r \\ \frac{dR_r}{dt} = \omega I_r - (\varphi+\mu_r)R_r \end{cases} \quad (1)$$

The definitions of above model variables and parameters are listed in Tables 1 and 2.

Table 1: Description of the state variables of the model

Variable	Description
S_h	Number of Susceptible humans
E_h	Number of Exposed humans
I_h	Number of Infectious humans
T_{h1}	Early stage treatment group
T_{h2}	Late stage treatment group
R_h	Number of Recovered humans
S_r	Number of Susceptible rodents
I_r	Number of Exposed rodents
R_r	Number of Infectious rodents

Table 2: Description of the parameters of the Lassa fever model

Parameters	Description	Values	Source
Λ_h	Recruitment level of humans	20	Assumed
Λ_r	Recruitment level of rodents	200	Assumed
δ	Per capita Lassa-induced death rate	0.2	[17]
ψ	Recovered human loss of immunity	0.9	[16]
ϕ	Spontaneous individual recovery	0.001	[17]
β_1	Transmission rate per contact by an infectious rodent	0.00002	[18]
β_2	Transmission rate per contact by an infectious throu sexual activity	0.002	[18]
β_3	Transmission rate per contact from an infectious hum through other means	0.08	[18]

η	Relative infectiousness of individuals with aerosol	0.03	[15]
μ_h	Natural mortality rate for humans	0.02041	[4]
μ_r	Natural mortality rate for rodents	0.06	[17]
κ	Progression rate of human from exposed to infected	0.05	Estimated
α	Progression rate of rodents from exposed to infected	0.42	Estimated
σ	Contact rate of rodent per human per unit time	0.005	[6]
ϑ	Relative human-to-rodent transmissibility of infect humans	0.6	Estimated
ε	Relative human-to-human transmissibility of infect humans	0.5	Estimated
r	Number of doses of airborne viral particles	Negligible	[6]
ω	Recovery rate of rodents	0.00001	[18]
φ	Recovered rodent loss of immunity	0.001	Estimated
ϖ	Rate of early stage treated group who recovers	0.2	Estimated
α	Fraction of early stage treatment group	(0,1)	Assumed
θ	Treatment rate of humans	0.8	Estimated

Source: [6]

2 Qualitative properties of the model

Since the model (1) monitors human population, all its associated parameters and state variables are assumed to be non-negative for all t ≥ 0. Before analysing the model, it is instructive to show that the state variables of the model remain non-negative for all non-negative initial conditions. Thus, we claim the following result.

Theorem 1: Let the initial data $S_h(0), E_h(0), I_h(0), T_{h1}(0), T_{h2}(0), R_h(0), S_r(0), I_r(0), R_r(0)$ be non-negative. Then, the solutions $(S_h, E_h, I_h, T_{h1}, T_{h2}, R_h, S_r, I_r, R_r)$ of model (1) are positive and bounded for all $t > 0$, whenever they exists.

Proof:

The first equation (1) of the system can be written as:

$$\frac{dS_h}{dt} + \left[\lambda(S_h) + \mu_h\right] S_h = \Lambda_h + \phi(1-\nu) I_h + \psi R_h$$

$$\frac{d}{dt}\left[S_h(t)\eta(t)\right] = \left(\Lambda_h + \phi(1-\nu) I_h + \psi R_h\right)\eta(t),$$

where $\eta(t) = \exp\left(\int_0^t \left[\lambda(S_h) + \mu_h\right] t dt\right) > 0$ is the integrating factor. Hence, integrating this last relation from 0 to t_1, we have

$$\left[S_h(t_1)\eta(t_1)\right] - S_h(0)\eta(0) = \int_0^{t_1} \left(\Lambda_h + \phi(1-\nu) I_h + \psi R_h\right) \exp\left[\left(\lambda(S_h) + \mu_h\right) p\right] dp,$$

$$\eta(0) = 1,$$

$$\left[S_h(t_1)\eta(t_1)\right] - S_h(0) = \int_0^{t_1} \left(\Lambda_h + \phi(1-\nu) I_h + \psi R_h\right) \exp\left[\left(\lambda(S_h) + \mu_h\right) p\right] dp$$

$$S_h(t_1)\eta(t_1) = S_h(0) + \int_0^{t_1} \left(\Lambda_h + \phi(1-\nu) I_h + \psi R_h\right) \exp\left[\left(\lambda(S_h) + \mu_h\right) p\right] dp$$

so that the division of both side by $\eta(t)$ yields

$$S_h(t_1)=\left[S_h(0)+\int_0^{t_1}\left(\Lambda_h+\phi(1-\nu)I_h+\psi R_h\right)\exp\left[\left(\lambda(S_h)+\mu_h\right)p\right]dp\right]\times\eta^{-1}(t_1)>0.$$

The same arguments can be used to prove that $E_h(t),I_h(t),R_h(t),S_r(t),I_r(t),R_r(t)\geq 0$ for all $t>0$.

Furthermore, let $N=S_h+E_h+I_h+T_{h1}+T_{h2}+R_h$. Then,

$$\begin{aligned}&\dot{N}(t)=\dot{S}_h+\dot{E}_h+\dot{I}_h+\dot{T}_{h1}+\dot{T}_{h2}+\dot{R}_h\\&=\Lambda_h+\phi(1-\nu)I_h+\psi R_h-\lambda_h S_h-\mu_h S_h+\lambda_h S_h-(\kappa+\mu_h)E_h+\kappa E_h-(\phi+\theta+\delta+\mu_h)I_h\\&+\alpha\theta I_h-\mu_h T_{h1}+(1-\alpha)\theta I_h-(\delta+\mu_h)T_{h2}+\phi\nu I_h+\varpi T_{h1}-(\psi+\mu_h)R_h\\&\leq\ \Lambda_h-\mu_h N_h-\delta I_h,\ \ \delta=0\end{aligned}$$

This implies that as $t\to\infty,\sup N_h\leq\frac{\Lambda_h}{\mu_h}$. Also from (1), we have that as: $t\to\infty,\sup S_r(t)\leq\frac{\Lambda_r}{\mu_r}$.
This completes the proof.

Combining Theorem 1 with the trivial existence and uniqueness of a local solution for the model (1), we have established the following theorem which ensures the mathematical and biological well-posedness of system (1).

Theorem 2: The dynamics of model (1) is a dynamical system in the biological feasible compact set

$$\Gamma:=\left\{\left(S_h,E_h,I_h,T_{h1},T_{h2},R_h,S_r,I_r,R_r\right)\in\mathbb{R}_+^9:0\leq S_h\leq\frac{\Lambda_h}{\mu_h};N_h\leq\frac{\Lambda_h}{\mu_h},N_r\leq\frac{\Lambda_r}{\mu_r}\right\}\tag{2}$$

2.1 Reproduction number and stability analysis

The model (1) has a disease-free equilibrium (DFE), obtained by setting the right hand sides of the equations in the model to zero, given by

$$\xi_0=\left(S_h^0,E_h^0,I_h^0,T_{h1}^0,T_{h2}^0,R_h^0,S_r^0,I_r^0,R_r^0\right)=\left(\frac{\Lambda_h}{\mu_h},0,0,0,0,0,\frac{\Lambda_r}{\mu_r},0,0\right).\tag{3}$$

The stability of ξ_0 can be established using the next generation operator method on the system (1). Using the notations in [16], the matrices F *and* V, for the new infection terms and the remaining transfer terms respectively, are given by

$$F=\begin{pmatrix}0&\beta_2\varepsilon S_h^0&0&0&\beta_1\sigma S_h^0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&\beta_3\vartheta S_r^0\end{pmatrix};\ V=\begin{pmatrix}(\mu_h+\kappa)&0&0&0&0\\-\kappa&(\phi+\mu_h+\delta+\theta)&0&0&0\\0&\alpha\theta&\mu_h&0&0\\0&-\alpha(1-\theta)&0&(\delta+\mu_h)&0\\0&0&0&0&(\omega+\mu_r)\end{pmatrix}$$

In the calculation of matrices F *and* V, we took the infection variables to be E_h,I_h,T_{h1},T_{h2} and I_r as explained in [5]. Thus

$$R_T=R_h+R_r=\frac{\beta_2\kappa\varepsilon\Lambda_h}{\mu_h(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)}+\frac{\beta_3\vartheta\Lambda_r}{\mu_r(\omega+\mu_r)},$$

where R_T is obtained from $\rho\left(FV^{-1}\right)$ with ρ being the spectral radius of the matrix FV^{-1} Computing the partial derivatives of R_T with respect to the parameters under investigation (θ, ϖ) further reveals the effect of these parameters on Lassa fever control in the community. This gives

$$\frac{\partial R_T}{\partial \theta} = -\frac{\beta_2 \varepsilon \kappa \Lambda_h}{\mu_h (\mu_h + \kappa)(\phi + \mu_h + \delta + \theta)^2} < 0 . \quad (4)$$

Clearly, it follows from (4) that the partial derivative is less than zero, unconditionally. Hence, treatment rate of humans will have a positive impact in reducing the burden of LF in the population, irrespective of the values of the other parameters in the expressions on the right-hand sides of (4). Assuming treatment control can reduce the basic reproduction number reduces R_0 by a factor q, then effective reproduction number [6]:

$$R_T = (1-q) R_0 . \quad (5)$$

This implies that no outbreak if $R_T \le 1, ie, \ if \ q \ge 1 - \frac{1}{R_0}$, we obtain that

$$\frac{R_T}{R_0} = \frac{\phi + \mu_h + \delta}{\phi + \mu_h + \delta + \theta} = 0.88 \Rightarrow R_T = (1-q) R_0$$
$$\therefore 1 - q = 0.88 \Rightarrow q = 0.12$$

Thus, introduction of treatment control significantly reduces the transmission of LF required for successful eradication of the disease (requirement of at least 12% effectiveness). This means that the treatment reproduction number R_T is 12% smaller than the basic reproduction number R_0, indicating that the use of treatment as a control strategy is 12% effective on reducing LF transmission. This implies that early detection is of paramount importance in the control of Lassa fever in a population. However, this is an indication that the early stage group treatment program alone at this rate is not enough to eliminate the disease. So, multiple control strategy requires urgent assessment.

Lemma 1: The DFE of the model (1), ξ_0, is locally asymptotically stable if $R_T < 1$ and unstable if $R_T > 1$. The threshold quantity R_T is the effective reproduction number under treatment control for the Lassa fever model. Biologically speaking, Lemma (1) implies that Lassa fever can be eliminated from the community (when $R_T < 1$) if the initial sizes of the subpopulation of the model are in the basin of attraction of ξ_0 in the presence of treatment.

2.2 Global Asymptotic Stability: Special Case $\sigma = \eta = 0$.

Consider the model (1) with $\sigma = \eta = 0$. We claim the following:

Theorem 3: The DFE of the model (1) with $\sigma = \eta = 0$ is globally-asymptotically stable (GAS) whenever $R_T < 1$.

Proof:

Consider the model (1) with $\sigma = \eta = 0$. Further, consider the following linear Lyapunov function

$F = \kappa E_h + (\kappa + \mu_h) I_h$

with Lyapunov derivative (where a dot represents differentiation with respect to t):

$$\dot{F} = \kappa \dot{E}_h(t) + (\kappa + \mu_h)\dot{I}_h(t)$$
$$= \kappa\left(\lambda_h S_h - (\kappa + \mu_h)E_h\right) + (\kappa + \mu_h)\left(\kappa E_h - (\phi + \theta + \delta + \mu_h)I_h\right)$$
$$= (\kappa + \mu_h)(\phi + \theta + \delta + \mu)\left[\frac{\beta_2 \varepsilon \kappa S_h}{(\kappa + \mu_h)(\phi + \theta + \delta + \mu)} - 1\right] I_h$$
$$\dot{F} \le (\kappa + \mu_h)(\phi + \theta + \delta + \mu_h)[R_T - 1] I_h$$

Hence, $\dot{F} \le 0$ if $R_T \le 1$ with $\dot{F} = 0$ if and only $I_h = 0.$ Therefore F is a Lyapunov function in Γ and it follows Salle's Invariance Principle [6], that every solution to the equations in (1) (with $\sigma = \eta = 0$) with initial conditions in Γ converges to ξ_0 as $t \to \infty.$ i.e.,

2.3 Effect of the effective treatment rate

In order to investigate the impact of the effective treatment rate on the spread of LF, we carry out some numerical simulations to show the contribution of effective treatment rate during the whole infection. From Figure 2, we can observe that infected individuals reach a lower point level as θ increases. This Figure 1 illustrates the great influence of the effective treatment rate as shown in the threshold analysis.

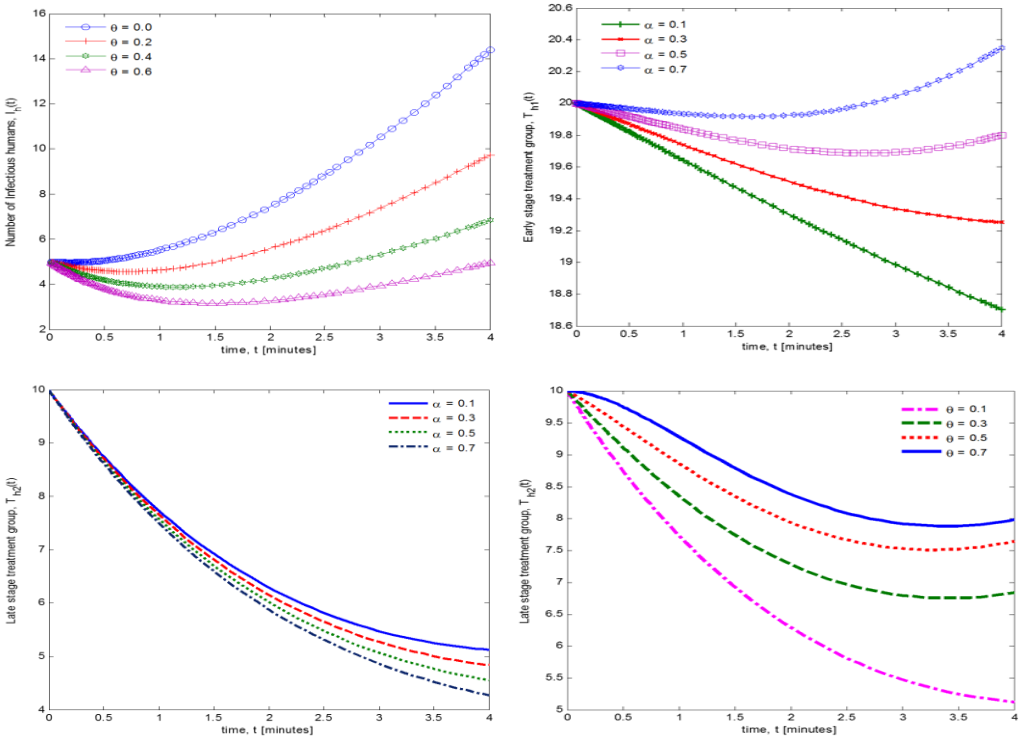

Figure 1. Impact of effective treatment rate on the spread of LF.

3 Sensitivity analysis

Sensitivity indices allow us to measure the relative change in a variable when a parameter changes. If the model is simple, it may be possible to differentiate the outcome with respect to each parameter in turn. The derivatives are the rate of changes of predictions with respect to the parameters. This work adopts the normalized forward sensitivity index to conduct the sensitivity index of a variable with respect to a parameter is the ratio of the change in the parameter. When the variable is a differentiable function of the parameter, the sensitivity index may be alternatively defined using partial derivative

[5]. For instance, the normalized forward sensitivity index on R_T which depends differentially on a parameter φ, is defined by

$$\Pi_{\varphi}^{R_T} = \frac{\partial R_T}{\partial \varphi} \times \frac{\varphi}{R_T} \tag{6}$$

The sensitivity index of parameters with respect to R_T is given as:

$$\Pi_{\Lambda_h}^{R_T} = \frac{\beta_2 \varepsilon \kappa}{\mu_h(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)} \times \frac{\Lambda_h(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)\mu_h}{\beta_2 \varepsilon \kappa \Lambda_h} = 1 > 0$$

$$\Pi_{\beta_2}^{R_T} = \frac{\varepsilon \kappa \Lambda_h}{\mu_h(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)} \times \frac{\beta_2(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)\mu_h}{\beta_2 \varepsilon \kappa \Lambda_h} = 1 > 0$$

$$\Pi_{\varepsilon}^{R_T} = \frac{(1-u_1)\kappa \Lambda_h}{\mu_h(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)} \times \frac{\varepsilon(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)\mu_h}{\beta_2 \varepsilon \kappa \Lambda_h} = 1 > 0$$

$$\Pi_{\kappa}^{R_T} = \frac{\beta_2 \varepsilon \Lambda_h \mu_h}{(\kappa+\mu_h)^2\left[(\phi+\mu_h+\delta+\theta)\mu_h\right]} \times \frac{\kappa(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)\mu_h}{\beta_2 \varepsilon \kappa \Lambda_h} = 0.667 > 0$$

$$\Pi_{\theta}^{R_T} = -\frac{\beta_2 \varepsilon \kappa \Lambda_h}{\mu_h(\mu_h+\kappa)(\phi+\mu_h+\delta+\theta)^2} \times \frac{\theta(\phi+\mu_h+\delta+\theta)\mu_h}{\beta_2 \varepsilon \kappa \Lambda_h} = -1.784 < 0$$

$$\Pi_{\delta}^{R_T} = \frac{-\beta_2 \varepsilon \kappa \Lambda_h}{\mu_h(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)^2} \times \frac{\delta(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)\mu_h}{\beta_2 \varepsilon \kappa \Lambda_h} = -0.743 < 0$$

$$\Pi_{\mu_h}^{R_T} = \frac{-\beta_2 \varepsilon \kappa \Lambda_h\left[3\mu_h + 2\mu_h(2\kappa(\phi+\delta+\theta)+\kappa(\phi+\delta+\theta)+2\kappa(\phi+\delta+\theta)\right]}{\left[\mu_h(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)\right]^2}$$

$$\times \frac{\mu_h^{\,2}(\kappa+\mu_h)(\phi+\mu_h+\delta+\theta)}{\beta_2 \varepsilon \kappa \Lambda_h} = -1.603 < 0$$

$$\Pi_{\phi}^{R_T} = -\frac{\beta_2 \varepsilon \kappa \Lambda_h}{\mu_h(\mu_h+\kappa)(\phi+\mu_h+\delta+\theta)^2} \times \frac{\phi(\phi+\mu_h+\delta+\theta)\mu_h}{\beta_2 \varepsilon \kappa \Lambda_h} = -1.482 < 0$$

Table 3: Sensitivity indices of the effective reproduction number to model parameters

Parameter	Estimated values	Sensitivity index
μ_h	0.02041	–1.603
θ	0.8	–1.784
δ	0.2	–1.784
ϕ	0.001	–1.482
Λ_h	20	1
β_2	0.002	1
ε	0.5	1
κ	0.01	–0.667

Source: Researchers' Computation

It is observed in Table 3 that these parameters have either positive or negative effects on the effective reproduction number. Positive index indicates that the value of R_T will increase with increase in the parameter and the one with negative index will decrease the value of R_T with increase in the parameter values.

4 Discussion of results

This study is concerned with an epidemiological treatment model of Lassa fever (LF). The effect of treatment rate of humans on the model is investigated and the main results are given in terms of local and global stability. It has been shown that that treatment rate of humans will have a positive impact in reducing the burden of LF in the population. Our model predicts that treatment control can reduce the population level transmission by up to 12% alone without existing interventions. Therefore, treatment has significant effect on LF transmission, but it may not be able to eliminate the disease unless a multiple control strategy is adopted. Accordingly, we should reduce the reproduction number periodically so as to keep the number of infected below the capacity for treatment. Finally, some numerical simulations are carried out to support our theoretical results. Based on this, we carry out some numerical simulations to show the contribution of effective treatment rate during the whole infection. From the simulation results, we observed that infected individuals reach a lower point level as θ increases. This figure illustrates the great influence of the effective treatment rate as shown in the threshold analysis. It is hoped that this study would motivate public health epidemiologists to collect relevant data for further and better understanding of the effect of treatment on the dynamics of LF.

From Table 3, the most significant parameters affecting the effective reproduction number are per capital induce death rate, treatment rate and natural recovery rate. These parameters should be targeted by control intervention strategies. Increasing these parameters will decrease the effective reproduction number for the control model. Similarly, if the values are decreased, the effective reproduction number will be increased and as such the spread of Lassa fever increases. An increase in the positively indexed parameters like rate of exposure to aerosol, transmission rate per contact by an infective through sexual activity and relative human-to-human transmissibility rate of infected humans increase the value of the effective reproduction number and as such the disease invades the population. The results in Table 3 have far reaching implications to Lassa fever transmission and control. Firstly, it shows that treatment plays an indispensable role in curtailing the spread of Lassa fever outbreak. This collaborates the earlier works of [5, 10, 13] who concluded that the best strategies against the disease are maintaining good community hygiene to control the mastomys and taking care of the infective under specific isolation precaution. This result shows that with individuals' total adherence to effective use of personal protection against disease-induce death, little treatment efforts will then be required by the community in the control of the spread of the disease. Secondly, the lowest sensitivity index, which is the relative infectiousness of individuals with aerosol does not imply that this parameter is not relevant to Lassa fever control. It implies that Lassa fever prevention is achievable provided that relative infectiousness of individuals with aerosol is low enough to cause outbreak. This could be done by keeping the environment very ventilated. Ventilation represents a primary infectious disease control strategy through dilution of room air around a source and removal of infectious agents [5].

5 Conclusion

In this paper, we have successfully carried out the sensitivity analysis of treatment model of Lassa fever transmission in human and rodent population. Mathematically, we have shown that it is possible to control Lassa fever disease in Nigeria by treatment measure. We therefore conclude that early treatment measures have some influence in reducing the burden of Lassa fever. Also, it was concluded that the most sensitive parameter is the treatment rate, followed by human recovery rate, Lassa fever induced-death then person-to-person contact. The following recommendations are made:

- We recommend possible public health interventions based on the result of this study. This result obtained in this work can be a useful guide for the local national control programme on Lassa fever, decide on the framework for planning and designing the cost-effective strategies for the best intervention packages in eliminating Lassa fever in Nigeria.

- We recommend that relevant health agencies of government (such as NCDC) should bring about rapid control of Lassa fever through the results of this study as well as increasing capacity for early detection. More so, we recommend a research collaboration between mathematical epidemiologists and other medical researchers such as epidemiologists, virologists, microbiologists, etc.

The result of the sensitivity analysis shows that the most sensitive parameter is the treatment rate, followed by human recovery rate, then person to person contact. This suggests that control strategies should target effective drugs for treatment and education to reduce person to person contact.

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