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A mathematical modeling of the impact of proliferation of arms and weapons on Nigeria and control

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Abstract

The proliferation of arms and weapons is a kind of terrorism against humanity that has claimed the lives of many Nigerians, rendered many homeless, made widows and widowers, and forced the closure of companies as a result of ransom demanded for hostages by kidnappers. As a result, a detailed investigation into the cause of the growth of light weapons utilized and how it assists kidnapping activities is in great demand. In this paper, we utilize a compartmental deterministic model to investigate the dynamics of small and light weapon proliferation and kidnapping in Nigeria. A mathematical model was developed on the impact of the proliferation of arms and weapons used in kidnapping activities, as a system of nonlinear differential equations. We established the existence of solution and equilibrium state for the model and discussed its global stability. The model was analyzed numerically using the fourth-order Runge-Kutta method. Graphical results showing the effects of detention, imprisonment, and rehabilitation on kidnapping activities are given. The results show that kidnapping activities can be reduced through the control of the importation of small arms and light weapons

Keywords: proliferation; arms; weapons; terrorism; kidnapping

1 Introduction

Small arms, as defined by the ECOWAS Convention in 2006, are firearms and other destructive arms or devices such as exploding bombs, incendiary bombs or gas bombs, grenades, rocket launchers, missiles, missile systems, or landmines, revolvers, and pistols with automatic loading, rifles and carbines, machine guns, assault rifles, and light machine guns.

Light weapons, on the other hand, are portable arms designed to be used by several persons working together in a team, and which include notable heavy machine guns, portable grenade launchers, mobile or mounted portable anti-aircraft cannons, portable anti-tank cannons, non-recoil guns, portable anti-tank missile launchers, and mortars with a calibre of less than 100 millimeters.
The proliferation of arms and weapons is a kind of terrorism against humanity that has claimed the lives of many Nigerians, rendered many homeless, made widows and widowers, and forced the closure of companies as a result of ransom demanded for hostages by kidnappers. As a result, a detailed

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investigation into the cause of the growth of light weapons utilized and how it assists kidnapping activities is in great demand. In this paper, we utilize a compartmental deterministic model to investigate the dynamics of small and light weapon proliferation and kidnapping in Nigeria.

2 Literature

Many scholars have utilized differential equation mathematical models to examine the effects of many physical phenomena and systems; hence, they may be used to simulate and predict the dynamics of small arms and light weapons proliferation and kidnapping in Nigeria. This dissertation will provide material that will be useful in future studies and recommendations.

[1] suggest a brand-new mathematical model that incorporates information on the use of illegal drugs and acts of terrorism. They explore how drug usage might aid in the spread and maintenance of terrorism within a region using this model. The authors use actual data on drug use and terrorism attacks, presumably gathered from different regions or nations, to verify the model's effectiveness. The mathematical simulations offer insightful analyses into how illegal drugs might affect the dynamics of terrorism, as well as a quantitative foundation for decision-makers to create targeted actions. The results of this study shed light on the complex connection between using illegal drugs and terrorism, highlighting the necessity of addressing both problems at once. Based on the model's predictions, effective mitigation methods can be developed with the goal of reducing drug-related actions that could support terrorism. Such initiatives could help prevent the spread of terrorism, protect local populations, and advance international security. [3] emphasizes the necessity for multifaceted measures to ensure public safety, increase security, and protect societal well-being by adding to our understanding of the co-dynamics between illegal drug use and terrorism. [6] bridged the gap between theoretical modeling and real-world applications by providing a complete set of tools for small arms assessments. It improves the field of small arms research and emphasizes the significance of accuracy and precision in the design and application of firearms.

By including the idea of deradicalizing and rehabilitating kidnappers in a system of ordinary differential equations that describes the evolution and spread of abduction as a crime in human society, [5] created a straightforward mathematical model on kidnapping. It explains how kidnappers communicate with helpless people and eventually kidnap them for the primary goal of demanding ransom. We define the crime propagation number, Cpn, where a Cpn 1 ensures the asymptotically stable local and global absence of kidnappings. The combination of different levels of kidnapper recruiting and rehabilitation is simulated. According to the analysis, the best and most efficient approach to guarantee a society free from abduction is to increase the rate of rehabilitation for kidnappers.

Recently, kidnapping has developed into a lucrative business for young people in Nigeria. This criminal activity, which began as hostage taking in the Niger-Delta region of the country to draw the government's attention to the region's marginalization, appears to have evolved into a sophisticated form of armed robbery that kidnaps people illegally for ransom. Several residents, businesspeople, foreign investors, law enforcement agencies, and the government are now puzzled by the rise of this security danger. Therefore, this study examines this new wave of crime critically as well as its effects on the country. According to the study, kidnapping for ransom violates the fundamental human right to life. Security personnel must act promptly and pro-actively to stop such citizens. However, it was determined that effective governance remained the most important factor in addressing the danger of insecurity posed by abduction for ransom in Nigeria, [7].

Many people have died recently in numerous nations as a result of using illegal drugs and terrorism. This study examined the dynamics of illegal drug use and terrorism in a population using a mathematical modeling technique. The equilibrium points were established when a model was created and its fundamental attributes were investigated. The threshold (R_0) between illegal drug usage and terrorism was determined using the next generation approach. Analytically, it was demonstrated that the current equilibrium point between terrorism and illegal drug usage was asymptotically stable. This was done by building a suitable Lyapunov function. The model displays backward bifurcation, which was investigated using center manifold theory. This highlights the significant defeat in the fight against illicit drug use. even when R0 1 if a few conditions are met, use and terrorism in the populace. The contributing impacts of each parameter on the scourge of terrorism and the spread of illegal drug usage in a population were calculated using the normalized forward-sensitivity index of the variables. With the aid of MATLAB, numerical simulations were carried out in order to validate these analytical conclusions. The outcome identifies the parameters that should be strengthened and those that should be corrected, [3].

The idea that militancy and violence serve as a trigger for kidnapping in Nigeria is examined in this study. Although the study agrees that militancy can be both violent and nonviolent, kidnapping is always done violently and with force. Nigeria has seen and still experiences many different types of militancy and bloodshed, which has led to the flourishing new business of abduction for ransom. Government initiatives to address these issues have been biased, emotive, and racial in nature. The government's partiality and complacency in combating this threat have given rise to militants' aspirations for self-determination and sovereignty. Social injustice, oppressive practices, marginalization, and resource domination are examples of militancy-inspiring factors, but they don't appear to have any influence on government agenda and policy. There is mistrust amongst Nigerian citizens due to skepticism of the government's intentions in curbing the profile of militants and kidnappers, who are on the rise. Reorienting toward values, promoting good governance based on justice and the rule of law, and changing the criminal justice system are some of the recommendations we make, [2].

Although it has a long history, terrorism is frequently wrongly conceived of as a new occurrence. A History of Terrorism traces the development of political terror from nineteenth-century Europe through contemporary Arab and other groups' international operations. What will be its actual effect now and in the future? Laqueur looks closely at the sociology of terrorism, including funding, intelligence gathering, weapons and tactics, informers and countermeasures, and the crucial role played by the media's portrayal of the "terrorist personality." She also addresses long-ignored psychological issues regarding the causes of and motivations for terrorism. Systematic terrorism and contemporary conceptions of terrorism, as well as its recurring themes, motivations, and objectives, are unapologetically addressed and explicated. In the end, Laqueur assesses the effectiveness of terrorism. the terrifying potential for nuclear blackmail. This book, which was first published in 1977, is one of the two on terrorism that are frequently cited. This revised edition includes a fresh introduction and significant updates from renowned security expert Bruce Hoffman that adapt Laqueur's timeless classic to modern terrorism concerns, [4].

3 Model equations

The impact on the proliferation of arms and weapons used and kidnapping activities in the population was studied with the total population denoted by $N(t)$, at time t, which was sub-divided into seven well-defined classes of individuals that are susceptible $S(t)$ (someone who does not indulge in the use of illegal proliferation of arms and weapons but move with the users), Prisoner $P(t)$ (someone who is in correctional center due to the use or proliferation of arms and suspected kidnappers), $D(t)$ someone who is in the police cell due to the use of proliferation of illicit weapons and a suspected kidnapper, suspected Kidnapper $\mathrm{K}(t)$, arms and weapons smugglers $W_S(t)$ (someone who smuggle small and light weapons into the nation and $(R_k(t)$) educate or rehabilitate the population of arrested kidnappers (someone who is under rehabilitation due to use or proliferation of weapons and arms or art of being involved in kidnapping). Q(t) is the quitters individuals.

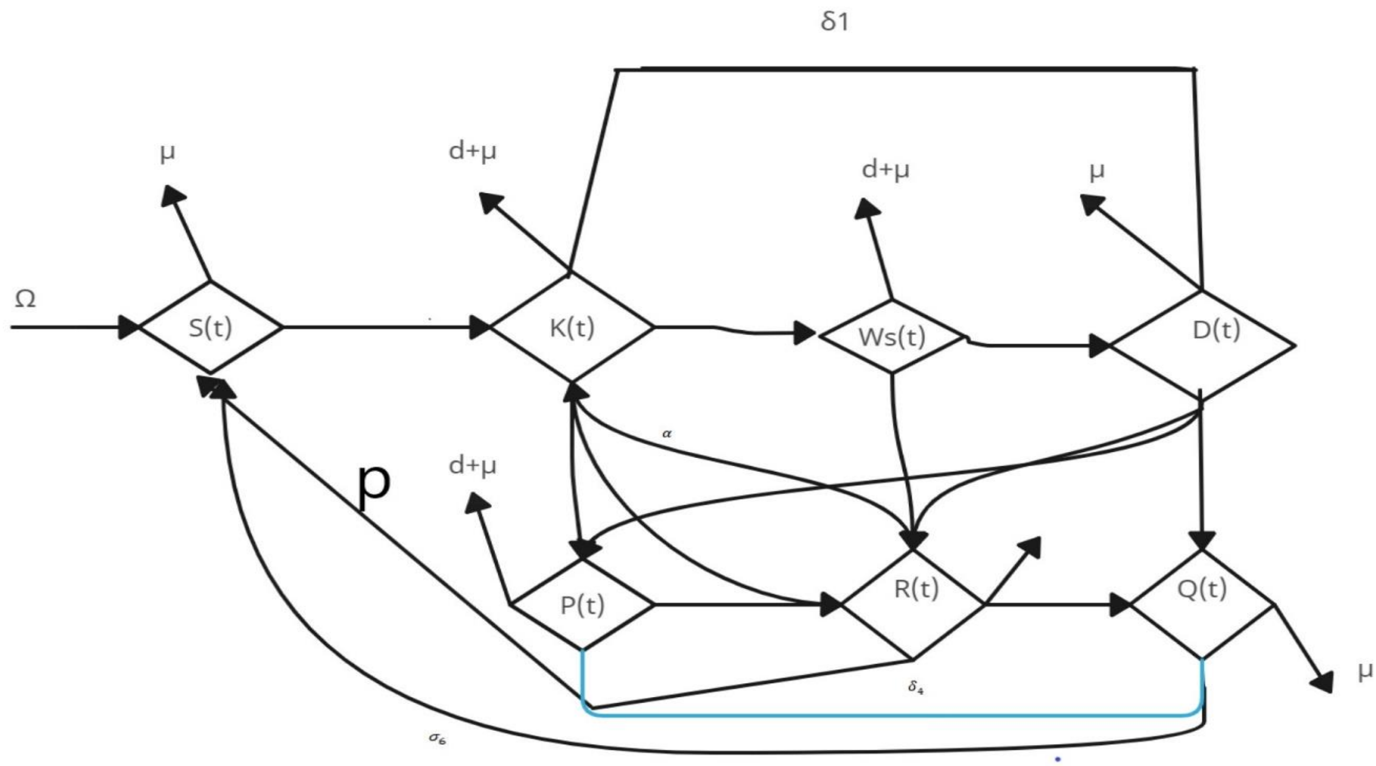

Fig. 3.1: Schematic diagram for the control of proliferation of arms and weapons used and kidnapping activities

$\frac{dS}{dt} = \Omega - \frac{\beta K(t)}{N(t)} S(t) + (\alpha + \sigma_6) D(t) + \rho R_k(t) - \mu S(t)$ (3.1)

$\frac{dW_s}{dt} = \frac{\beta(K(t))}{N(t)} S(t) + (\alpha + \sigma_5) R_k(t) - (\sigma_1 + \tau_1 + \gamma_1 + \delta_1 + \mu) W_s(t)$ (3.2)

$\frac{d\mathrm{K}}{dt} = \gamma_1 W_s(t) - (\sigma_2 + \tau_2 + \gamma_2 + \delta_2 + \mu)\mathrm{K}(t)$ (3.3)

$\frac{dD}{dt} = \sigma_1 W_s(t) + \sigma_2 \mathrm{K}(t) - (\sigma_3 + \gamma_3 + \tau_3 + \mu) D(t)$ (3.4)

$\frac{dP}{dt} = \gamma_2 \mathrm{K}(t) + \gamma_3 D(t) - (\sigma_4 + \tau_4 + \delta_4 + \mu) P(t)$ (3. 5)

$\frac{dR_k}{dt} \tau_1 W_s(t) + \tau_2 \mathrm{K}(t) + \tau_3 D(t) + \tau_4 P(t) - (\sigma_5 + \alpha + \rho + \mu) R_k(t)$ (3. 6)

$\frac{dQ}{dt} = \sigma_3 D - \sigma_6 Q - \mu Q$ (3.7)

with initial conditions at $t = 0.1$:

$S(0) = S_0, \mathrm{K}(0) = K_0, W_s(0) = W_{s_0}, D(0) = D_0, P(0) = P_0, R_k(0) = R_{k_0}$ (3.8)

The total population is obtained by adding all the equations (3.1) to (3.) to resulted to:

$\frac{dN}{dt} \leq \Omega - \mu N$ (3.9)

Table 3.2: Parameters.

Variables	Descriptions
S(t)	Susceptible: someone who does not indulge in the use of illegal proliferation of small arms and light weapons but move with the users
P(t)	Prisoner: someone who is in correctional center due to the use or proliferation of arms and suspected kidnappers
D(t)	Detention: someone who is in the police cell due to the use of proliferation of illicit weapons and a suspected kidnapper
K(t)	A kidnapper: someone who uses smuggled arms for illegal activities.
$W_s(t)$	someone who smuggle small and light weapons into the nation
$R_k(t)$	Rehabilitation and education: someone who is under rehabilitation or being educate due to use or proliferation of light weapons and small arms or art of being involved in kidnapping.
Q(t)	Quitters at time t
Ω	Recruitment rate into the susceptible population
β	Effective influence rate
α	Movement of rehabilitation individuals to smugglers of arms and weapons or to users
ρ	Movement of rehabilitation individuals to susceptible population
σ_1	Detention rate of kidnappers
σ_2	Detention rate of kidnappers
σ_3	Quitting rate of detained individuals in police custody
σ_4	Quitting rate of individuals in the prison rehabilitation and education of members of public
γ_1	Progression rate of illegal arms and weapons users into kidnaping.
γ_2	Movement of Kidnappers and arms and weapons smugglers into prison

γ_3	Exponential rate of movement those in police custody to prison
μ	Natural rate
τ_1	Education and rehabilitation of Kidnappers and the population respectively
δ_1	Death rate of the smugglers of arms and weapons
δ_4	Death rate of those that have been sentence to prison
δ_2	Kidnappers and arms/weapon smugglers death rate
τ_2	kidnappers and smugglers of arms/weapon rate of rehabilitation
τ_3	Rehabilitation rate of detained individual
τ_4	Education rate of those in the correctional centers
δ_4	death rate of smugglers
σ_5	Treatment rate of those in the rehabilitation centers
σ_6	Movement rate of quitter to susceptible

3.1 Basic properties

Given that all parameters are non-negativity and that the defined model (3.1) monitors the human population, it is important to demonstrate the non-negativity of the state variables for all periods.

3.1.1. Positivity solutions

Theorem 1. The state variables, $S(t), W_s(t), \mathrm{K}(t), D(t)$, $P(t), R_k(t)$ and $Q(t)$, of the proliferation of arms and weapons use and Kidnappers model (1), with the non-negative initial condition, remain positive for all $t > 0$. Proof. From the first equation (3.1)

$$\frac{dS}{dt} \geq -(\lambda + \mu)S(t) \tag{3.10}$$

it follows that,

$$\frac{d}{dt}\left(S(t)\exp\left(\mu t + \int_0^t \lambda(\Psi)d\Psi\right)\right) \geq 0\ . \tag{3.11}$$

Then

$$S(t) \geq S(0)\exp\left(-\left(\mu t + \int_0^t \lambda(\Psi)d\Psi\right)\right) > 0, \forall t > 0. \tag{3.12}$$

The other state variables $W_S(t), \mathrm{K}(t), D(t), P(t), R_K(t)$ and $Q(t)$, will be non-negativity for all $t > 0$ when the same approach been used for $S(t)$ is employed.

3.1.2. Invariant region

The biologically feasible region, define by $\mathrm{F} \subset \mathbb{R}_+^7$ was considered, such that $\mathrm{F} = \left\{(S, W_S, \mathrm{K}, D, P, R_K, Q) \in \mathbb{R}_+^7 : N \leq \frac{\Omega}{\mu}\right\}$. It was proved that F is positively invariant in that region.

Theorem 2. The region F is positively invariant with respect to the model (3.1)

Proof. Rate of change of the total population of model (3.1) is given by (3.3) resulting into the solution $N(t) = N(0)\exp(-\mu t) + \frac{\Omega}{N}(1 - \exp(-\mu t))$. Following that $N(t) \longrightarrow \frac{\Omega}{\mu}$ as $\mathrm{t} \longrightarrow \infty$ in particular, $\mathrm{N(t)} \leqslant \frac{\Omega}{\mu}$ if $N(0) \leqslant \frac{\Omega}{\mu}$ with respect to the impact on the proliferation of arms and weapons used and kidnapping activities model (3.1). Hence, it suffices to consider the dynamics of the model in F. In this region, the impact on the proliferation of arms and weapons used and kidnapping activities model can be considered as being mathematically and biologically well-posed.

3.2 Existence and stability analysis of equilibria

This part establishes the model's steady-state solution's existence and examines the type of bifurcation the model displays.

3.2.1 Kidnapper-free equilibrium

Population without arms and weapons users or kidnappers is defined as a proliferation of arms and weapons-use and kidnapping-free population. After that, there will be a growth of light weapons use and an equilibrium without kidnapping $W_s = \mathrm{K} = 0$. Solving model (3.1) gives,

$$S = \frac{\Omega}{\mu}, D = 0, C = 0, R = 0. \tag{3.13}$$

Therefore, the existence of the impact on the proliferation of arms and weapons used and kidnapping activities free equilibrium is given by $\mathcal{D}_0$

$$\mathcal{D}_0 = \left(S_0, W_{s_0}, K_0, D_0, P_0, R_{K_0}, Q\right) = \left(\frac{\Omega}{\mu}, 0,0,0,0,0,0\right) \tag{3.14}$$

3.2.2 Kidnapper-present equilibrium

The population's current presence of people using illegal weapons and kidnapping is referred to as the proliferation of small arms and light weapon use. The steady-state solution of model (3.1) is the point where all the state variables are positive, indicated by D* and defined as the proliferation of small arms and light weapon usage and kidnapper-present equilibrium point.

$$\mathcal{D}^* = (S^*, W_S^*, K^*, D^*, P^*, R_K^*, Q^*). \tag{3.15}$$

Then, setting the right-hand sides of (3.1) to zero, the following are obtained in terms of λ:

$$S_\lambda^* = \frac{\Omega b_1 k_6}{k_6(\lambda+\mu)b_1 + \sigma_6 \lambda b_2} \tag{3.16}$$

$$K_\lambda^* = \frac{\Omega k_2 k_3 k_4 k_5 k_6 \lambda}{k_6(\lambda+\mu)b_1 + \sigma_6 \lambda b_2} \tag{3.17}$$

$$W_{S\lambda}^* = \frac{\gamma_1 \mathrm{A} k_3 k_4 k_5 k_6 \lambda}{k_6(\lambda+\mu)b_1 + \sigma_6 \lambda b_2} \tag{3.18}$$

$$D_\lambda^* = \frac{B\Omega k_4 k_5 k_6 \lambda}{k_6(\lambda+\mu)b_1 + \sigma_6 \lambda b_2} \tag{3.19}$$

$$P_\lambda^* = \frac{M\Omega k_5 k_6 \lambda}{k_6(\lambda+\mu)b_1 + \sigma_6 \lambda b_2} \tag{3.20}$$

$$R_{k\lambda}^* = \frac{G\Omega k_6 \lambda}{k_6(\lambda+\mu)b_1 + \sigma_6 \lambda b_2} \tag{3.21}$$

$$Q_\lambda^* = \frac{b_3 \Omega \lambda}{k_6(\lambda+\mu)b_1 + \sigma_6 \lambda b_2} \tag{3.22}$$

where

$$A = \tau_1 k_2 + \tau_2 \gamma_1, \tag{3.23}$$

$$B = \sigma_1 k_2 + \sigma_2 \gamma_1 \tag{3.24}$$

$$F = \gamma_1 \gamma_2 k_3 + \gamma_1 B, \tag{3.25}$$

$G = Ak_3k_4 + Bk_4\tau_3 + \tau_4 F,$ (3.26)
$H = \sigma_3 Bk_4k_5 + \sigma_4 k_5 F,$ (3.27)
$M = \gamma_1\gamma_2k_2 + \gamma_3\sigma_1k_2 + \gamma_1\gamma_3\sigma_2$ (3.28)
$b_1 = k_1k_2k_3k_4k_5 - \alpha G,$ (3.29)
$b_2 = H + \sigma_5 G$ (3.30)
$b_3 = \sigma_3 Bk_4k_5 + \sigma_4 k5F + \sigma_5 G.$ (3.31)
$\lambda^* = \frac{\beta(I^* + \eta S_E^*)}{N^*}$ (3.32)

and

$$N^* = S^* + W_S^* + S_E^* + D^* + C^* + R_K^* + Q^*. \quad (3.33)$$

Substituting gives the following

$$\lambda = 0 \text{ or } \lambda = \mu(\mathcal{R}_\mathrm{n} - 1). \quad (3.34)$$

When $\lambda = 0$ from system means proliferation of small arms and light weapons use and Kidnapping-present equilibrium does not exist, but we have proliferation of small arms and light weapons use and Kidnapping-free equilibrium, and when $\lambda = \mu(\mathcal{R}_\mathrm{n} - 1)$, we have a unique proliferation of small arms and light weapons use and Kidnapping-present equilibrium if $\mathcal{R}_\mathrm{n} > 1$. The following unique proliferation of small arms and light weapons use and Kidnapping-present equilibrium $\mathcal{D}^*$ are obtained in terms of $\mathcal{R}_\mathrm{n}$ as follows

$S^* = \frac{\Omega k_6}{k_6\mu(\mathcal{R}_\mathrm{n} - 1)}$ (3.35)
$K^* = \frac{\Omega\beta(k_3k_4k_5)^2(\eta\gamma_1 + k_2)}{\mathcal{R}_\mathrm{n}}$ (3.36)
$W_s^* = \frac{\gamma_1\Omega\beta(k_3k_4k_5)^2(\eta\gamma_1 + k_2)}{\mathcal{R}_\mathrm{n}}$ (3.37)
$D^* = \frac{\Omega\beta k_3(k_4k_5)^2(\eta\gamma_1 + k_2)}{\mathcal{R}_\mathrm{n}}$ (3.38)
$P^* = \frac{M\Omega\beta k_3k_4k_5^2(\eta\gamma_1 + k_2)}{\mathcal{R}_\mathrm{n}}$ (3.39)
$R_k^* = \frac{G\Omega\beta k_3k_4k_5(\eta\gamma_1 + k_2)}{\mathcal{R}_\mathrm{n}}$ (3.40)
$Q^* = \frac{b_3\Omega\beta k_3k_4k_5(\eta\gamma_1 + k_2)}{k_6\mathcal{R}_\mathrm{n}}$ (3.41)

3.5. Global stability

Theorem 5. The global stability of proliferation of small arms and light weapons use and kidnapping model is globally asymptotically stable in

$\Gamma \setminus \Gamma_0$ if $\mathcal{R}_0 > 1$, $R_\mathrm{k}K^{**} \leq R_\mathrm{k}{}^{**}K, W_s R_\mathrm{k}{}^{**} \leq W_s^{**}R_\mathrm{k}$ and $KW_s^{**} \leq K^{**}W_s$ (3.42)

Proof. Consider the non-linear Lyapunov function $F\colon \Gamma\ \Gamma_0 \to \mathbb{R}$ defined by

$$F= S-S^{**} - S^{**}\ln\frac{S}{S^{**}} + \left(K - K^{**} - K^{**}\ln\frac{K}{K^{**}}\right) + \frac{\alpha\tau_3(\gamma_3+k_4)}{k_3k_4k_5}\left(D - D^{**} - D^{**}\ln\frac{D}{D^{**}}\right) + \frac{\tilde{\beta}S(I+\eta S_E)k_3k_4k_5+p_5}{k_2k_3k_4k_5}\left(W_s - W_s^{**} - W_s^{*}\ln\frac{W_S}{W_S^{**}}\right) + \frac{\alpha\tau_3}{k_4k_5}\left(P - P^{**} - P^{**}\ln\frac{P}{P^{**}}\right)\frac{\alpha}{k_5}\left(R_k - {R_k}^{**} - {R_k}^{**}\ln\frac{R_k}{{R_k}^{**}}\right) \tag{3.40}$$

Let $p_5 = \alpha(\tau_3(\sigma_2(\gamma_3 + k_4) + \gamma_2 k_3) + k_3k_4\tau_2)$. Equation (28) is of Goh-Volterra type. The Lyapunov derivative is given by

$$\frac{dF}{dt} = \frac{dS}{dt} - \frac{S^{**}}{S}\frac{dS}{dt} + \left(\frac{dP}{dt} - \frac{P^{**}}{P}\frac{dP}{dt}\right) + \frac{\alpha\tau_3(\gamma_3+k_4)}{k_3k_4k_5}\left(\frac{dD}{dt} - \frac{D^{**}}{D}\frac{dD}{dt}\right) + \frac{\alpha}{k_5}\left(\frac{dR_k}{dt} - \frac{{R_k}^{**}}{R_k}\frac{dR_k}{dt}\right) + \frac{\alpha\tau_3}{k_4k_5}\left(\frac{dP}{dt} - \frac{P^{**}}{P}\frac{dP}{dt}\right) + \frac{\tilde{\beta}S(P+\eta W_S)k_3k_4k_5+p_5}{k_2k_3k_4k_5}\left(\frac{dW_S}{dt} - \frac{W_S^{**}}{W_S}\frac{dW_S}{dt}\right) \tag{3.43}$$

$$\text{Let } \frac{\beta(I(t)+\eta W_S(t))}{N(t)} = \tilde{\beta}(P(t) + \eta W_S(t)), \tag{3.44}$$

then putting the appropriate equations of the system (3.1) into (3.42), one obtains

$$\frac{dF}{dt} = \left(1 - \frac{S^{**}}{S}\right)(\Omega - \tilde{\beta}\left(K(t) + \eta W_S(t)\right)S(t) + \rho R_k(t) + \left(1 - \frac{K^{**}}{K}\right) - \mu S(t)$$
$$\left(\tilde{\beta}(P(t) + \eta W_S(t))S(t) + \alpha R_k(t) - k_1 K(t)\right) + \frac{\tilde{\beta}S(K+\eta W_S)k_3k_4k_5+p_5}{k_2k_3k_4k_5}$$
$$\left(1 - \frac{W_S^{**}}{W_S}\right)\left(\gamma_1 K(t) - k_2 W_S(t)\right) + \frac{\alpha\tau_3(\gamma_3 + k_4)}{k_3k_4k_5}\left(1 - \frac{D^{**}}{D}\right)$$
$$\left.\left(\sigma_1 K(t) + \sigma_2 W_S(t) - k_3 D(t)\right) + \frac{\alpha\tau_3}{k_4k_5}\left(1 - \frac{P^{**}}{P}\right)(\gamma_2 W_S(t) + \gamma_3 D(t) - k_4)P(t)\right)$$
$$+\frac{\alpha}{k_5}\left(1 - \frac{{R_k}^{**}}{R_k}\right)(\tau_1 K(t) + \tau_2 W_S(t) + \tau_3 D(t) + \tau_4 P(t) - k_5 R_k(t)) \tag{3.45}$$

At the proliferation of small arms and light weapons use and kidnapping-present equilibrium, the following relations hold from the system

$$\Omega = \tilde{\beta}(K^{**} + \eta W_S^{**})S^{**} - \rho {R_k}^{**} + \mu S^{**} \tag{3.46}$$
$$k_1 = \frac{\tilde{\beta}\left(K^{**}+\eta W_S^{**}\right)}{K^{**}} + \frac{\alpha_1 {R_k}^{**}}{K^{**}} \tag{3.47}$$
$$k_2 = \frac{\gamma_1 K^{**}}{W_S^{**}} \tag{3.48}$$
$$k_3 = \frac{\sigma_1 K^{**}+\sigma_2 W_S^{**}}{D^{**}} \tag{3.49}$$
$$k_4 = \frac{\gamma_2 W_S^{**}+\gamma_3 D^{**}}{P^{**}} \tag{3.50}$$
$$k_5 = \frac{\tau_1 K^{**}+\tau_2 W_S^{**}+\tau_3 D^{**}+\tau_4 P^{**}}{{R_k}^{**}} \tag{3.51}$$

Using the relations (3.21) in (3.20) and simplifying yields

$$\frac{dF}{dt} = \tilde{\beta}S^{**}(K^{**} + \eta W_S^{**})\left(2 - \frac{S^{**}}{S} - \frac{S}{S^{**}}\right) + \mu S^{**}\left(2 - \frac{S}{S^{**}} - \frac{S^{**}}{S}\right) + \alpha\rho {R_k}^{**}\left(1 - \frac{R_k K^{**}}{R^{**}K}\right) + \frac{\sigma_2\alpha\tau_3(\gamma_3+k_4)}{k_3k_4k_5}W_S^{**}\left(2 - \frac{KD^{**}}{K^{**}D} - \frac{P^{**}D}{PD^{**}}\right) + \frac{\alpha\tau_3\gamma_2}{k_4k_5}W_S^{**}\left(2 - \frac{W_S P^{**}}{W_S^{**}P} - \frac{D^{**}W_S}{DW_S^{**}}\right) + \frac{\alpha\tau_2}{k_5}W_S^{**}\left(1 - \frac{W_S {R_k}^{**}}{W_S^{**}R_k}\right) + k_1K^{**}\left(1 - \frac{KW_S^{**}}{K^{**}W_S}\right) + \frac{\alpha\tau_3}{k_5}P^{**}\left(2 - \frac{P{R_k}^{**}}{P^{**}R_k} - \frac{PD^{**}}{P^{**}D}\right) \tag{3.52}$$

Since $R_kK^{**} \le {R_k}^{**}K, W_S{R_k}^{**} \le W_S^{**}R_k$ and $KW_S^{**} \le K^{**}W_S$, then $1 - \frac{{R_k}^{**}K}{R_kK^{**}} \le 0, 1 - \frac{W_S^{**}R_k}{W_S{R_k}^{**}} \le 0$ and $1 - \frac{{R_k}^{**}W_S}{KW_S^{**}} \le 0$ with equality if $R_kK^{**} = {R_k}^{**}K, W_S{R_k}^{**} = W_S^{**}R_k$ and $KW_S^{**} = K^{**}W_S$

Consequently, (3.2) becomes

$$\frac{dF}{dt} = \tilde{\beta}S^{**}(K^{**} + \eta W_S^{**})\left(2 - \frac{S^{**}}{S} - \frac{S}{S^{**}}\right) + \mu S^{**}\left(2 - \frac{S}{S^{**}} - \frac{S^{**}}{S}\right) + \frac{\sigma_2\alpha\tau_3(\gamma_3+k_4)}{k_3k_4k_5}W_S^{**}\left(2 - \frac{KD^{**}}{K^{**}D} - \frac{P^{**}D}{PD^{**}}\right) + \frac{\alpha\tau_3\gamma_2}{k_4k_5}W_S^{**}\left(2 - \frac{W_S P^{**}}{W_S^{**}P} - \frac{D^{**}W_S}{DW_S^{**}}\right) + \frac{\alpha\tau_3}{k_5}P^{**}\left(2 - \frac{P{R_k}^{**}}{P^{**}R_k} - \frac{PD^{**}}{P^{**}D}\right). \tag{3.53}$$

3.3 Numerical solution of the model

We now employ the Runge-Kutta Method of Order Four to get our numerical solutions. The Runge-Kutta Method of Order Four is defined as follows:

$$w_0 = a \tag{3.54}$$

$$W_{i+1} = W_i + \frac{1}{6}(F_1 + 2F_2 + 2F_3 + F_1) \tag{3.55}$$

Where

$$F_1 = hf(t_{i,}, w_i) \tag{3.56}$$

$$F_2 = hf\left(t_{i+}\frac{h}{2}, w_{i+}\frac{F_1}{2}\right) \tag{3.57}$$

$$F_3 = hf\left(t_{i+}\frac{h}{2}, w_{i+}\frac{F_2}{2}\right) \tag{3.58}$$

$$F_4 = hf\left(t_{i+}\frac{h}{2}, w_{i+}F_3\right) \tag{3.59}$$

We have F1; F2; F3; and F4 for each population assessed in the model for our $S_0K_0, W_s, D\ P, R_k, Q$ model using this way. In general, the Runge-Kutta Method of Order Four has the following algorithm:

INPUT: endpoints t_{min}, t_{max}; integer N; initial condition a

OUTPUT: approximations w to y at the (N + 1) values of t

Step 1: set

$$h = \frac{\text{tmin} - \text{tmax};}{N} \tag{3.60}$$

$$t_0 = t_{min}, w_0 = a \tag{3.61}$$

Step 2: for I = 1, 2, …, N do and repeat step 3 and step 4:

Step 3: Set

$F_1 = hf\ (t_i, w_i)$ (3.62)

$F_2 = hf\ (t_{i+}\frac{h}{2}, w_{i\ +}\ \frac{F_1}{2})$ (3.63)

$F_3 = hf\ (t_{i+}\frac{h}{2}, w_{i\ +}\ \frac{F_2}{2})$ (3.64)

$F_4 = hf\ (t_{i+}\frac{h}{2}, w_{i\ +}\ F_3)$ (3.65)

Step 4: OUTPUT (ti; w_i).

Step 5: STOP.

Thus, for our SEIQR Model, we have:

Step 1: For the systems of equations set the initial conditions and the parameters: α, β, σ, μ, S(0) = S0, W_s (0) = W_s0, K(0) = K0, D(0) = D0, R_k (0) = R_k0, P(0) = P0, Q(0) = Q0

Step 2: For i = 1; 2…, N do and repeat Steps 3 and 4.

Step 3: Set

$t_i = t_{min} + ih$ (3.66)

With

$FS_1 = hf1(S_{i-1}; K_{i-1})$ (3.67)

$Fw_{s1} = hf2(Si\text{-}1; W_si\text{-}1; Ki\text{-}1)$ (3.68)

$FK_1 = hf3(W_si\text{-}1; Ki\text{-}1)$ (3.69)

$FD_1 = hf4(Ki\text{-}1; Qi\text{-}1)$ (3.70)

$FR_{S1} = hf5(Qi\text{-}1)$

FP1 = hf7(Si-1); with f_i defined as in Euler's Method, and

$F_{S2} = h\ f_1\left(\mathrm{Si} - 1 + \frac{1}{2}\ \mathrm{FS1}, \mathrm{Ki} - 1\ + \frac{1}{2}\ \mathrm{FK1}\right)$ (3.71)

$F_{W_s2} = h\ f_2\left(\mathrm{Si} - 1 + \frac{1}{2}\ \mathrm{FS1}, W_s\mathrm{i} - 1\ + \frac{1}{2}\ \mathrm{F}W_s1\right)$ (3.72)

$F_{K2} = h\ f_3\left(W_s\mathrm{i} - 1 + \frac{1}{2}\ \mathrm{F}W_s1, \mathrm{Ki} - 1\ + \frac{1}{2}\ \mathrm{FK1}\right)$ (3.73)

$F_{D2} = h\ f_4\left(\mathrm{Ki} - 1 + \frac{1}{2}\ \mathrm{FK1}, \mathrm{Di} - 1\ + \frac{1}{2}\ \mathrm{FD1}\right)$ (3.74)

$F_{P2} = h\ f5\left(\mathrm{Di} - 1 + \frac{1}{2}\ \mathrm{FD1},\right)$ (3.75)

$F_{R_K2} = h\ f6\left(S\mathrm{i} - 1 + \frac{1}{2}\ \mathrm{FS1}\right)$ (3.76)

And

$F_{S3} = h\ f_1\left(\mathrm{Si} - 1 + \frac{1}{2}\ \mathrm{FS2}, \mathrm{Ki} - 1\ + \frac{1}{2}\ \mathrm{FK2}\right)$ (3.77)

$F_{W_s3} = h\ f_2\left(\mathrm{Si} - 1 + \frac{1}{2}\ \mathrm{FS2}, W_s\mathrm{i} - 1\ + \frac{1}{2}\ \mathrm{F}W_s2\right)$ (3.78)

$F_{K3} = h\ f_3\left(W_s\mathrm{i} - 1 + \frac{1}{2}\ \mathrm{F}W_s2, \mathrm{Ki} - 1\ + \frac{1}{2}\ \mathrm{FK2}\right)$ (3.79)

$F_{D3} = h\ f_4\left(\mathrm{Ki} - 1 + \frac{1}{2}\ \mathrm{FK2}, \mathrm{Di} - 1\ + \frac{1}{2}\ \mathrm{FD2}\right)$ (3.80)

$F_{P3} = h\ f5\left(\mathrm{Di} - 1 + \frac{1}{2}\ \mathrm{FD2},\right)$ (3.81)

$F_{R_K3} = h\ f6\left(S\mathrm{i} - 1 + \frac{1}{2}\ \mathrm{FS2}\right)$ (3.82)

And finally,

F_{S4} = h f₁$\left(\mathrm{Si}-1+\frac{1}{2}\ \mathrm{FS3}, \mathrm{Ki}-1\ +\frac{1}{2}\ \mathrm{FK3}\right)$ (3.83)

$F_{W_s 4}$ = h $f_2\left(\mathrm{Si}-1+\frac{1}{2}\ \mathrm{FS3}, W_s\mathrm{i}-1\ +\frac{1}{2}\ \mathrm{F}W_s 3\right)$ (3.84)

F_{K4}=h$f_3\left(W_s\mathrm{i}-1+\frac{1}{2}\ \mathrm{F}W_s 3, \mathrm{Ki}-1\ +\frac{1}{2}\ \mathrm{FK3}\right)$ (3.85)

F_{D4} = h $f_4\left(\mathrm{Ki}-1+\frac{1}{2}\ \mathrm{FK3}, \mathrm{Di}-1\ +\frac{1}{2}\ \mathrm{FD3}\right)$ (3.86)

F_{P4} = h f5$\left(\mathrm{Di}-1+\frac{1}{2}\ \mathrm{FD3},\right)$ (3.87)

$F_{R_K 4}$ = h f6$\left(S\mathrm{i}-1+\frac{1}{2}\ \mathrm{FS3}\right)$ (3.88)

And together,

$S_i = S_{i-1}+\frac{1}{6}(\mathrm{FS1}\ +2\mathrm{FS2}+2\mathrm{FS3}+\mathrm{FS4})$ (3.89)

$W_{s_i} = W_{s_{i-1}}+\frac{1}{6}(\mathrm{F}W_s 1\ +2\mathrm{F}W_s 2+2\mathrm{F}W_s 3+\mathrm{F}W_s 4)$ (3.90)

$K_i = K_{i-1}+\frac{1}{6}(\mathrm{FK1}\ +2\mathrm{FK2}+2\mathrm{FK3}+\mathrm{FK4})$ (3.91)

$D_i = D_{i-1}+\frac{1}{6}(\mathrm{FD1}\ +2\mathrm{FD2}+2\mathrm{FD3}+\mathrm{FD4})$ (3.92)

$P_i = P_{i-1}+\frac{1}{6}(\mathrm{FP1}\ +2\mathrm{FP2}+2\mathrm{FP3}+\mathrm{FP4})$ (3.93)

$R_{k_i} = R_{k_{i-1}}+\frac{1}{6}(\mathrm{F}R_k 1\ +2\mathrm{F}R_k 2+2\mathrm{F}R_k 3+\mathrm{F}R_k 4)$ (3.94)

Step 4: OUTPUT ($t_i, S_i, W_{s_{i,}} K_i, D_i, P_i, R_{k_i}$)

Step 5: STOP.

3.4 Application of Runge-Kutta fourth order method for $\boldsymbol{S_0 K_0 W_s D\ P R_k\ Q}$ model

We develop a framework to estimate the fractional temporal proliferation of weapons usage in the Nigerian population. In this model, a steady population is separated into distinct groups based on the beginning conditions.

$S(0)0=1, W_s(0)=0, K(0)=1, D(0)=0, P(0)=0,\ R_k(0)=0\ \&\ \mathrm{Q}(0)=0$ (3.95)

The numerical solution of SEIQR (ODEs) can be determined by Runge-Kutta fourth order method. In Table 3.1 and 3.2, we establish the parameters of model (3.1), the table below display the corresponding values of the parameters.

Table 4.1: Parameters values and source.

Parameter	Range	Baseline value
Ω	$70-100$	90
τ_1	$0.7-0.95$	0.85
μ	$0.009-0.04$	0.007
τ_3	$0.7-0.95$	0.85
σ_1	$0.7-1.0$	0.95
σ_3	$0.6-0.9$	0.7
σ_4	$0.4-0.9$	0.2

γ_2	$0.5 - 0.8$	0.65
γ_3	$0.6 - 0.9$	0.7
δ_1	$9 - 14$	0.14
δ_2	$0.09 - 0.2$	0.14
δ_3	$0.08 - 0.2$	0.14
ρ	$0.06 - 0.32$	0.77

In Table 3.2 we derive the values of the parameters base on the facts presented from certain scholars who had worked on issues related to kidnapping, banditry and terrorism.

Table 4.2: Parameters values and source.

Parameters	Range	Values
σ_2	$0.5 - 0.8$	0.6
τ_2	$0.05 - 0.2$	0.1
β	$0.002 - 0.005$	0.64
τ_4	$0.05 - 0.2$	0.1
α	$0.6 - 0.9$	0.9
σ_5	$0.4 - 0.9$	0.55
σ_6	$0.4 - 0.9$	0.55
γ_1	$0.5 - 0.8$	0.6

Put $\Omega = 100, \beta = 0.5, \sigma = 0.75, \mu = 0.04, \alpha = 0.6, \eta = 0.6, \sigma_1 = 0.5, \sigma_2 = 0.5, \sigma_3 = 0.7, \sigma_4 = 0.1, \sigma_5 = 0.4, \Upsilon_1 = 0.5, \Upsilon_2 = 0.9, \Upsilon_3 = 0.5, \delta_1 = 0.2, \delta_2 = 0.2, \delta_3 = 0.2, \delta_4 = 0.2, \delta_6 = 0.2, \mathrm{P} = 0.8, \tau_1 = 0.2, \tau_2 = 0.02, \tau_3 = 0.4, \tau_4 = 0.04$.

$$\frac{dS}{dt} = 30 - 0.25S + 0.4Q + 0.8R_k - 0.04S \quad (3.96)$$

$$\frac{dW_s}{dt} = 0.25S + 0.6R_k - 1.26W_s \quad (3.97)$$

$$\frac{d\mathrm{K}}{dt} = 0.5W_s - 1.26\mathrm{K} \quad (3.98)$$

$$\frac{dD}{dt} = 0.5W_s + 0.5\mathrm{K} - 1.9D \quad (3.99)$$

$$\frac{dP}{dt} = 0.5\mathrm{K} + 0.5D - 0.38P \quad (3.100)$$

$$\frac{dR_k}{dt} = 0.2W_s + 0.02\mathrm{K} + 0.4D + \mathrm{o.o}4P - 3.9R_k \quad (3.101)$$

$$S(0) = 1, W_s(0) = 0, K(0) = 1\ D(0) = 0\ P(0) = 0 \text{ And } R_k(0) = 0 \quad (3.102)$$

By using MATLAB, we have generated the numerical solution of equation (3.3) using the Runge-Kutta of order 4.

Discussion and Results

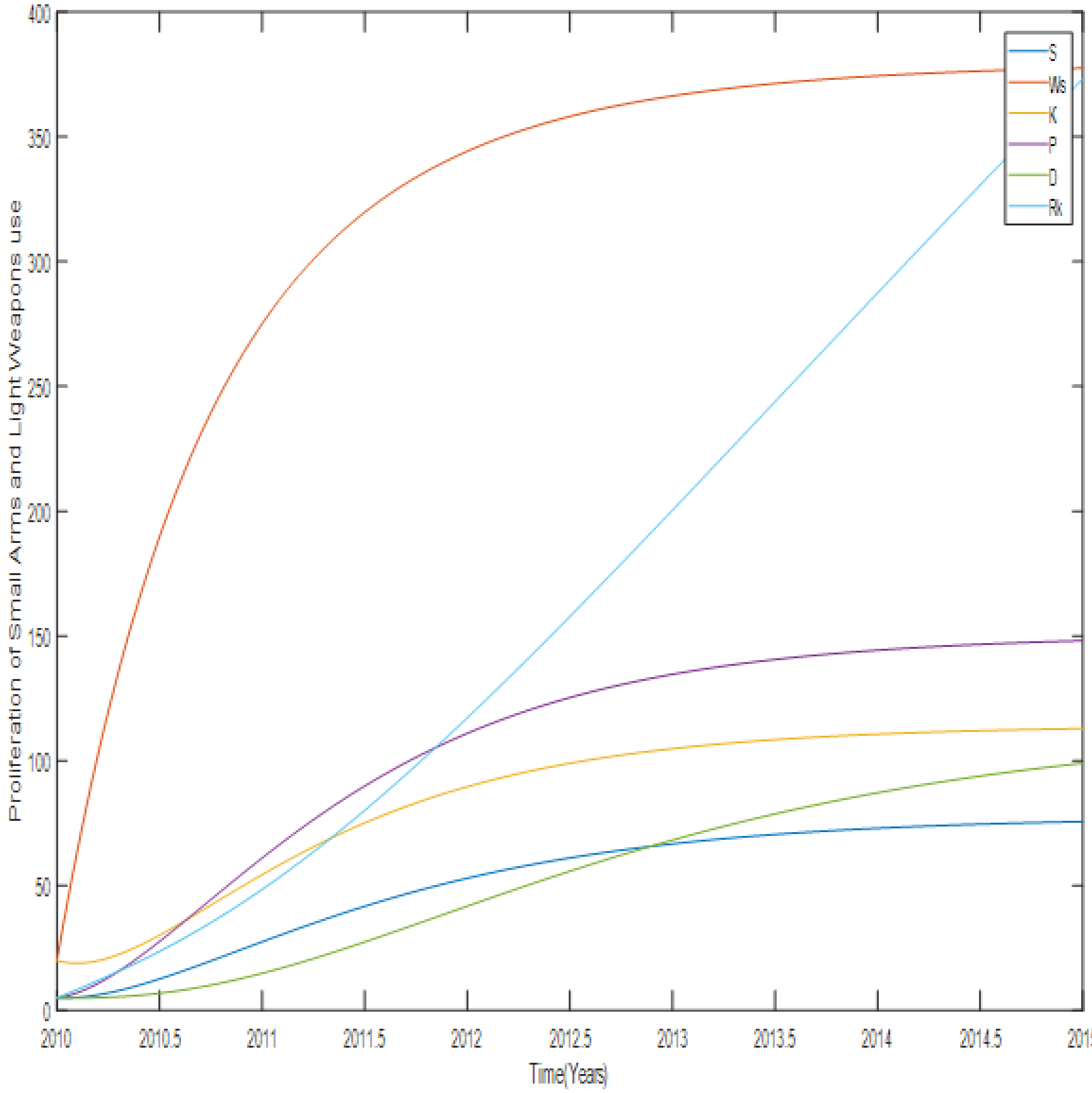

Figure 3.1: The state arms and weapons inflows between 2010 and 2013.

Figure 3.1 show the initial situation of the proliferation of small arms and light weapons as at 2010, 2011, 2012 and 2013. We observed an exponential inflow of weapons, susceptible, imprisonment, Kidnapping and detention of the state actors.

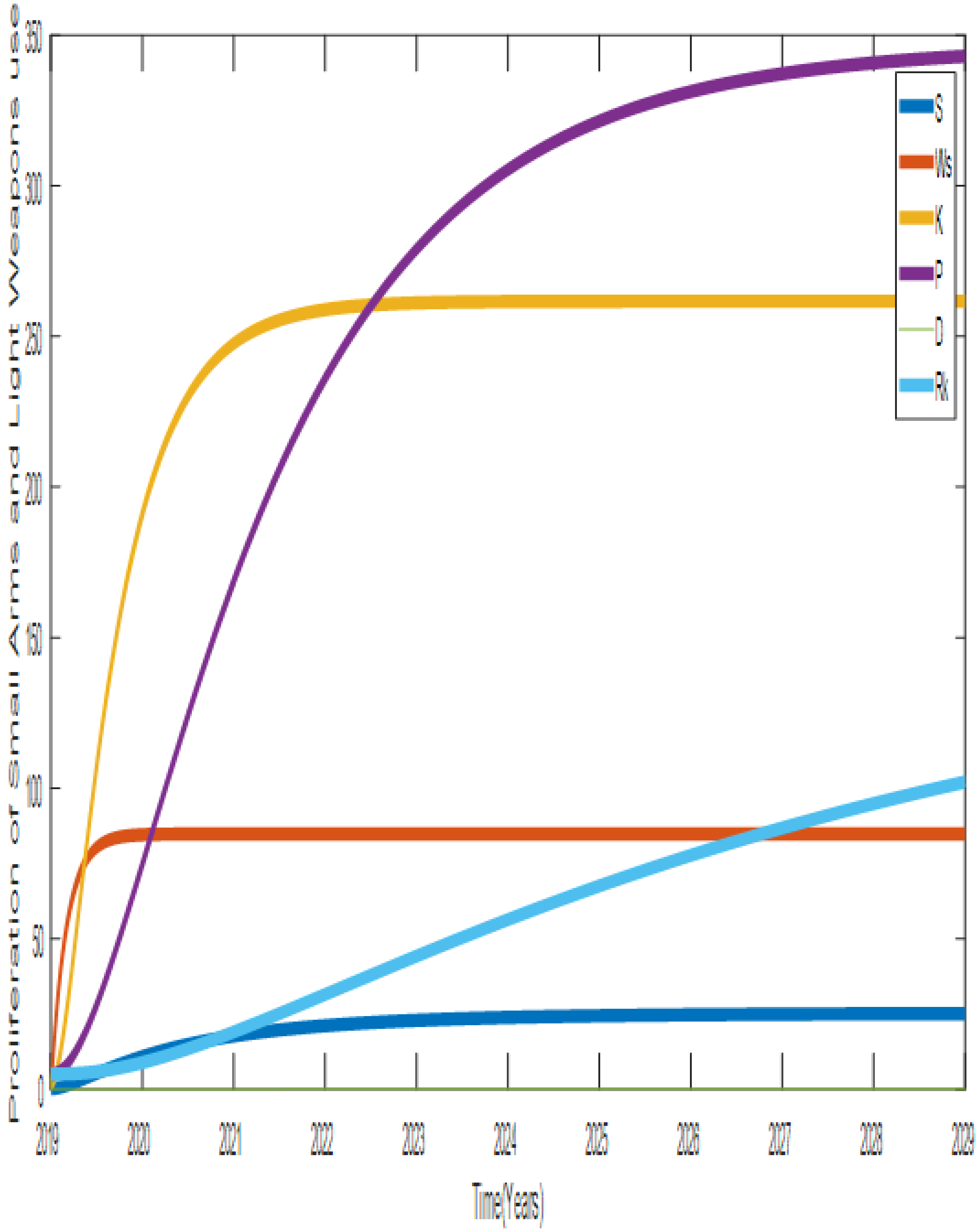

Figure 3.2: The state arms and weapons inflows between 2019 and 2022.

Figure 3.2 shows that in 2019, 2020, 2021 and late 2022, the threshold of smuggling of arms, kidnapping activities and rate of imprisonment increases respectively. The control parameter i.e., rehabilitation of inmate (sentence to prison as a result of their involvement on illegal activities of weapons smuggling and kidnapping) and education of the susceptible population.

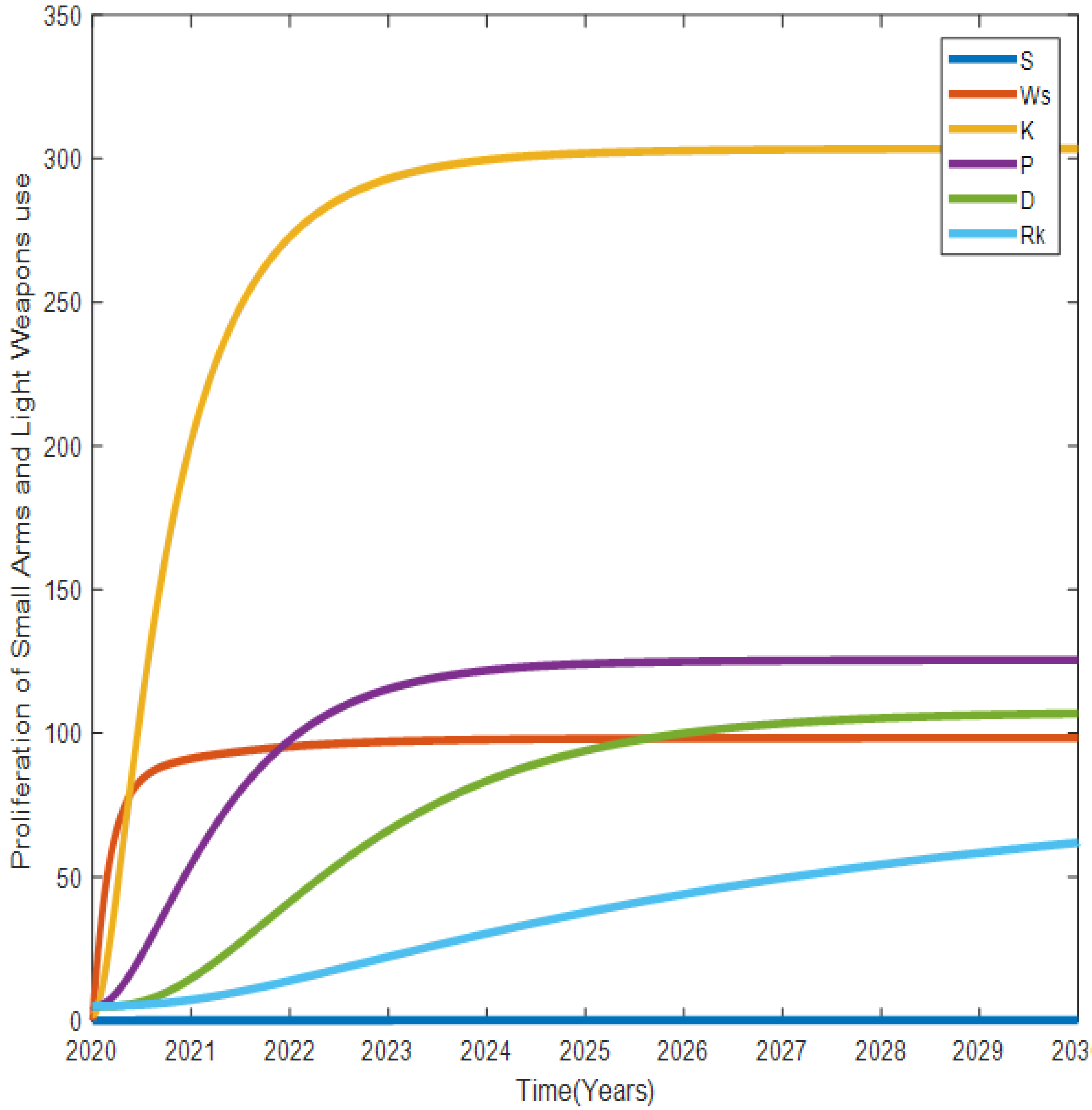

Figure 3.3: The state arms and weapons inflows between 2020 and 2022.

The above figure (3.3) reveals the situation proliferation of small arms and light weapons, kidnaping, those who has been sentence to prison and those in police custody, within 2020 to 2022 there is an exponential increase of the parameter under consideration. The red line which signifies the smuggle of small arms and light weapon, reveals from the graph that, the more the inflow of the illegal arms, the more the imprisonment, kidnapping activities and also the number of suspected kidnappers and smugglers in the police custody which accurately interpreted the overcrowded situation of all the largest prison in Nigeria.

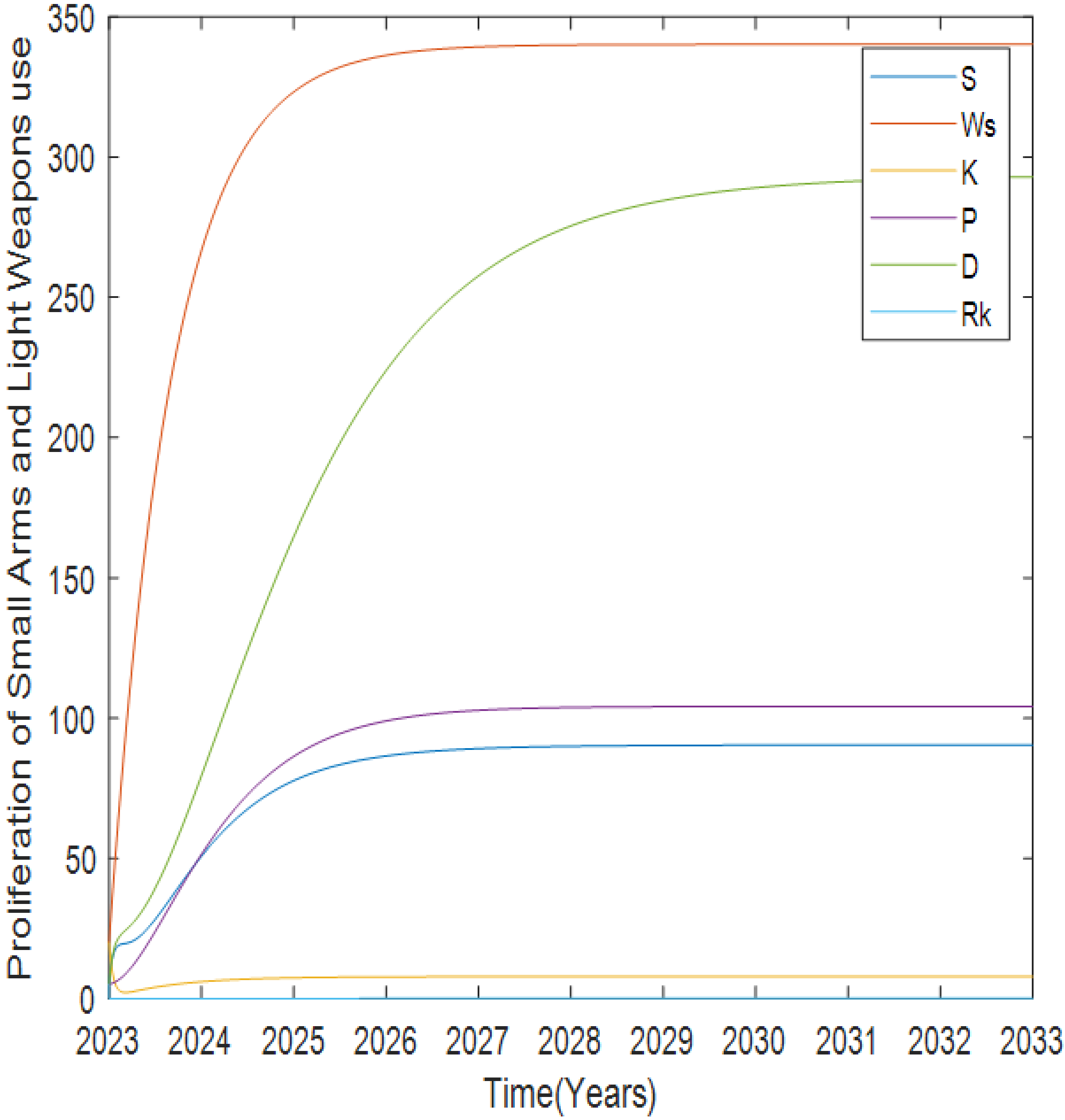

Figure 3.4: Arms and weapons inflows between 2022 and 2027.

Figure 3.4 demonstrates the solution to the SEQIR by using the Runge-Kutta of order 4 method. It also reveals the presence condition. Here, the proliferation of small arms and light weapons, Detention in Police custody, susceptible (someone living among those who smuggle arms) and Kidnapping raises and then approaches close to zero.

This is as result of decrease of the rate of rehabilitation and education of the people of the danger of smuggling illegal arms. Clearly, from the graph we can predict that the rate of Ws, D, S and kidnapping was increasing as a result of the decline of Rehabilitation of the smugglers and education of the entire population.

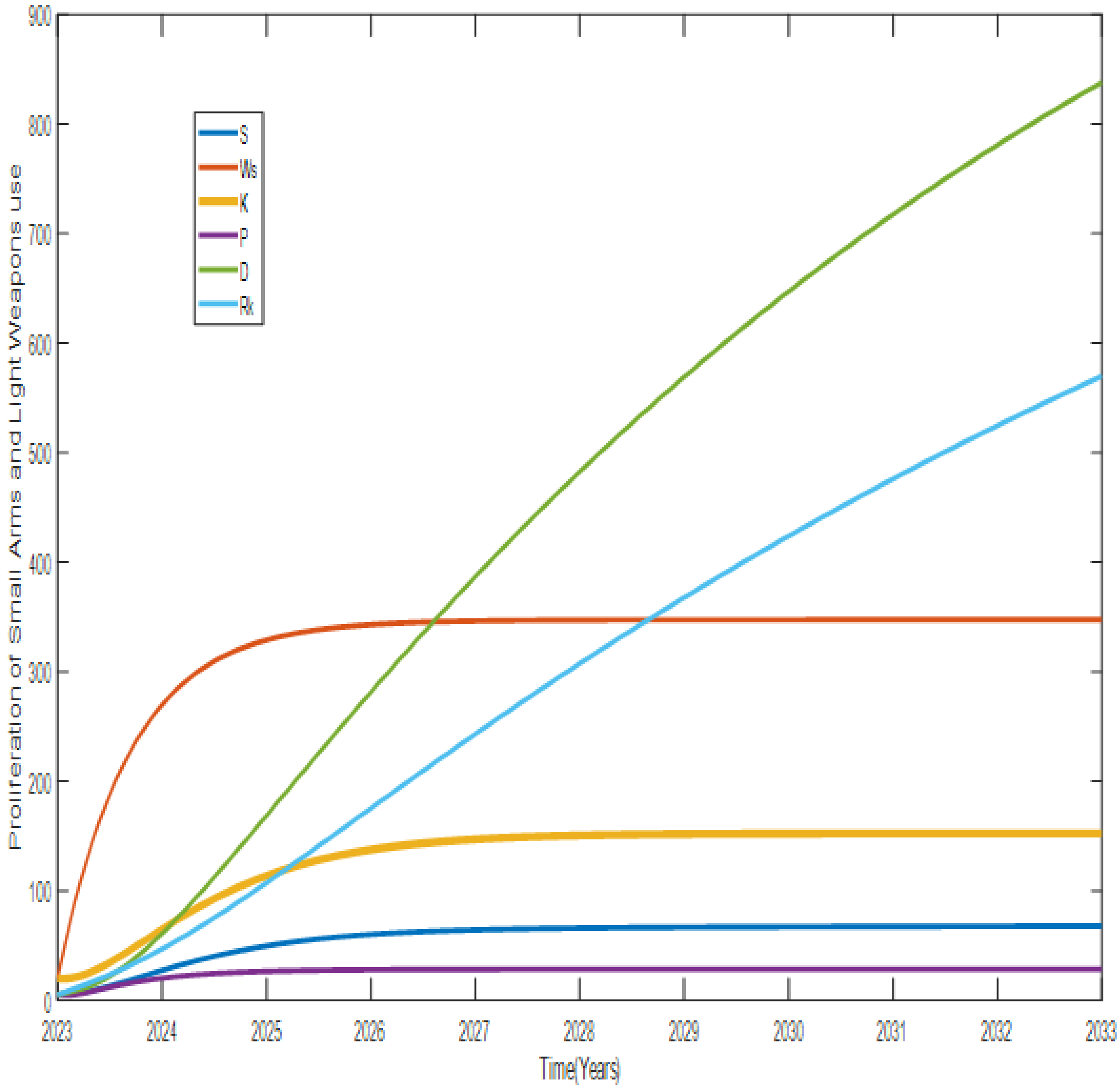

Figure 3.5: The state arms and weapons inflows between 2023 and 2033.

Figure 3.5 reveals the presence condition of the proliferation of small arms and light weapons and kidnaping activities. It also demonstrates the solution to the SEQIR by using the Runge-Kutta of order 4 method, at a certain time, the number of smugglers detent in the police custody and rehabilitation increase exponentially. Also, the smugglers of small arms and light weapons, Kidnappers, those who have been already sentence to prison and the susceptible raises and becomes stable. This tends to explain the inverse of the law of demand. The higher the number of arm and weapon smuggling into the country, the higher the kidnapping, suspect in the police custody, prisoners and other related crimes that requires the use of weapons.

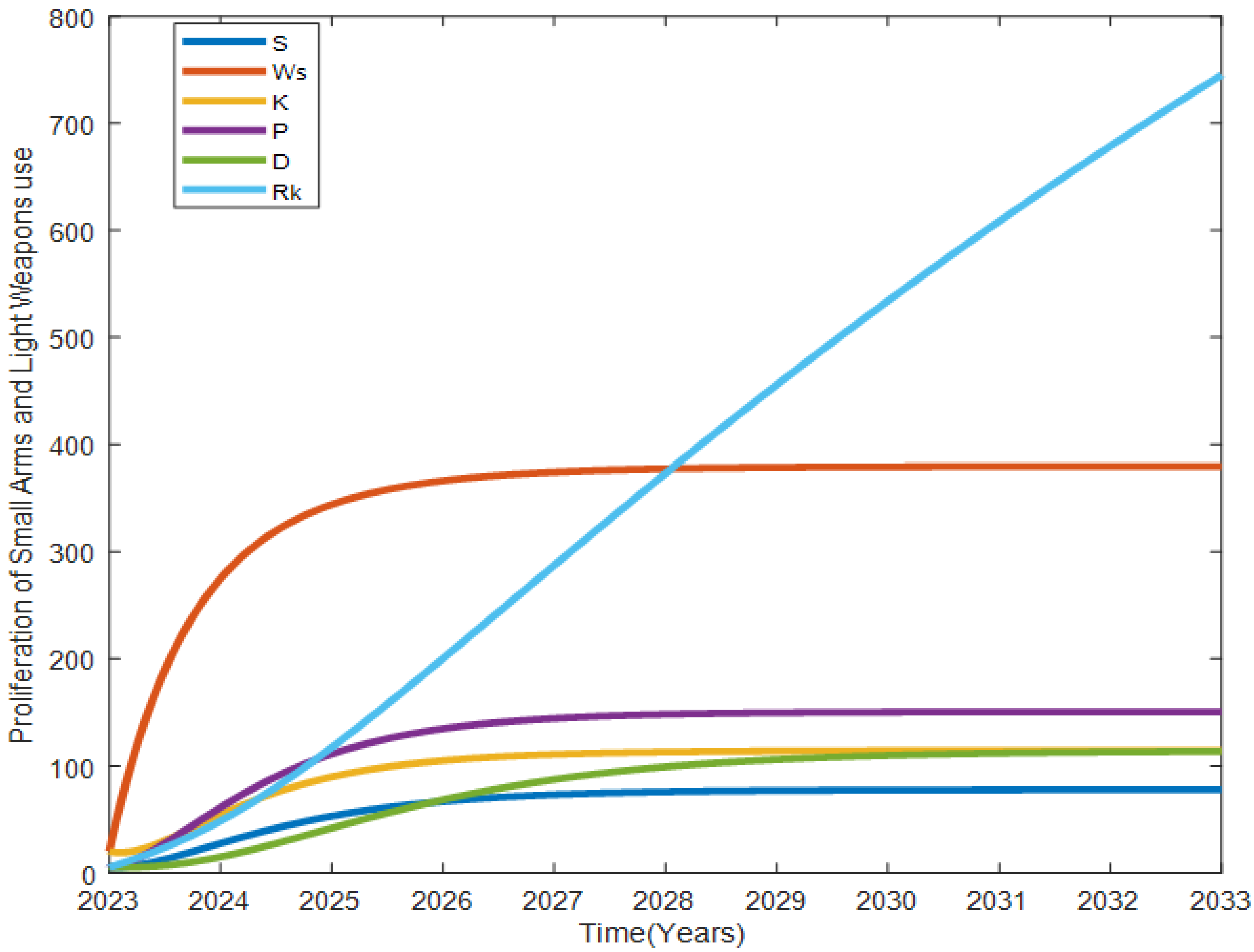

Figure 3.6: The impact of rehabilitation and education of inmates and the public.

In figure 3.6, we adjust all the parameters except the rehabilitation and education of the people, the susceptible, proliferation of small arms and light weapons, kidnapping, prison, detention and rehabilitation at a certain time were growing exponentially, to a point all of the components became stable except the line of rehabilitation and education of the inmate and the entire population. This proposes that, the increase of rehabilitation and education of the inmate and the member of the publics lowers the t rate of every other variable.

From the numerical solutions, we discovered in figure 3.6 that as long as the rate of rehabilitation and education of the inmate in detention and also the member of the public respectively of the danger; the illegal proliferation of small arms and light weapons used, kidnaping activities, and imprisonment raise for sometimes and then become stable. This implies that, if people can be educated of the danger of the illegal smuggling of small arms and light weapon, then certainly, they will not be involved in the illegal activities.

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