



International Journal of Mathematical Analysis and Modelling

**(Formerly Journal of the Nigerian Society
for Mathematical Biology)**

Volume 6, Issue 1 (July), 2023

ISSN (Print): 2682 - 5694

ISSN (Online): 2682 - 5708

On the volatility estimation of daily price returns of Nigerian Stock Market

S.C. Emenyonu^{*†}, B.O. Osu[‡], and C. Olunkwa[†]

Abstract

The stock market is exposed to risk and this risk which is the risk of losses in position brought about the movements on market variables like prices and volatility. Modelling the volatility in daily stock prices entails studying the particular error distribution that is most appropriate for the model. Considering a particular Nigeria stock market, this study estimates both the symmetric and asymmetric volatility models. The ARMA-GARCH, ARMA-EGARCH and ARMA-TGARCH. These models are employed with the error distributions such as normal distribution, student t-distribution and skewed student t- distribution. The ARMA (2,1)-EGARCH (1,1) with student t-distribution was seen to be the most appropriate model. A volatility forecasting accuracy was determined by using the mean absolute scaled error (MASE) to predict the values of the stock market prices for the next 20 years and the result showed that the model was appropriate for predicting volatility. Hence volatility prediction would help in achieving a sound policy decision. R-Code is used to fit the ARMA-GARCH, ARMA-EGARCH and ARMA-TGARCH models (as in the appendix).

Keywords: GARCH; EGARCH; TGARCH; stochastic volatility model; error distribution

1 Introduction

The financial market is affected by volatility that is the rate at which the prices of assets rise and fall given a particular set of returns. In a study of testing volatility in Nigeria Stock Market using GARCH Models, [2] explains that through the mobilization of long-term money for future investment, the capital market section of the financial market contributes significantly to the process of economic expansion around the world. For instance, the stock market in Nigeria aids in the long-term financing of government development projects, acts as a funding source for long-term investments in the private sector, and acted as a catalyst for the consolidation of the banking system in 2004 and 2005.

A stock market is a recognized legal environment where shares of numerous businesses or organizations can be traded. A rising stock market is regarded as a sign of a developing economy. It is frequently seen as the main metric of a nation's economic health and development; see [4]. Similarly, another study of modelling volatility in selected Nigerian stock Market, [1] suggest that Investment analysts, economists, and policymakers place a lot of attention on the stock market since it can be used to gauge changes in overall economic activity using the stock prices of listed companies on the Nigerian Stock Exchange (NSE).

The stock market is exposed to risk and this risk which is the risk of losses in position brought about the movements on market variables like prices and volatility. In addition, volatility is the rate at which the price of assets rises and falls given a particular set of returns. Also, it is a measure which

^{*} Department of Mathematics and Statistics, College of Natural and Applied Sciences, Gregory University, Uturu, Abia State, Nigeria

[†] Department of Mathematics, Abia State University, Uturu, Abia State, Nigeria

[‡] Corresponding author

investors look at before making any trading decisions. [5] explains that in these situations, the assumption of constant variance (homoscedasticity) is unsuitable since the stock market demonstrates fluctuations in variance over time. The variance in the financial statistics can be as a result of volatility in the financial market. On the other hand, in a study, [3] examines volatility prediction as a crucial instrument in financial economics for risk management and asset allocation since knowledge of upcoming volatility can help investors reduce losses.

Volatility has been a major concern to stock market; due to this fact many studies have modelled volatility with several volatility models. Just like in the study of modelling and forecasting of all India monthly average wholesale price volatility of onion. The GARCH, EGARCH and TGARCH techniques were applied. While another study on fitting the Nigeria stock market return series using GARCH models made use of several. However, this current study will estimate the first order symmetric and asymmetric volatility models Nigeria stock market from April 1, 2016 to December 16 2022, each model in Normal, Student's-t and skewed student's -t error distributions then selecting the best forecasting volatility model with the most appropriate error distribution. And comparing the best model to the stochastic volatility model.

2 Methodology

2.1 ARCH (q) Model

ARCH models (Autoregressive Conditional Heteroscedastic are formed on the basis of the variance of the error term at time t which is dependent on the realized values of the squared error terms in past time periods. That is, it is able to capture the behaviour of volatility as a function of time. More so, it takes into account the number of stylized facts that characterize the majority of financial series such as persistence, volatility clusters and leptokurtic behaviour of data. The model is specified as

Let Y_t the return be explained by some variables X_t which can be an ARMA (p, q) model. $Y_t = X_t\beta + \varepsilon_t$, where $\varepsilon_t / I_{t-1} \sim N(0, \delta_t)$. I_{t-1} is all the information available in the returns data up to the time $t-1$

The ARCH model allows you to set the conditional variance δ_t^2 that is,

$$\delta_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots + \alpha_q \varepsilon_{t-q}^2$$

$$\varepsilon_t = \delta_t z_t \quad z_t \sim N(0,1),$$

where,

δ_t is the conditional variance

z_t is the standard residue

ε_t is the conditional errors of the asset at the time t

$\alpha_0, \dots, \alpha_i$ are real parameters

q is order of ARCH process.

The value of the conditional variance δ_t^2 must be positive and negative variance at any point in time would be meaningless.

2.2 Generalized-ARCH model (GARCH)

Let

$$Y_t = X_t \beta + \varepsilon_t.$$

The return be described by some variables X_t . Where $\varepsilon_t / I_{t-1} \sim N(0, \delta_t)$.

The GARCH process allows you to set the conditional variance δ_t^2 .

That is

$$\delta_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j}$$

α_0 the parameter representing the long-term average variance

α_i the parameter that measures the sensitivity to conditional volatility

β_j the parameter that measures the persistence of conditional volatility

To get a positive condition variance, one must adhere to the conditions

$\alpha_0 > 0$ and $\alpha_i \geq 0$ for all $i = 1, 2, 3, \dots, q$

$\beta_j \geq 0$ for all $j = 1, 2, 3, \dots, p$ $q > 0$ and $p \geq 0$

2.3 Exponential GARCH (EGARCH) model

The EGARCH is an asymmetric GARCH model that identifies not only the conditional variance but the logarithm of the conditional volatility. It is generally recognized that EGARCH model gives a much more in sample in-sample fit than other types of GARCH models. The EGARCH model was put forward by Nelson and the model includes the leverage effects in its equation.

$$\ln(\delta_t^2) = \Omega + \beta \ln(\delta_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} + \alpha \left[\frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right].$$

Variance is always positive because $\delta_t^2 = \log(\delta_t^2)$.

Even if the parameters are negative, the variance will still be positive. Leverage effect indicates that if the relationship between volatility and returns is negative then $\gamma < 0$.

The α shows the symmetric effect, the β measures the persistence in conditional volatility shock and reflects the asymmetric performance.

2.4 Threshold GARCH (TGARCH) model

Another asymmetric model proposed by Glosten Jagannathan and Runke known as the TGARCH (Threshold GARCH) model.

$$\delta_t^2 = c + \sum_{j=1}^p B_j \delta_{t-j}^2 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{k=1}^r \gamma_k \varepsilon_{t-k}^2 - k I_{t-k} (\varepsilon_{t-k} < 0)$$

$$I_t = \begin{cases} 1, & \text{if } \varepsilon_t < 0, \text{ bad news} \\ 0, & \text{if } \varepsilon_t \geq 0, \text{ good news} \end{cases}$$

where I_t is an indicator function, when the residual (ε_t) is less than zero, the indicator term $I_t = 1$ or $I_t = 0$ when the residual is not less than zero. Where $I_t = 1$ if ε_t is negative and 0 otherwise. In the TGARCH (1,1) model, volatility is likely to rise with bad news ($\varepsilon_{t-1} < 0$) and declines with good news ($\varepsilon_{t-1} \geq 0$). Good news has an influence of α_1 while bad news

has an influence of $\alpha_1 + \gamma$. If leverage effect parameter $\gamma > 0$ and statistically significant then the leverage effect exists. If $\gamma \neq 0$, the shock is asymmetric, and if $\gamma = 0$, the shock is symmetric. The persistence of shocks to volatility is measured by $\alpha_1 + 1 + \gamma/2$ see [3].

2.5 Error distribution

This study introduces both the normal and the non-normal innovations. That is we are going to make use of three distributions, the normal distribution, the student-t distribution and the skewed student-t distribution.

Normal distribution

The probability density function of X_t is shown as

$$f(X_t) = \frac{1}{\sqrt{2\pi}\delta} \exp\left\{-\frac{1}{2} \left(\frac{X_t - \mu}{\delta}\right)^2\right\}$$

where μ is mean and δ is the standard deviation

Student t-distribution

The probability density function X_t is given as

$$f(X_t) = \frac{\gamma^{\frac{v+1}{2}}}{\gamma^{\frac{v}{2}} \sqrt{(v-2)\pi}} \left(1 + \frac{x_t^2}{v-2}\right)^{-\frac{1}{2}(v+1)}$$

where v is the number of degrees of freedom, $2 < v \leq \infty$, and is gamma function. When $v \rightarrow \infty$, the student t-distribution is almost equals to the normal distribution. The smaller the v the heavier the tails.

Skewed student-t distribution

The Student t distribution is more effective in generating values that lie far from its mean and capturing heavy tail data sets. On the other hand, it is a symmetric distribution that cannot capture asymmetry. To make room for asymmetry and long tailed data. Hansen (1994) put forward the skewed t distribution whereas retaining the property of a zero mean and variance equal to the Skewed t distribution with degree of freedom = v degrees of freedom have the following density, where $f(x)$ is the density of the t distribution, with $= v$ degrees of freedom:

$$f(x) = \frac{2}{\gamma+1} f(\gamma x) \quad \text{for } x < 0$$

$$\text{and} \quad f(x) = \frac{2}{\gamma+1} f\left(\frac{x}{\gamma}\right) \quad \text{for } x \geq 0$$

Selection of model

Selection of model when making comparison among several specification of ARMA-GARCH models. the appropriate model is selected by making use of the Akaike Information Criteria (AIC) and the Bayesian Information Criteria (BIC). The AIC and BIC can be calculated as

$$\text{AIC} = -2\ln(\text{residual sum of squares}) + 2k$$

$$\text{BIC} = -2\ln(\text{residual sum of squares}) + \ln(N)k$$

where N is the number of observations and k is the parameters estimated or estimated coefficients. The least value of AIC and BIC is selected as the more appropriate model when making comparison among models.

Evaluation of model

In evaluating the volatility forecasting performance of models, some statistical measures are considered here.

3 Data

A daily time series daily price data of Fidelity Bank Nigeria from April-2016 to December- 2022 (total number of observation 1664). Which the GARCH models are applied for prediction. In other to treat volatility, we compute the daily yield as follows:

$$r_t = \left(\frac{p_t}{p_{t-1}} \right) \times 100$$

where p_t is the Fidelity bank stock price from April 1 2016 to December 16 2022.

Table 1. Descriptive statistics of daily price returns of Fidelity Bank Nigeria

N	Mean	Min	Max	SD	Skew	Kurt
1664	0.03	-9.48	116.93	3.84	0.20	2.59

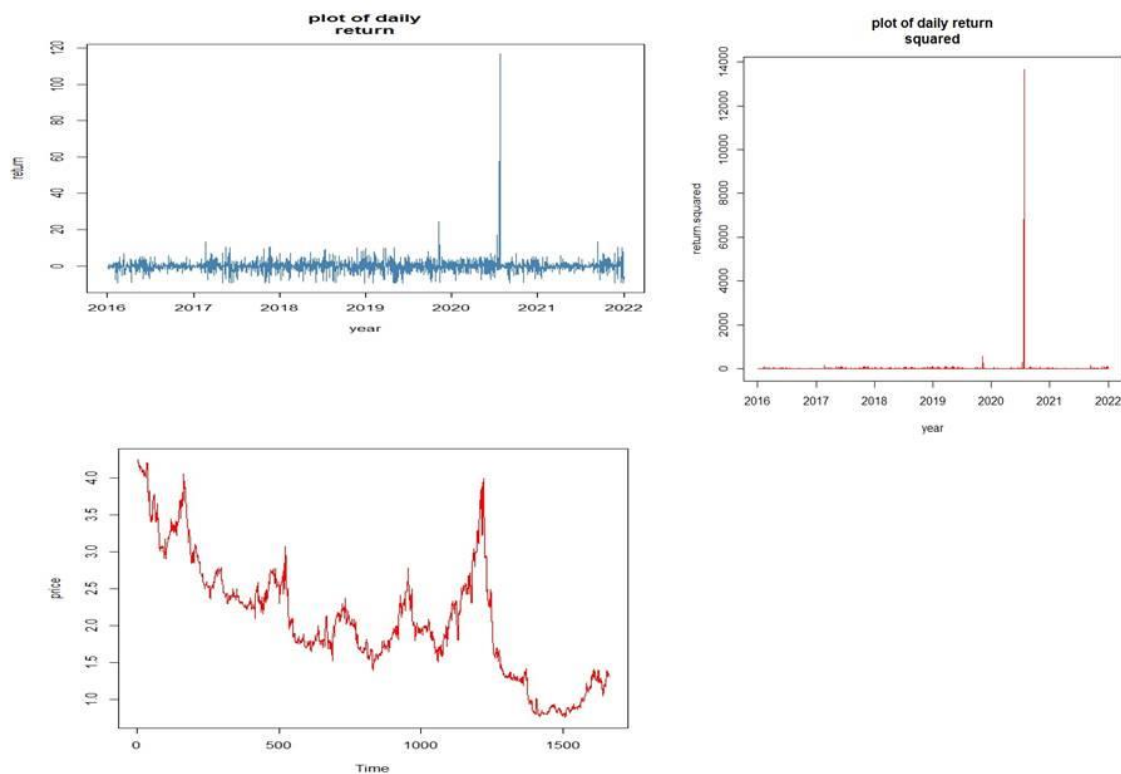


Figure 1. Plots of daily returns

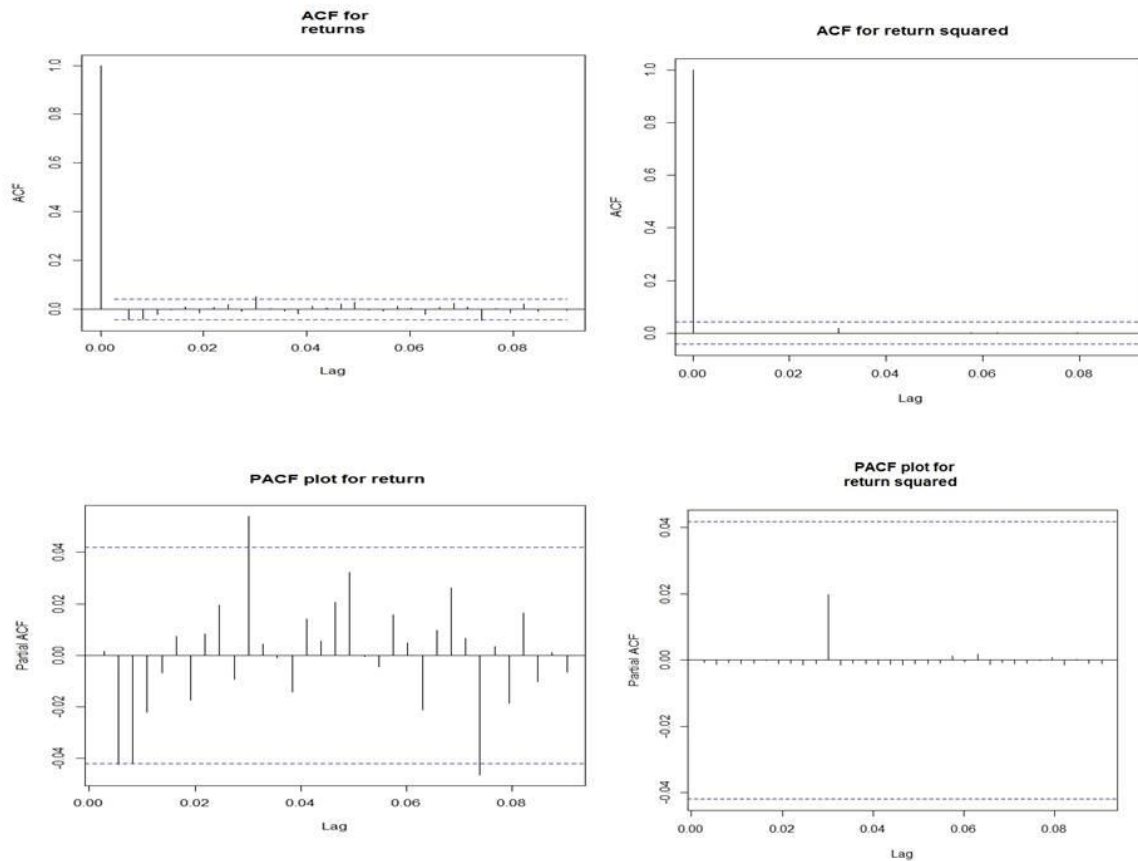


Figure 2. The autocorrelation of the series sample

Table 1 shows the skewness is 0.20 that means is positively skewed, it is not zero, we can say that the return is asymmetric. While the kurtosis is seen to be 2.59 which is less than three and this means that the distribution of return is said platykurtic that is the distribution tends to produce fewer and less outliers than the normal distribution.

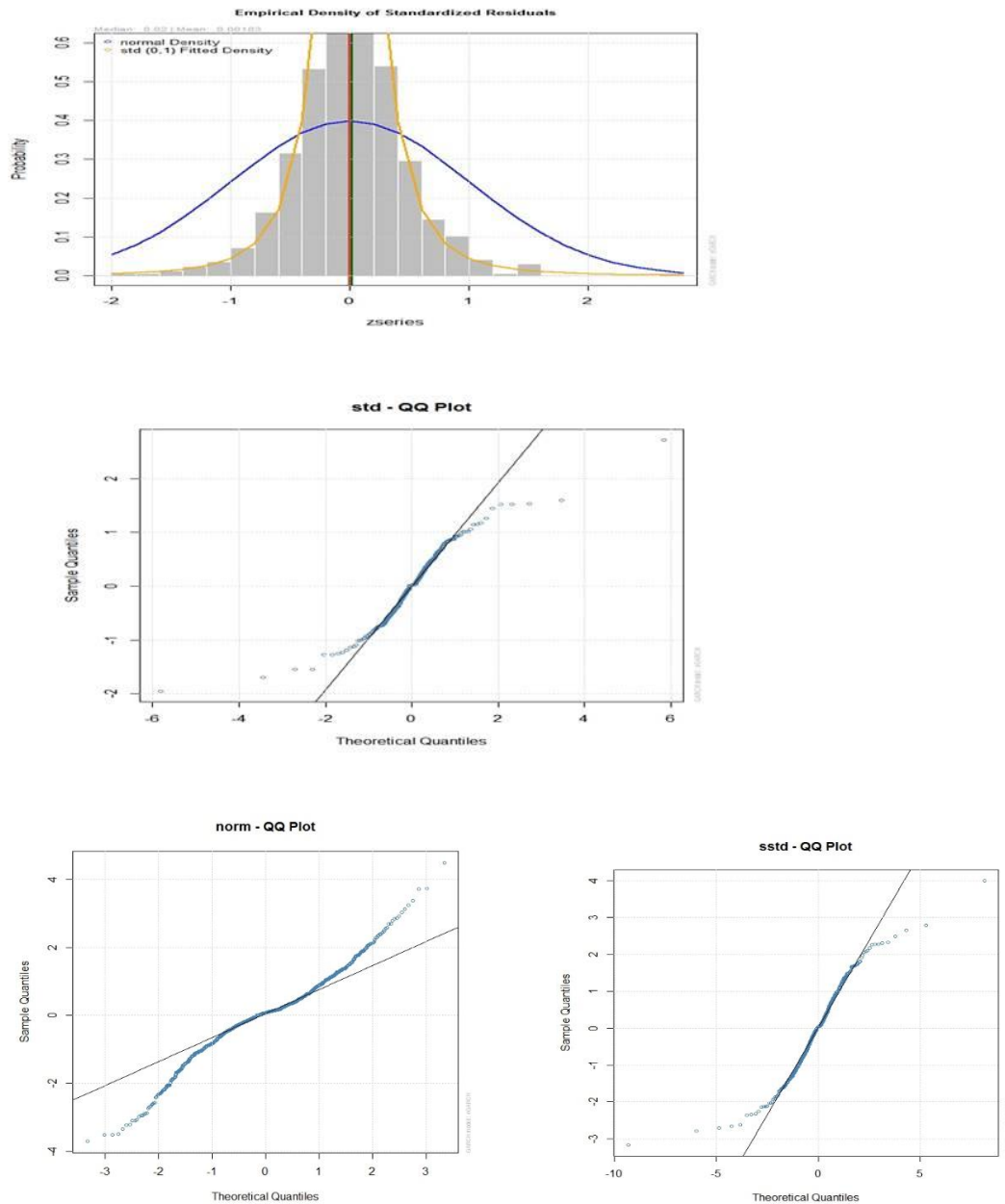


Figure 3. The Density and the QQ plot of Standardized Residuals of ARMA (2.1)-EGARCH (1, 1) with Normal, Student t-Distribution and Skewed student t-distribution

4 Results

In this result, ARMA-GARCH, ARMA-EGARCH and ARMA-TGARCH models are used in estimating and forecasting the daily returns of the aforementioned stock market in Nigeria under different error distributions. That is the normal distribution, student t-distribution and the skewed student t-distribution. Comparison is made using forecasting measures such as MASE, then choosing the appropriate volatility prediction model.

4.1 Model selection for Mean Equation and Variance Equation

In selecting the appropriate mean equation ARMA (p, q) and Variance equation GARCH (p, q), EGARCH (p, q) and TGARCH (p, q) model for the daily price returns. The autocorrelation and the partial autocorrelation of the daily returns are observed. Thus the other of p and q can be obtained. The Autoregressive integrated moving average (ARIMA 2,0,1) is selected within the candidate models by thoroughly examining the selection of parameters. Comparing ARMA (2,1)- GARCH(1,1), ARMA(2,1)-EGARCH(1,1) and ARMA(2,1)-TGARCH(1,1) models under three error distributions by using the AIC and BIC values for different models, the EGARCH(1,1) with student t-distribution is seen to be the best possible candidate model. The estimated parameters of ARMA(2,1)-EGARCH(1,1) model with student t-distribution is represented in Table 4.

The variance equation of the estimated EGARCH (1,1) model is shown below

$$\ln(\delta_t^2) = 0.120 + 0.958 \ln(\delta_{t-1}^2) + 0.025 \frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} + 0.387 \left[\frac{u_{t-1}}{\sqrt{\delta_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

Table 3. Selection criteria of model

Model	Distribution	AIC	BIC
ARMA (2,1)-GARCH (1,1)	Normal distribution	4.808	4.837
ARMA (2,1)-GARCH (1,1)	Student t-distribution	4.686	4.720
ARMA (2,1)-GARCH (1,1)	Skewed student t-distribution	4.686	4.720
ARMA (2,1)-EGARCH (1,1)	Normal Distribution	4.791	4.826
ARMA (2,1)-EGARCH (1,1)	Student t-distribution	4.675	4.713
ARMA (2,1)-EGARCH (1,1)	Skewed student t-distribution	4.674	4.717
ARMA (2,1)-TGARCH (1,1)	Normal distribution	4.809	4.843
ARMA (2,1)-TGARCH (1,1)	Student t-distribution	4.688	4.726
ARMA (2,1)-TGARCH (1,1)	Skewed student t-distribution	4.688	4.731

Table 4. Coefficient Estimates of ARMA (2,1) –EGARCH (1.1) Model with Student t=distribution

	Parameter	Standard Error	t-Statistics	P-Value
Mean Equation				
μ_0	-0.066	0.042	-1.573	0.002
AR (1)	0.605	0.039	15.691	<0.001
AR (2)	-0.048	0.024	-2.024	0.043
MA (1)	-0.602	0.038	-15.744	<0.001
Variance Equation				
Ω	0.120	0.031	3.877	<0.001
α_1	0.025	0.044	0.561	<0.001
β_1	0.958	0.011	84.397	<0.001
γ	0.387	0.085	4.573	<0.001
ν	2.593	0.317	8.173	<0.001
$\alpha_1 + \beta_1$	0.983			

The Table 4 shows that all the ARMA(2,1)-EGARCH(1,1) model parameters are statistically significant and the shock persistence parameter ($\beta_1 = 0.958$) is near to the unity which means that the conditional variance has a long memory and the volatility shock is reasonably persistent. More so, the leverage effect shown by the EGARCH (1,1) model, indicates that the leverage effect coefficient ($\gamma = 0.387$) is positive and statistically significant, implying that the positive shocks have small influence on the future volatility unlike the negative shocks which have a greater influence or impact on the future volatility. That is to say, the more leverage you have the more risk you face. So if there is a drop in the value of stock (negative shock/return) there is an increase in financial leverage which makes the stock riskier thus increasing volatility.

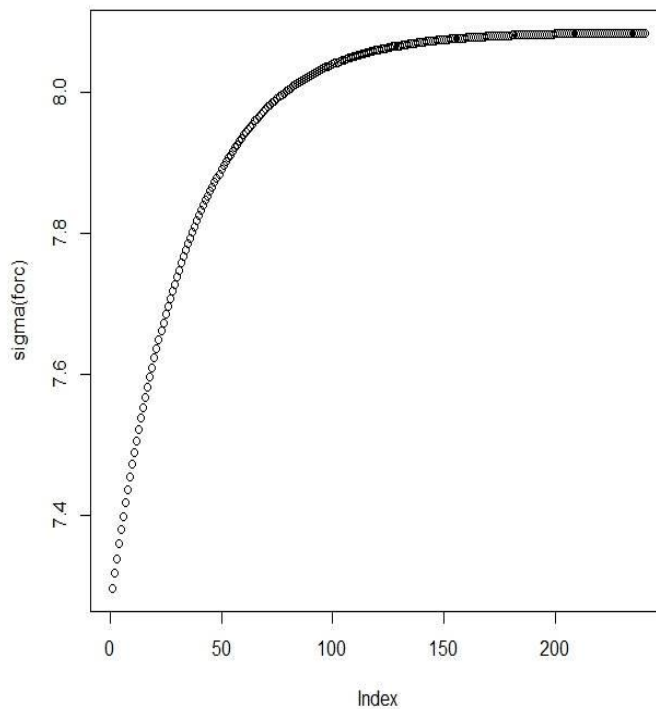


Figure 4. Plot showing the Forecast of Volatility for the next 20 years

The Figure 3 show the volatility forecast for the next 20 years (240 months) of the Fidelity bank stock return for the next 20 years. Here we can observe that based on this model, we expect the volatility to potentially increase in the next 120 months (10 years) and remains at the same level for the remaining years as seen in the plot.

4.2 Diagnostic check for the selected model

In determining whether the model is appropriate, the Ljung-Box test and the ARCH-LM are used in testing serial correlation (autocorrelation). Basically if the p-value is lower than the 0.05, then the null hypothesis is rejected, as we can see from Table 5 and Table 6, the p-value of the two tests are seen to be greater than 0.05. that is, no enough evidence to reject the null hypothesis for all the lags, that is no serial correlation and no ARCH effect in ARMA (2,1)-EGARCH(1,1) model.

Table 5. Ljung- Box squared residual test for ARMA(2,1) –EGARCH(1,1) model with Student t-distribution

ARMA(2,1) –EGARCH(1,1) model with Student t- distribution		
Ljung-Box squared residual		
	Statistics	p-value
Lag [3]	4.57	0.10
Lag [5]	6.02	0.16
Lag [7]	8.15	0.12

Table 6. ARCH Effect test for ARMA(2,1) –EGARCH(1,1) model with Student t- distribution

ARMA(2,1) –EGARCH(1,1) model with Student t- distribution		
ARCH test		
	Statistics	p-value
Lag [3]	1.60	0.21
Lag [5]	2.46	0.38
Lag [7]	4.15	0.33

4.3 Forecast accuracy

In other to determine the volatility forecasting performance of models, the Mean absolute scaled error (MASE) is used. The MASE measures the forecast error which is compared to the error of naïve forecast as in [6]. In this study, we predicted the values of fidelity prices for the next 20 years and the MASE calculated with observed values is given as one (1) approximately, which indicates that the ARMA (2,1)-EGARCH (1,1) with model is appropriate in predicting volatility.

5 Conclusion

The study evaluated the daily stock prices of Fidelity Bank from April 2016 to December 2022. Both the symmetric and the Asymmetric GARCH models were considered with three different error distributions. The Autoregressive integrated moving average (ARIMA 2, 0,1) was selected within the candidate models by thoroughly examining the selection of parameters. Then the ARMA-GARCH, ARMA-EGARCH and ARMA-TGARCH models were employed with the error distributions such as normal distribution, student t-distribution and skewed student t- distribution. The ARMA (2,1)-EGARCH (1,1) with student t-distribution was seen to the most appropriate model. A volatility forecasting accuracy was determined by using the mean absolute scaled error to predict the values of the stock market prices for the next 20 years and the result showed that the model was appropriate for predicting volatility. Furthermore, serial correlation was checked by using the Ljung-Box test and the ARCH effect test and there was no autocorrelation or serial correlation. Finally the descriptive statistics were estimated and the results were shown in Table 1 also the Density and QQ plots of the standardizes residuals for ARMA-EGARCH model are seen in Figure 3.

Conflicting Interests

Authors have declared that there is no conflicting of interest

Appendix

R code for the GARCH models

```
Y<- (data= fidelity)
price<-(data=fidelity[,2])
fid=ts(as.vector(price), start=2016,end=2022,frequency=365)
return =diff(log(fid)) *100 # daily return fidelity
summary(return)
return squared=return^2
par(mfrow=c(1,1))
plot(return,main="fid daily return",xlab="year",type="l", col="blue")
ACF and PACF plots
par(mfrow=c(1,1))
acf(return, ci.type="ma",main=" ACF for
returns")
pacf(return, main=" PACF plot for returns")
acf(return squared, main=" ACF for return squared")
pacf(return squared, main=" PACF plot for return squared ")
#####
Model fit
## GARCH with normal distribution
library(rugarch)
garch_spec <- ugarchspec (variance.model=list(model="sGARCH", garchOrder=c(1,1)),
mean.model=list(armaOrder=c(2,1)),distribution.model="norm")
fit_garch <- ugarchfit(spec = garch_spec, data = return ,out.sample=20)
#####
## GARCH with t- distribution
library(rugarch)
mod_specify <- ugarchspec(variance.model=list
(model="sGARCH", garchOrder=c(1,1)), mean.model=list
(armaOrder=c(2,1)),distribution.model="std")
mod_fitting <- ugarchfit(spec = mod_specify, data = return,out.sample=20)
mod_fitting
#####
## GARCH with Skewed t- distribution
library(rugarch)
mod_specify <- ugarchspec(variance.model=list (model="sGARCH", garchOrder=c(1,1)),
mean.model=list
(armaOrder=c(2,1)),distribution.model="sstd")
mod_fitting <- ugarchfit(spec = mod_specify, data = return,out.sample=20)
#####
Model fit for EGARCH(1, 1)
## Nelson's egarch model with normal distribution
library(rugarch)
egarch1 <- ugarchspec(variance.model=list (model="eGARCH", garchOrder=c(1,1)),
mean.model=list
(armaOrder=c(2,1)),distribution.model="norm")
```

```

egarchfit1 <- ugarchfit(spec = egarch1, data = return,out.sample=1000)
## EGARCH with t- distribution
library(rugarch)
egarch2 <- ugarchspec(variance.model=list (model="eGARCH", garchOrder=c(1,1)),
mean.model=list
(armaOrder=c(2,1)),distribution.model="std")
Egarchfit2 <- ugarchfit(spec = egarch2, data = return,out.sample=1000)
## EGARCH with Skewed t- distribution
library(rugarch)
egarch3 <- ugarchspec(variance.model=list (model="eGARCH", garchOrder=c(1,1)),
mean.model=list
(armaOrder=c(2,1)),distribution.model="sstd")
Egarchfit3 <- ugarchfit(spec = egarch3, data = return,out.sample=1000)

tgarch1 <- ugarchspec(variance.model=list
(model="girGARCH", garchOrder=c(1,1)), mean.model=list
(armaOrder=c(2,1)),distribution.model="norm")

tgarchfit1 <- ugarchfit(spec = tgarch1, data = return ,out.sample=1000)


#TGARCH Model with t distribution
####TGARCH Model Secification

library(rugarch)

tgarch2 <- ugarchspec(variance.model=list
(model="girGARCH", garchOrder=c(1,1)), mean.model=list
(armaOrder=c(2,1)),distribution.model="std")

tgarchfit2 <- ugarchfit(spec = tgarch2, data = return ,out.sample=1000)

#####

library(rugarch)

tgarch3 <- ugarchspec(variance.model=list
(model="girGARCH", garchOrder=c(1,1)), mean.model=list
(armaOrder=c(2,1)),distribution.model="sstd")

tgarchfit3 <- ugarchfit(spec = tgarch3, data = return ,out.sample=1000)

```

References

- [1] Ekum, M. I. and Owolabi, T. O. (2018). Modelling volatility in selected Nigerian Stock Market. International Journal of Economics and Financial Management, 3(1).
www.iiardpub.org
- [2] Atoi, N. V. (2014). Testing volatility in Nigeria Stock Market using GARCH Models: CBN Journal of Applied Statistics, 5(2).
- [3] Pushpa, S. and Preeti, P. (2022). Modelling and forecasting of all India monthly average wholesale price volatility of Onion: Application of GARCH and EGARCH Techniques. Current Journal of Applied Science and Technology, 41(48): 91-100.
- [4] Simeyo O., Edgar, O.O., and Nyabwanga, R.N. (2015). Application of Markov Chain to model and forecast Stock Market trend: a study of Safaricom shares in Nairobi Securities Exchange, Kenya. International Journal of Current Research, 7(4): 14712-14721.
- [5] Usman, U., Auwal, H.M., and Abdulmuhyi, M.A. (2017). Fitting the Nigeria Stock Market return series using GARCH models. Theoretical Economics Letters, 7: 2159-2176.
- [6] Emenyonu, S. C., Osu, B.O., and Olunkwa, C. (2023). Estimating volatility of daily price returns of Nigerian Stock Market. J Math Techniques Comput Math, 2(4): 163-169.
- [7] Hassan, B.E. (1994). Autoregressive conditional density estimation. International Economic Review, 35: 705 – 730.