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Mathematical analysis of Ebola virus population dynamics

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Abstract

In this paper, a non-linear mathematical model for the population dynamics of Ebola virus disease in existence of vaccination was developed and analyzed. Existence of equilibrium point was carried out, which shows us that the model consists of two equilibrium point. Local stability analyses of DFE have been carried out, it was found that the DFE is locally asymptotically stable if the threshold called reproduction number can be brought to a number less than unity. Comparison theorem was used to access the globally Stability of DFE, it was found that the DFE is globally asymptotically stable if the reproduction number can be brought to a number less than unity. LAS of endemic equilibrium point was done using Centre manifold theorem, it was established that the endemic equilibrium point is locally asymptotically stable if the reproduction number is greater than one. We were able to prove the global stability of endemic equilibrium point using Lyapunov function of Goh-Volterra type, it shows us that the endemic equilibrium point is globally asymptotically stable if the reproduction number is less than one. The numerical simulation shows that vaccinating a good number of people in a society can mitigate the Ebola virus disease (EVD).

Keywords and Phrases: Ebola virus; mathematical model; population dynamics; mathematical; population.

1 Introduction

EVD is a serious infection which is referred to as deadly, harmful and infectious disease which claimed numerous humans and other mammals' lives that causes huge damages economically in the West African Countries. A river in DRC was the main origin of EBV and it was named after the

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DRC (Democratic Republic of Congo) river and it was in the year 1976 Ebola virus disease was discovered which can also harm primates and humans (Abdulrahman, 2016; Deepaa, 2015 and Tae and Lee, 2016).

The virus transmits into human population through blood contacts, secretions, other bodily fluids of infected animals or organs (Herick *et al.*, 2020). Characterization of EBV is by flu which quickly progress to vomiting, diarrhea, rash, internal and external bleeding (Birmingham and Cooney, 2002; Eric *et al.*, 2020 and WHO, 2014). It is said that, Ebola virus can transmit from human to human, bats (fruit bats) to human and from other animals such as Monkeys, Gorillas and Chimpanzees etc (Amenoghawan and Aboubakary, 2015 and Ustun and Ozguller, 2004).

The recent 2018 to 2020 outbreak in Democratic republic of Congo was very multifaceted due to lack of security which affected the public health workers badly rejoiner activities, sequencing of samples suggests that cases in this outbreak are connected to cases in the area throughout the year 2018 to 2020 outbreak and possibly resulted from continual infection in a survivor that led to either a reversion or sexual transmission of the virus (CDC, 2019; Eric, *et al.*, 2020 and WHO, 2019).

Recent researches in mathematical modeling of Ebola virus (EBV) suggest that, tracing of infected /exposed individuals must be efficiently done in order to quarantine/isolate the affected population and educate community on the various ways in which the disease transmit, symptoms and preventive measures in order to minimize human to human transmissions (Herick *et al.*, 2020; Hussain *et al.*, 2017 and Rachah and Torres, 2015).

Bats as a main reservoir of numerous viral infections are widely accepted (Pourrut *et al.*, 2009; Leroy *et al.*, 2005; Hayman, 2016 and Zineb *et al.*, 2018). Hayman (2016) think that the disease can be contracted by eating of contaminated fruits that is touched by bodily fluids of infectious bats (Zineb *et al.*, 2018). Over quite some years ago, there were several models which were developed mathematically to understand the dynamics of Ebola virus disease (EVD) (Azuaba *et al.*, 2017; Berge *et al.*, 2018; Eric, *et al.*, 2020 Espinoza *et al.*, 2015; Herick *et al.*, 2020; Kalu *et al.*, 2016; Nourridine, 2020; Lekone and Finkenstadt, 2006 and Murtaza *et al.*, 2016). Though, the said systems have normally considered the spread of the disease in human populace, only (Berge *et al.*, 2018 and Zineb *et al.*, 2018) considered bats population. In addition, they do not take into account the vaccinated humans and natural recovery due to immune response. For these precise and biological considerations, we propose a universal deterministic model for EVD.

2 Model formulation and fundamental properties

We build a model in a mathematical form of an Ebola virus dynamics by considering natural recovery and vaccination. We subdivided the system compartmentally in nine places, namely: Susceptible individuals (S_h), Latently infected individuals (L_h), Infectious individuals (I_h), Isolated individuals (J_h), Removed persons as a result of permanent recovery from the disease (R_h), Dead bodies of humans induced by Ebola virus disease before burial (D_h), environmental virus (V), Non-Carrier bats (N_b), Carrier-bats (C_b), humans Total population (T_h) and bats Total population (T_b). Figure 1 is the schematized representation of the model.

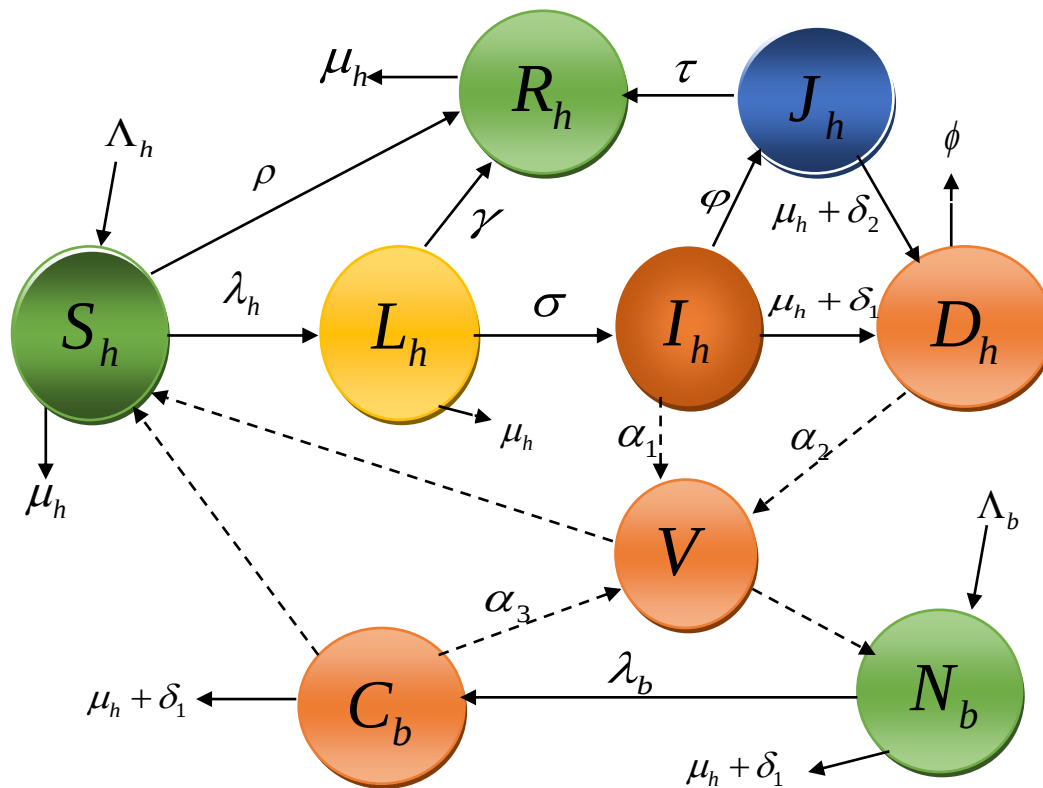


Figure 1. Schematized EVD transmission dynamics diagram

The S_h compartment is a representation of humans that are prone to the disease. This class are increased as a result of daily recruitment due to birth/immigration at the rate Λ_h . They get infected and transfer to L_h class by intermingling with those humans in the I_h and D_h compartments, virus in the environment and bats in the C_b compartment given by the term $\lambda_h = \frac{\beta_1(1-\varepsilon)(I_h + \eta D_h) + \beta_2(1-\varepsilon)V + \beta_3(1-\varepsilon)C_b}{T_h}$. The parameters β_1 , β_2 and β_3 are the effective contact rate of human to human, virus in the environment to human and bats to humans respectively, η is the modification parameters which is tight to reduced contacts with the human dead body compare to humans who are infectious, the term $(1-\varepsilon)$ is the enhancement of public enlightenment on personal hygiene in respect of the transmission of the disease; $0 < \varepsilon < 1$. The class decreases due to people that have been vaccination at rate ρ .

$$\frac{dS_h}{dt} = \Lambda_h - (\lambda_1 + \lambda_2 + \lambda_3)S_h - (\rho + \mu_h)S_h \quad (1)$$

The L_h class represents the already infected persons with the virus but the clinical symptoms of Ebola not yet developed and are not capable of infecting others. L_h class is increased by effective/actual contact between S_h with I_h , D_h , V and C_b given by the term $\lambda_h = \frac{\beta_1(1-\varepsilon)(I_h + \eta D_h) + \beta_2(1-\varepsilon)V + \beta_3(1-\varepsilon)C_b}{T_h}$ as earlier explained. They decrease at the rates γ and σ due to natural recovery and disease progression respectively.

$$\frac{dL_h}{dt} = (\lambda_1 + \lambda_2 + \lambda_3)S_h - (\sigma + \gamma + q + \mu_h)L_h \quad (2)$$

I_h class is a representation of individuals who are symptomatic, infected as well as infectious. I_h class are increased at the rate σ from the L_h class as a result of clinical symptoms of Ebola developed by those in the L_h class. The I_h class diminishes at rates φ and δ_1 due to isolation and disease induced dead respectively.

$$\frac{dI_h}{dt} = \sigma L_h - (\varphi + \delta_1 + \mu_h)I_h \quad (3)$$

The J_h class is a representation of humans who clinical symptoms are developed and have been isolated for treatment. The compartment is generated from I_h at the rate φ due to isolation. The class diminishes at the rates τ and δ_2 due to treatment and disease-induced death.

$$\frac{dJ_h}{dt} = \varphi I_h - (\tau + \delta_2 + \mu_h)J_h \quad (4)$$

The removed humans' compartment, R_h represents humans that are effectively vaccinated as well as those that recovered from the disease and they possess lasting immunity against EVD as assumed. R_h class is increased from L_h , J_h and S_h classes at the rates γ , τ and ρ due to natural recovery of individuals in latent class, recovery due to effective treatment of individuals in J_h class and effective vaccination of individuals in S_h class respectively.

$$\frac{dR_h}{dt} = \rho S_h + \gamma L_h + \tau J_h - \mu_h R_h \quad (5)$$

The D_h class is a representation of human dead bodies of those that die due to Ebola virus from both J_h and I_h classes. The compartment decreases due to proper burial at the rate ϕ . The human's natural mortality can take place in all the existing humans' classes (in exception of D_h) at the rate μ_h .

$$\frac{dD_h}{dt} = (\delta_1 + \mu_h)I_h + (\delta_2 + \mu_h)J_h - \phi D_h \quad (6)$$

The V class is a representation of the virus in the environment. The class is generated as a result of the virus shade by I_h , D_h and C_b in the environment denoted by α_1 , α_2 and α_3 respectively. The compartment diminishes due to natural death of virus ξ .

$$\frac{dV}{dt} = \alpha_1 I_h + \alpha_2 D_h + \alpha_3 C_b - \xi V \quad (7)$$

The N_b class is a representation of non-career bats. The class is increased as a result of constants recruitment at the rate Λ_b . They got infected and moved to C_b class by getting contacts with carrier bats in the C_b compartment and virus in the environment, given by the term $\lambda_b = \frac{\beta_4 C_b}{T_b} + \frac{\beta_5 V}{T_b}$ where β_4 and β_5 is rate of effective mingling of bats and environment to bats respectively.

$$\frac{dN_b}{dt} = \Lambda_b - (\lambda_4 + \lambda_5) N_b - (\delta_b + \mu_b) N_b \quad (8)$$

The C_b class is a representation of the Carrier bats who are capable of getting others infected. The class is increased by effective contact between N_b with C_b and V given by the term $\lambda_b = \frac{\beta_4 C_b}{T_b} + \frac{\beta_5 V}{T_b}$ as earlier explained. Both N_b and C_b classes diminish at the rates δ_b and μ_b due to human activities such as hunting and natural death respectively.

$$\frac{dC_b}{dt} = (\lambda_4 + \lambda_5) N_b - (\delta_b + \mu_b) C_b \quad (9)$$

The following assumptions and conditions are considered during the development of the system:

- Personal hygiene is enhancing by public enlightenment.
- Effective vaccinated and recovered individuals are permanently immune from the Ebola virus.

In synopsis the model consists of the following structure of nonlinear (deterministic) differential equations.

$$\begin{aligned}
 \dot{S}_h &= \Lambda_h - \lambda_h S_h - (\rho + \mu_h) S_h \\
 \dot{L}_h &= \lambda_h S_h - (\sigma + \gamma + q + \mu_h) L_h \\
 \dot{I}_h &= \sigma L_h - (\varphi + \delta_1 + \mu_h) I_h \\
 \dot{J}_h &= \varphi I_h - (\tau + \delta_2 + \mu_h) J_h \\
 \dot{R}_h &= \rho S_h + \gamma L_h + \tau J_h - \mu_h R_h \\
 \dot{D}_h &= (\delta_1 + \mu_h) I_h + (\delta_2 + \mu_h) J_h - \phi D_h \\
 \dot{V} &= \alpha_1 I_h + \alpha_2 D_h + \alpha_3 C_b - \xi V \\
 \dot{N}_b &= \Lambda_b - \lambda_b N_b - (\delta_b + \mu_b) N_b \\
 \dot{C}_b &= \lambda_b N_b - (\delta_b + \mu_b) C_b.
 \end{aligned} \tag{10}$$

where

$$T_h = S_h + L_h + I_h + J_h + R_h + D_h,$$

and

$$T_b = N_b + C_b.$$

3 Basic analysis of the model (10)

The total population of human T_h is giving by adding the first six equations of the model (10) gives

$$\frac{dT_h}{dt} = \Lambda_h - \mu_h (T_h - I_h - J_h) - \phi D_h, \text{ so that } T_h \rightarrow \frac{\Lambda_h}{\mu_h} \text{ as } t \rightarrow \infty. \text{ Thus, } \frac{\Lambda_h}{\mu_h} \text{ is an upper bound of}$$

$T_h(t)$ provided that $T_h(0) \leq \frac{\Lambda_h}{\mu_h}$. Furthermore, if $T_h(0) > \frac{\Lambda_h}{\mu_h}$, then $T_h(t)$ will decrease to this

level. Similarly, calculation for the bats equations shows that $T_b \rightarrow \frac{\Lambda_b}{\mu_b}$ as $t \rightarrow \infty$.

Thus, the following feasible region:

$$\Pi = \left\{ (S_h, L_h, Q_h, I_h, J_h, D_h, R_h, N_b, C_b) \in \mathbb{R}_+^9 : T_h \leq \frac{\Lambda_h}{\mu_h}, T_b \leq \frac{\Lambda_b}{\mu_b} \right\}$$

is positively invariant set under the flow described in (1). Hence, no solution path leaves through and boundary of Π . Also, since solution paths cannot leave Π , solutions remain non-negative for non-negative initial conditions. Solutions exist for all time t . In this region, the model (10) is said to be well posed mathematically and epidemiologically.

3.1 Existence of equilibria (E^*)

At any giving equilibrium state the rate of change of each variable is equal to zero. That is

$$\frac{dS_h}{dt} = \frac{dL_h}{dt} = \frac{dI_h}{dt} = \frac{dJ_h}{dt} = \frac{dR_h}{dt} = \frac{dD_h}{dt} = \frac{dV}{dt} = \frac{dN_b}{dt} = \frac{dC_b}{dt} = 0. \quad (11)$$

At any arbitrary equilibrium state, let

$$(S_h^*, L_h^*, I_h^*, J_h^*, R_h^*, D_h^*, V^*, N_b^*, C_b^*) = (S_h, L_h, I_h, J_h, R_h, D_h, V, N_b, C_b). \quad (12)$$

Thus, the state variables give

$$\begin{aligned} S_h^* &= \frac{\Lambda_h}{\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1}, \quad L_h^* = \frac{(\lambda_1^* + \lambda_2^* + \lambda_3^*)\Lambda_h}{K_2(\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1)}, \quad I_h^* = \frac{\sigma(\lambda_1^* + \lambda_2^* + \lambda_3^*)\Lambda_h}{K_2 K_3 (\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1)} \\ J_h^* &= \frac{\sigma\phi(\lambda_1^* + \lambda_2^* + \lambda_3^*)\Lambda_h}{K_2 K_3 K_4 (\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1)}, \\ R_h^* &= \frac{K_2 K_3 K_4 \rho \Lambda_h + K_3 K_4 \gamma \Lambda_h (\lambda_1^* + \lambda_2^* + \lambda_3^*) + \tau \sigma \phi \Lambda_h (\lambda_1^* + \lambda_2^* + \lambda_3^*)}{\mu_h K_2 K_3 K_4 ((\lambda_1^* + \lambda_2^* + \lambda_3^*) + K_1)} \\ D_h^* &= \frac{\sigma K_3 K_5 \Lambda_h (\lambda_1^* + \lambda_2^* + \lambda_3^*) + \sigma \phi K_6 \Lambda_h (\lambda_1^* + \lambda_2^* + \lambda_3^*)}{\phi K_2 K_3 K_4 (\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1)}, \quad N_b^* = \frac{\Lambda_b}{(\lambda_4^* + \lambda_5^* + K_7)} \\ C_b^* &= \frac{(\lambda_4^* + \lambda_5^*)\Lambda_b}{K_7 (\lambda_4^* + \lambda_5^* + K_7)}, \quad V^* = \frac{\left(\sigma K_7 \Lambda_h (\lambda_1^* + \lambda_2^* + \lambda_3^*) (\lambda_4^* + \lambda_5^* + K_7) (\alpha_1 \phi K_4 + \alpha_2 (K_3 K_5 + \phi K_6)) \right.}{\xi \phi K_2 K_3 K_4 K_7 (\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1) (\lambda_4^* + \lambda_5^* + K_7)} \\ &\quad \left. + \alpha_3 \phi K_2 K_3 K_4 \Lambda_b (\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1) (\lambda_4^* + \lambda_5^*) \right) \end{aligned}$$

where

$$\lambda_2^* = \frac{\beta_2 (1 - \varepsilon) V^*}{T_h^*} \quad (14)$$

$$\lambda_3^* = \frac{\beta_3 (1 - \varepsilon) C_b^*}{T_h^*} \quad (15)$$

$$\lambda_4^* = \frac{\beta_4 C_b^*}{T_b^*} \quad (16)$$

$$\lambda_5^* = \frac{\beta_5 V^*}{T_b^*} \quad (17)$$

substituting the state variable concern in (13), we have

$$\lambda_1^* = \frac{\frac{\sigma\beta_1(1-\varepsilon)\Lambda_h(\lambda_1^* + \lambda_2^* + \lambda_3^*)}{K_2 K_3 (\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1)} + \frac{\sigma\eta\beta_1(1-\varepsilon)\Lambda_h(K_3 K_5 + \phi K_6)(\lambda_1^* + \lambda_2^* + \lambda_3^*)}{\phi K_2 K_3 K_4 (\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1)}}{T_h^*} \quad (18)$$

simplifying (18), gives

$$(P_3 \lambda_1^* + P_3 \lambda_2^* + P_3 \lambda_3^* + P_3 K_1 - (P_1 + P_2)) \lambda_1^* - (P_1 + P_2) \lambda_2^* - (P_1 + P_2) \lambda_3^* = 0 \quad (19)$$

where $P_1 = \sigma\beta_1(1-\varepsilon)\Lambda_h\phi K_4$, $P_2 = \sigma\eta\beta_1(1-\varepsilon)\Lambda_h(K_3 K_5 + \phi K_6)$, $P_3 = \phi K_2 K_3 K_4 T_h^*$.

Substituting the state variable concern in (14), gives

$$\lambda_2^* = \frac{\left(\sigma\beta_2(1-\varepsilon)K_7\Lambda_h(\lambda_1^* + \lambda_2^* + \lambda_3^*)(\lambda_4^* + \lambda_5^* + K_7)(\alpha_1\phi K_4 + \alpha_2(K_3 K_5 + \phi K_6)) \right.}{\xi\phi K_2 K_3 K_4 K_7(\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1)(\lambda_4^* + \lambda_5^* + K_7)} \left. + \beta_2(1-\varepsilon)\alpha_3\phi K_2 K_3 K_4 \Lambda_b(\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1)(\lambda_4^* + \lambda_5^*) \right) \quad (20)$$

Simplifying (20), we have

$$(P_6 \lambda_2^* + P_6 \lambda_1^* + P_6 \lambda_3^* + P_6 K_1 - (P_4 + P_5)) \lambda_2^* - (P_4 + P_5) \lambda_1^* - (P_4 + P_5) \lambda_3^* - P_5 K_1 = 0, \quad (21)$$

where

$$P_4 = \sigma\beta_2(1-\varepsilon)K_7\Lambda_h(\lambda_4^* + \lambda_5^* + K_7)(\alpha_1\phi K_4 + \alpha_2(K_3 K_5 + \phi K_6))$$

$$P_5 = \beta_2(1-\varepsilon)\alpha_3\phi K_2 K_3 K_4 \Lambda_b(\lambda_4^* + \lambda_5^*), P_6 = \xi\phi K_2 K_3 K_4 K_7(\lambda_4^* + \lambda_5^* + K_7)$$

Substituting the state variable concern in (15), gives

$$\lambda_3^* = \frac{\beta_3(1-\varepsilon)(\lambda_4^* + \lambda_5^*)\Lambda_b}{T_h^* K_7 (\lambda_4^* + \lambda_5^* + K_7)}. \quad (22)$$

Substituting the state variable concern in (16), we have

$$\lambda_4^* = \frac{\beta_4(\lambda_4^* + \lambda_5^*)\Lambda_b}{T_b^*K_7(\lambda_4^* + \lambda_5^* + K_7)}. \quad (23)$$

Simplifying (23), gives

$$(T_b^*K_7\lambda_4^* + T_b^*K_7\lambda_5^* + T_b^*K_7K_7 - \beta_4\Lambda_b)\lambda_4^* - \beta_4\Lambda_b\lambda_5^* = 0 \quad (24)$$

Substituting the state variables concern into (17), we have

$$\lambda_5^* = \frac{\beta_5 \left(\sigma K_7 \Lambda_h (\lambda_1^* + \lambda_2^* + \lambda_3^*) (\lambda_4^* + \lambda_5^* + K_7) (\alpha_1 \phi K_4 + \alpha_2 (K_3 K_5 + \phi K_6)) \right.}{\xi \phi K_2 K_3 K_4 K_7 T_b^* (\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1) (\lambda_4^* + \lambda_5^* + K_7)} \quad (25)$$

Simplifying (25), we have

$$\lambda_5^* = \frac{(P_7 + P_8)\lambda_4^* + (P_7 + P_8)\lambda_5^* + P_7K_7}{P_9\lambda_4^* + P_9\lambda_5^* + P_9K_7} \quad (26)$$

$$(P_9\lambda_4^* + P_9\lambda_5^* + P_9K_7 - (P_7 + P_8))\lambda_5^* - (P_7 + P_8)\lambda_4^* - P_7K_7 = 0, \quad (27)$$

where

$$P_7 = \sigma\beta_5K_7\Lambda_h(\lambda_1^* + \lambda_2^* + \lambda_3^*)(\alpha_1\phi K_4 + \alpha_2(K_3K_5 + \phi K_6)),$$

$$P_8 = \beta_5\alpha_3\phi K_2K_3K_4\Lambda_b(\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1), P_9 = \xi\phi K_2K_3K_4K_7T_b^*(\lambda_1^* + \lambda_2^* + \lambda_3^* + K_1)$$

Consider a scenario where $\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^* > 0$, at endemic all the force of infection are positive.

Also, consider

$$\lambda_5^* = 0. \quad (28)$$

Substituting (28) into (24), gives

$$(T_b^*K_7\lambda_4^* + T_b^*K_7K_7 - \beta_4\Lambda_b)\lambda_4^* = 0. \quad (29)$$

Equation (29) gives

$$\lambda_4^* = 0 \quad (30)$$

or

$$T_b^*K_7\lambda_4^* + T_b^*K_7K_7 - \beta_4\Lambda_b = 0. \quad (31)$$

Simplifying (31), we have

$$\lambda_4^* = \frac{\beta_4 \Lambda_b - T_b^* K_7 K_7}{T_b^* K_7} \quad (32)$$

Considering the last model equation in system (1), it is easy to deduce that $K_7 < 1$, therefore $K_7 K_7$ will become smaller, which makes $\lambda_4^* > 0$.

$$\lambda_4^* > 0 \quad (33)$$

Substituting (28) and (30), into (22), gives

$$\lambda_3^* = 0 \quad (34)$$

Substituting (28) while $\lambda_5^* > 1$ into (22), we have

$$\lambda_3^* > 0 \quad (35)$$

if

$$\lambda_1^* = 0 \quad (36)$$

Substituting (28), (30), (34) and (36) into (21), gives

$$(P_6 \lambda_2^* + P_6 K_1 - (P_4 + P_5)) \lambda_2^* = 0 \quad (37)$$

either

$$\lambda_2^* = 0 \quad (38)$$

or

$$\lambda_2^* = \frac{(P_4 + P_5) - P_6 K_1}{P_6} \quad (39)$$

For λ_2^* to be greater than zero ($\lambda_2^* > 0$) it is very important to compare the values of P_4, P_5 and P_6 within mind that multiplication of numbers less than zero is smaller.

$$P_4 + P_5 - P_6 K_1 = P_4 = \sigma \beta_2 (1 - \varepsilon) K_7 K_7 \Lambda_h (\alpha_1 \phi K_4 + \alpha_2 (K_3 K_5 + \phi K_6)) - \xi \phi K_1 K_2 K_3 K_4 K_7 K_7 > 0 \quad (40)$$

with the above reason clearly $\lambda_2^* > 0$, therefore

$$\lambda_2^* > 0 \quad (41)$$

Substituting (28), (30), (34), (36) and (39) into (19), we have

$$(P_3 \lambda_1^* + P_3 K_1 - (P_1 + P_2)) \lambda_1^* = 0 \quad (42)$$

either

$$\lambda_1^* = 0 \quad (43)$$

or

$$P_3 \lambda_1^* + P_3 K_1 - (P_1 + P_2) = 0 \quad (44)$$

Simplifying (44), gives

$$\lambda_1^* = \frac{(P_1 + P_2) - P_3 K_1}{P_3} \quad (45)$$

For λ_1^* to be greater than zero ($\lambda_1^* > 0$) it is very important to compare the values of P_1, P_2 and P_3 within mind that multiplication of numbers less than zero is smaller.

$$P_1 + P_2 - P_3 = \sigma\beta_1(1-\varepsilon)\Lambda_h\phi K_4 + \sigma\eta\beta_1(1-\varepsilon)\Lambda_h(K_3 K_5 + \phi K_6) - \phi K_2 K_3 K_4 T_h^* \quad (46)$$

Therefore, based on the explanation above, is greater than zero. Thus

$$\lambda_1^* > 0. \quad (47)$$

Substituting (30), (34), (36), (39) and (43) into (27), gives

$$(P_9 \lambda_5^* + P_9 K_7 - P_8) \lambda_5^* = 0 \quad (48)$$

either

$$\lambda_5^* = 0$$

or

$$P_9 \lambda_5^* + P_9 K_7 - P_8 \quad (49)$$

Simplifying (49), we have

$$\lambda_5^* = \frac{\beta_5 \alpha_3 \phi K_1 K_2 K_3 K_4 \Lambda_b - \xi \phi K_1 K_2 K_3 K_4 K_7 K_7 T_b^*}{P_3} \quad (50)$$

for λ_5^* to be greater than zero, it is very important to consider the numerator of (50), $\xi K_7 K_7$ is far less than one because all the three parameters are less than one. Therefore,

$$\beta_5 \alpha_3 \phi K_1 K_2 K_3 K_4 \Lambda_b - \xi \phi K_1 K_2 K_3 K_4 K_7 K_7 T_b^* > 0 \quad (51)$$

Thus, we have

$$\lambda_5^* > 0 \quad (52)$$

Hence, considering (28), (30), (34), (36) and (39) the exist DFE, also considering (33), (35), (41), (47) and (52) there exist EEP.

3.2 Local stability of disease-free equilibrium (DFE)

There is no infection at disease-free equilibrium. Therefore, all the disease classes are zero at this point and population will reduced to non-Career bats and vaccinated individuals. The DFE of system (10) obtained by setting the right-hand side of the system (10) to zero is given by

$$E^0 = (S_h, L_h, I_h, J_h, R_h, D_h, V, N_b, C_b, T_h, T_b) = \left(\frac{\Lambda_h}{K_1}, 0, 0, 0, \frac{\rho \Lambda_h}{\mu_h K_1}, 0, 0, \frac{\Lambda_b}{K_8}, 0, \frac{\Lambda_h}{\mu_h}, \frac{\Lambda_b}{K_8} \right). \text{ Next generation}$$

operator method has been to prove the local stability of DFE (E^0) of system (10). Considering only infected classes ($L_h, I_h, J_h, D_h, V, C_b$) at DFE and taking into consideration the notation in Van den Driessche and Watmough (2002) the F and V matrices for those who are newly infected and those

remaining terms that are transferred from the infected compartments respectively, are respectively given as

$$F = \begin{pmatrix} 0 & \frac{\beta_1(1-\varepsilon)\mu_h}{K_1} & 0 & \frac{\eta\beta_1(1-\varepsilon)\mu_h}{K_1} & \frac{\beta_2(1-\varepsilon)\mu_h}{K_1} & \frac{\beta_3(1-\varepsilon)\mu_h}{K_1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_5 & \beta_4 \end{pmatrix} \quad (53)$$

and

$$V = \begin{pmatrix} K_2 & 0 & 0 & 0 & 0 & 0 \\ -\sigma & K_3 & 0 & 0 & 0 & 0 \\ 0 & -\phi & K_4 & 0 & 0 & 0 \\ 0 & -K_5 & -K_6 & \phi & 0 & 0 \\ 0 & -\alpha_1 & 0 & -\alpha_2 & \xi & -\alpha_3 \\ 0 & 0 & 0 & 0 & 0 & K_7 \end{pmatrix} \quad (54)$$

where $K_1 = \rho + \mu_h$, $K_2 = \sigma + \gamma + \mu_h$, $K_3 = \phi + \delta_1 + \mu_h$, $K_4 = \tau + \delta_2 + \mu_h$, $K_5 = \delta_1 + \mu_h$, $K_6 = \delta_2 + \mu_h$, $K_7 = \delta_b + \mu_b$

the product of (FV^{-1}) , gives

$$FV^{-1} = \begin{pmatrix} R_0^{hhv1} & R_0^{hhv2} & R_0^{hhv3} & R_0^{hhv4} & R_0^{hv} & R_0^{hvb} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ R_0^{bv1} & R_0^{bv2} & R_0^{bv3} & R_0^{bv4} & R_0^{bv5} & R_0^{bbv} \end{pmatrix} \quad (55)$$

where $R_0^{hhv1} = \frac{\sigma\beta_1(1-\varepsilon)\mu_h}{K_1K_2K_3} + \frac{\sigma\eta\beta_1(1-\varepsilon)\mu_h(\phi K_6 + K_4K_5)}{\phi K_1K_2K_3K_4} + \frac{\sigma\beta_2(1-\varepsilon)\mu_h(\phi\alpha_1K_4 + \phi\alpha_2K_6 + \alpha_2K_4K_5)}{\phi\xi K_1K_2K_3K_4}$,
 $R_0^{hhv2} = \frac{\sigma\beta_1(1-\varepsilon)\mu_h}{K_1K_3} + \frac{\eta\beta_1(1-\varepsilon)\mu_h(\phi K_6 + K_4K_5)}{\phi K_1K_3K_4} + \frac{\beta_2(1-\varepsilon)\mu_h(\phi\alpha_1K_4 + \phi\alpha_2K_6 + \alpha_2K_4K_5)}{\phi\xi K_1K_3K_4}$

$$\begin{aligned}
 R_0^{hhv3} &= \frac{\eta\beta_1(1-\varepsilon)\mu_h K_6}{\phi K_1 K_4} + \frac{\beta_2(1-\varepsilon)\mu_h \alpha_2 K_6}{\phi \xi K_1 K_4}, \quad R_0^{hhv4} = \frac{\eta\beta_1(1-\varepsilon)\mu_h}{\phi K_1} + \frac{\beta_2(1-\varepsilon)\mu_h \alpha_2}{\phi \xi K_1}, \quad R_0^{hv} = \frac{\beta_2(1-\varepsilon)\mu_h}{\xi K_1}, \\
 R_0^{hvb} &= \frac{\beta_2(1-\varepsilon)\mu_h \alpha_3}{\xi K_1 K_7} + \frac{\beta_3(1-\varepsilon)\mu_h}{K_1 K_7}, \quad R_0^{bv1} = \frac{\sigma\beta_5(\phi\alpha_1 K_4 + \phi\alpha_2 K_6 + \alpha_2 K_4 K_5)}{\phi \xi K_2 K_3 K_4}, \\
 R_0^{bv2} &= \frac{\beta_5(\phi\alpha_1 K_4 + \phi\alpha_2 K_6 + \alpha_2 K_4 K_5)}{\phi \xi K_3 K_4}, \quad R_0^{bv1} = \frac{\beta_5 \alpha_2 K_6}{\phi \xi K_4}, \quad R_0^{bv4} = \frac{\beta_5 \alpha_2}{\phi \xi}, \quad R_0^{bv5} = \frac{\beta_5}{\xi} \text{ and } R_0^{bbv} = \frac{\beta_5 \alpha_3}{\xi K_7} + \frac{\beta_4}{K_7}.
 \end{aligned}$$

Then the spectral radius is given by

$$\rho(FV^{-1}) = \max \left\{ \frac{R_0^{bbv} + R_0^{hhv1} + \sqrt{(R_0^{bbv} - R_0^{hhv1})^2 + 4R_0^{bv1}R_0^{hvb}}}{2}, \frac{R_0^{bbv} + R_0^{hhv1} - \sqrt{(R_0^{bbv} - R_0^{hhv1})^2 + 4R_0^{bv1}R_0^{hvb}}}{2} \right\} \quad (56)$$

$$R_c = \frac{R_0^{bbv} + R_0^{hhv1} + \sqrt{(R_0^{bbv} - R_0^{hhv1})^2 + 4R_0^{bv1}R_0^{hvb}}}{2} \quad (57)$$

The following result is established using Theorem 2 in Van den Driessche and Watmough (2002).

Lemma 1: The DFE of the system (10) is locally asymptotically stable if $R_c < 1$ and unstable if $R_c > 1$.

The epidemiological threshold R_c gives the average number of Ebola virus cases generated by the infected individual into an entirely susceptible population (Agusto *et al.*, 2017; Anderson and May, 1991; Andrawus *et al.*, 2017; Hethcot, 2000 and Van den Driessche and Watmough, 2002). The value called the threshold quantity R_c is the effective reproduction number for the model (10). The significance of Lemma 1 epidemiologically is that if R_c is less than unity, a small inflow of infected persons into the population would produce large outbreaks, and the disease dies out in time. However, the disease-free equilibrium might not be globally asymptotically stable even if $R_c < 1$ in the case of when backward bifurcation exists. That is, there is an existence of a stable EEP co-existing with the DFE.

3.3 GAS of DFE of model (10)

The stability of DFE at global level of the model (10) will be established. We claim the following result.

Theorem 1. The disease-free state of the model (10) is GAS whenever $R_c < 1$.

Proof: To prove the global stability of the disease-free equilibrium of model (10), we need to follow the approach in Castillo Chavez *et al.* (2002) that the two conditions (G1) and (G2) must be mate for $R_c < 1$. Let $X = (S_h, R_h, N_b)$ and $Y = (L_h, I_h, J_h, D_h, V, C_b)$ and separate model (10) into:

$$\frac{dX}{dt} = F(X, Y) \quad (58)$$

$$\frac{dY}{dt} = G(X, Y); G(X, 0) = 0 \quad (59)$$

with the components of $X \in R^3$ representing the uninfected compartment and the components $Y \in R^6$ representing the infected compartment.

The disease-free equilibrium is now represented as

$$E^0 = (X^*, 0) \quad (60)$$

where

$$X^* = (T_h^0 / T_b^0, 0) \quad (61)$$

Considering the first condition, that is global stability of X^* , gives

$$\frac{dX_1}{dt} = F(X_1, 0) = \begin{bmatrix} \Lambda_h - K_1 S_h^0 \\ \rho S_h^0 - \mu_h R_h^0 \\ \Lambda_b - \mu_b N_b^0 \end{bmatrix} \quad (62)$$

Now, using integrating factor (I.F)

$$S_h^0(t) = \frac{\Lambda_h}{K_1} - \frac{\Lambda_h}{K_1} e^{-K_1 t} + S_h^0(0) e^{-K_1 t} \quad (63)$$

$$R_h^0(t) = \frac{\rho S_h^0}{\mu_h} - \frac{\rho S_h^0}{\mu_h} e^{-\mu_h t} + R_h^0(0) e^{-\mu_h t} \quad (64)$$

$$N_b^0(t) = \frac{\Lambda_b}{\mu_b} - \frac{\Lambda_b}{\mu_b} e^{-\mu_b t} + N_b^0(0) e^{-\mu_b t}. \quad (65)$$

Now, obviously $S_h^0(t) + R_h^0(t) \rightarrow T_h^0(t)$ and $N_b^0 \rightarrow T_b^0$ as $t \rightarrow \infty$ regardless of the value of $S_h^0(0)$, $R_h^0(0)$ and $N_b^0(0)$. Thus, $X^* = (T_h^0 / T_b^0, 0)$ is a GAS.

Further, for the other criterion that is $\bar{G}(X, Y) = BY - G(X, Y)$ gives

$$B = \begin{pmatrix} -K_2 & \frac{\beta_1(1-\varepsilon)S_h^0}{T_h^0} & 0 & \frac{\eta\beta_1(1-\varepsilon)S_h^0}{T_h^0} & \frac{\beta_2(1-\varepsilon)S_h^0}{T_h^0} & \frac{\beta_3(1-\varepsilon)S_h^0}{T_h^0} \\ \sigma & -K_3 & 0 & 0 & 0 & 0 \\ 0 & \varphi & -K_4 & 0 & 0 & 0 \\ 0 & K_5 & K_6 & -\phi & 0 & 0 \\ 0 & \alpha_1 & 0 & \alpha_2 & -\xi & \alpha_3 \\ 0 & 0 & 0 & 0 & \frac{\beta_5 N_b^0}{T_b^0} & -K_7 + \frac{\beta_4 N_b^0}{T_b^0} \end{pmatrix} \quad (66)$$

The matrix (66) is M- matrix clearly (the off-diagonal element of B are non-negative).

$$\bar{G}(X, Y) = \begin{bmatrix} (\lambda_1 + \lambda_2 + \lambda_3)S_h^0 - K_2 L_h^0 \\ \sigma L_h^0 - K_3 I_h^0 \\ \varphi I_h^0 - K_4 J_h^0 \\ K_5 I_h^0 + K_6 J_h^0 - \phi D_h^0 \\ \alpha_1 I_h^0 + \alpha_2 D_h^0 + \alpha_3 C_b^0 - \xi V^0 \\ (\lambda_4 + \lambda_5)N_b^0 - K_7 C_b^0 \end{bmatrix} \quad (67)$$

then,

$$\bar{G}(X, Y) = [0, 0, 0, 0, 0, 0]^T. \quad (68)$$

It is thus clearly that $\bar{G}(X, Y) = 0$. Hence, the proof is complete.

Consider scenario where the endemic equilibrium exists.

3.4 Stability of endemic equilibrium at local level of model (10)

Theorem 2: The EEP of system (10) is LAS if the control reproduction number, $R_c > 1$.

Proof: Because of the high dimension of the system, standard linearization around the endemic equilibrium cannot be used. Therefore, the theory of Centre manifold which is clearly explained in Castillo-Chavez and Song (2004) for LAS analysis have been used in this work, in order to apply the theorem. Therefore, it is very important to make the following changes of variables. Let

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = (S_h, L_h, I_h, J_h, R_h, D_h, V, N_b, C_b) \quad (69)$$

Furthermore, using the vector notation

$$X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9)^T \quad (70)$$

The system (10) can be written in the form

$$\frac{dX}{dt} = F = (f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9)^T \quad (71)$$

such that

$$\begin{aligned} \dot{x}_1 &= \Lambda - \frac{\beta_1(1-\varepsilon)(x_3 + \eta x_6)}{T_h} x_1 - \frac{\beta_2(1-\varepsilon)x_7}{T_h} x_1 - \frac{\beta_3(1-\varepsilon)x_9}{T_h} x_1 - K_1 x_1 = f_1 \\ \dot{x}_2 &= \frac{\beta_1(1-\varepsilon)(x_3 + \eta x_6)}{T_h} x_1 + \frac{\beta_2(1-\varepsilon)x_7}{T_h} x_1 + \frac{\beta_3(1-\varepsilon)x_9}{T_h} x_1 - K_2 x_2 = f_2 \\ \dot{x}_3 &= \sigma x_2 - K_3 x_3 = f_3 \\ \dot{x}_4 &= \varphi x_3 - K_4 x_4 = f_4 \\ \dot{x}_5 &= \gamma x_2 + \rho x_1 + \tau x_4 - \mu_h x_5 = f_5 \\ \dot{x}_6 &= K_5 x_3 + K_6 x_4 - \phi x_6 = f_6 \\ \dot{x}_7 &= \alpha_1 x_3 + \alpha_2 x_6 + \alpha_3 x_9 - \xi x_7 = f_7 \\ \dot{x}_8 &= \Lambda_b - \frac{\beta_4 x_9}{T_b} x_8 - \frac{\beta_5 x_7}{T_b} x_8 - K_7 x_8 = f_8 \\ \dot{x}_9 &= \frac{\beta_4 x_9}{T_b} x_8 + \frac{\beta_5 x_7}{T_b} x_8 - K_8 x_9 = f_9. \end{aligned} \quad (72)$$

The Jacobian evaluated at DFE for the system (72) above is given by the Jacobian (73) as

$$J(E^0) = \begin{pmatrix} -K_1 & 0 & -\frac{\beta_1(1-\varepsilon)x_1^0}{T_h^0} & 0 & 0 & -\frac{\beta_1\eta(1-\varepsilon)x_1^0}{T_h^0} & -\frac{\beta_2(1-\varepsilon)x_1^0}{T_h^0} & 0 & -\frac{\beta_3(1-\varepsilon)x_1^0}{T_h^0} \\ 0 & -K_2 & \frac{\beta_1(1-\varepsilon)x_1^0}{T_h^0} & 0 & 0 & \frac{\beta_1\eta(1-\varepsilon)x_1^0}{T_h^0} & \frac{\beta_2(1-\varepsilon)x_1^0}{T_h^0} & 0 & \frac{\beta_3(1-\varepsilon)x_1^0}{T_h^0} \\ 0 & \sigma & -K_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varphi & -K_4 & 0 & 0 & 0 & 0 & 0 \\ \rho & \gamma & 0 & \tau & -\mu_h & 0 & 0 & 0 & 0 \\ 0 & 0 & K_5 & K_6 & 0 & -\phi & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & 0 & \alpha_2 & -\xi & 0 & \alpha_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\beta_5 & -K_7 & -\beta_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_5 & 0 & -K_7 + \beta_4 \end{pmatrix} \quad (73)$$

Suppose $\beta_5 = \beta_5^*$ is chosen as a bifurcation parameter at $R_c = 1$, we have that

$$\beta_5^* = \frac{\phi_5^\xi K_4 \left[\left(2 - (R_0^{bbv} + R_0^{hhv1}) \right)^2 - (R_0^{bbv} - R_0^{hhv1})^2 \right]}{4\alpha_2 K_6 R_0^{hvb}}. \quad (74)$$

Eigenvectors $J(E^0)_{\beta_5=\beta_5^*}$

Given W and V as a right and left eigenvectors associated with the zero eigenvalues of the Jacobian $J(E^0)_{\beta_5=\beta_5^*}$ (denoted by $J_{\beta_5^*}$) chosen such that $VJ(E^0)=0$ and $J(E^0)W=0$ with $VW=1$ where

$V = [v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9]$ and $W = [w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9]^T$. Thus,

$$v_1 = v_6 = v_8 = 0, \quad v_2 = v_4 > 0, \quad v_3 = \frac{\beta_1(1-\varepsilon)x_1^0}{T_h^0}v_2 + \phi v_2 + \frac{\beta_2(1-\varepsilon)x_1^0}{T_h^0}v_2 + \frac{\beta_3\beta_5(1-\varepsilon)x_1^0}{(K_7 - \beta_4)T_h^0}v_2$$

$$v_5 = \frac{K_4}{\tau}v_2, \quad v_7 = \frac{\beta_2(1-\varepsilon)x_1^0}{T_h^0}v_2 + \frac{\beta_3\beta_5(1-\varepsilon)x_1^0}{(K_7 - \beta_4)T_h^0}v_2, \quad v_9 = \frac{\beta_3\beta_5(1-\varepsilon)x_1^0}{(K_7 - \beta_4)T_h^0}v_2,$$

and

$$w_2 = w_2 > 0, \quad w_3 = \frac{\sigma}{K_3}w_2, \quad w_4 = \frac{\phi\sigma}{K_3K_4}w_2,$$

$$w_1 = - \left\{ \begin{aligned} & \frac{\sigma\beta_1(1-\varepsilon)x_1^0}{K_1K_3T_1^0} + \frac{\sigma\beta_1\eta(1-\varepsilon)x_1^0(K_4K_5 + K_6\phi)}{K_1K_3K_4T_1^0} \\ & + \frac{\beta_2(1-\varepsilon)x_1^0\xi(K_7 - \beta_4)}{K_1T_1^0(\xi(K_7 - \beta_4) - \alpha_3\beta_5)} \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\phi\sigma}{\xi\phi K_3K_4} \right) \\ & + \frac{\beta_2(1-\varepsilon)x_1^0\beta_5}{K_1T_1^0(\xi(K_7 - \beta_4) - \alpha_3\beta_5)} \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\phi\sigma}{\xi\phi K_3K_4} \right) \end{aligned} \right\} w_2,$$

$$w_5 = - \frac{\rho}{\mu_h} \left\{ \begin{aligned} & \frac{\sigma\beta_1(1-\varepsilon)x_1^0}{K_1K_3T_1^0} + \frac{\sigma\beta_1\eta(1-\varepsilon)x_1^0(K_4K_5 + K_6\phi)}{K_1K_3K_4T_1^0} \\ & + \frac{\beta_2(1-\varepsilon)x_1^0\xi(K_7 - \beta_4)}{K_1T_1^0(\xi(K_7 - \beta_4) - \alpha_3\beta_5)} \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\phi\sigma}{\xi\phi K_3K_4} \right) \\ & + \frac{\beta_2(1-\varepsilon)x_1^0\beta_5}{K_1T_1^0(\xi(K_7 - \beta_4) - \alpha_3\beta_5)} \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\phi\sigma}{\xi\phi K_3K_4} \right) \end{aligned} \right\} w_2 + \frac{\gamma}{\mu_h}w_2 + \frac{\tau\phi\sigma}{\mu_hK_3K_4}w_2,$$

$$w_6 = \frac{\sigma(K_4 K_5 + \phi K_6)}{\phi K_3 K_4} w_2, \quad w_7 = \frac{\xi(K_7 - \beta_4)}{(\xi(K_7 - \beta_4) - \alpha_3 \beta_5)} \left(\frac{\alpha_1 \sigma}{\xi K_3} + \frac{\alpha_2 K_5 \sigma}{\xi \phi K_3} + \frac{\alpha_2 K_6 \phi \sigma}{\xi \phi K_3 K_4} \right) w_2$$

$$w_8 = -\frac{\xi(K_7 - \beta_4)}{(\xi(K_7 - \beta_4) - \alpha_3 \beta_5)} \left(\frac{\beta_5}{K_7} + \frac{\beta_4 \beta_5}{K_7(K_7 - \beta_4)} \right) \left(\frac{\alpha_1 \sigma}{\xi K_3} + \frac{\alpha_2 K_5 \sigma}{\xi \phi K_3} + \frac{\alpha_2 K_6 \phi \sigma}{\xi \phi K_3 K_4} \right) w_2$$

$$w_9 = \frac{\beta_5}{(\xi(K_7 - \beta_4) - \alpha_3 \beta_5)} \left(\frac{\beta_5}{K_7} + \frac{\beta_4 \beta_5}{K_7(K_7 - \beta_4)} \right) \left(\frac{\alpha_1 \sigma}{\xi K_3} + \frac{\alpha_2 K_5 \sigma}{\xi \phi K_3} + \frac{\alpha_2 K_6 \phi \sigma}{\xi \phi K_3 K_4} \right) w_2$$

a and b computations.

For a, gives

$$a = \sum_{k,i,j=1}^n v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0,0). \quad (75)$$

For the system, gives $n=9$ and for $k=1,6,8$. $v_1 = v_6 = v_8 = 0 \Rightarrow a_1 = a_6 = a_8 = 0$, we therefore compute the associated non-zero partial derivatives of F at the DFE for f_2, f_3, f_4, f_5, f_7 and f_9 .

$$\frac{\partial^2 f_2}{\partial x_1^* \partial x_3^*}(0,0) = \frac{\partial^2 f_2}{\partial x_3^* \partial x_1^*}(0,0) = \frac{\beta_1^*(1-\varepsilon)}{T_h^0}, \quad \frac{\partial^2 f_2}{\partial x_1^* \partial x_6^*}(0,0) = \frac{\partial^2 f_2}{\partial x_6^* \partial x_1^*}(0,0) = \frac{\eta \beta_1^*(1-\varepsilon)}{T_h^0}$$

$$\frac{\partial^2 f_2}{\partial x_1^* \partial x_7^*}(0,0) = \frac{\partial^2 f_2}{\partial x_7^* \partial x_1^*}(0,0) = \frac{\beta_2^*(1-\varepsilon)}{T_h^0}, \quad \frac{\partial^2 f_2}{\partial x_1^* \partial x_9^*}(0,0) = \frac{\partial^2 f_2}{\partial x_9^* \partial x_1^*}(0,0) = \frac{\beta_3^*(1-\varepsilon)}{T_h^0},$$

$$\frac{\partial^2 f_9}{\partial x_7^* \partial x_8^*}(0,0) + \frac{\partial^2 f_9}{\partial x_8^* \partial x_7^*}(0,0) = \frac{\beta_4}{T_b^0}, \quad \frac{\partial^2 f_9}{\partial x_8^* \partial x_9^*}(0,0) = \frac{\partial^2 f_9}{\partial x_9^* \partial x_8^*}(0,0) = \frac{\beta_5}{T_b^0}.$$

Now, considering v 's, w 's and the partial derivative above, one can be able to show that

$$a_1 = a_3 = a_4 = a_5 = a_6 = a_7 = a_8 = a_9 = 0 \quad (76)$$

$$a = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = -(\Pi_1 \Pi_2 + P_1 P_2) < 0 \quad (77)$$

where

$$\Pi_1 = \left(\begin{aligned} & \frac{\sigma\beta_1(1-\varepsilon)x_1^0}{K_1K_3T_1^0} + \frac{\sigma\beta_1\eta(1-\varepsilon)x_1^0(K_4K_5 + K_6\varphi)}{K_1K_3K_4T_1^0} \\ & + \frac{\beta_2(1-\varepsilon)x_1^0\xi(K_7 - \beta_4)}{K_1T_1^0(\xi(K_7 - \beta_4) - \alpha_3\beta_5)} \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\varphi\sigma}{\xi\phi K_3K_4} \right) \\ & + \frac{\beta_2(1-\varepsilon)x_1^0\beta_5}{K_1T_1^0(\xi(K_7 - \beta_4) - \alpha_3\beta_5)} \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\varphi\sigma}{\xi\phi K_3K_4} \right) \end{aligned} \right) \quad (78)$$

$$\Pi_2 = \left(\begin{aligned} & \frac{2\varphi\sigma\beta_1(1-\varepsilon)}{K_3K_4T_h^0} + \frac{2\sigma(K_4K_5 + \varphi K_6)\eta\beta_1(1-\varepsilon)}{\phi K_3K_4T_h^0} \\ & + \frac{2\xi(K_7 - \beta_4)\beta_2(1-\varepsilon)}{(\xi(K_7 - \beta_4) - \alpha_3\beta_5)T_1^0} \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\varphi\sigma}{\xi\phi K_3K_4} \right) \\ & + \frac{\beta_5\beta_1(1-\varepsilon)}{(\xi(K_7 - \beta_4) - \alpha_3\beta_5)T_h^0} \left(\frac{\beta_5}{K_7} + \frac{\beta_4\beta_5}{K_7(K_7 - \beta_4)} \right) \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\varphi\sigma}{\xi\phi K_3K_4} \right) \end{aligned} \right) \quad (79)$$

$$P_1 = \frac{2\beta_3\beta_5(1-\varepsilon)x_1^0}{(K_7 - \beta_4)T_h^0} \frac{\xi(K_7 - \beta_4)\beta_4}{T_b^0(\xi(K_7 - \beta_4) - \alpha_3\beta_5)} \left(\frac{\beta_5}{K_7} + \frac{\beta_4\beta_5}{K_7(K_7 - \beta_4)} \right) \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\varphi\sigma}{\xi\phi K_3K_4} \right) v_2 w_2 \quad (80)$$

and

$$P_2 = \left(\begin{aligned} & \frac{\xi(K_7 - \beta_4)}{(\xi(K_7 - \beta_4) - \alpha_3\beta_5)} \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\varphi\sigma}{\xi\phi K_3K_4} \right) \\ & + \frac{\beta_5}{(\xi(K_7 - \beta_4) - \alpha_3\beta_5)} \left(\frac{\beta_5}{K_7} + \frac{\beta_4\beta_5}{K_7(K_7 - \beta_4)} \right) \left(\frac{\alpha_1\sigma}{\xi K_3} + \frac{\alpha_2K_5\sigma}{\xi\phi K_3} + \frac{\alpha_2K_6\varphi\sigma}{\xi\phi K_3K_4} \right) \end{aligned} \right) \quad (81)$$

equation (77) holds, since $v_2 > 0$, $w_2 > 0$ and all the parameters in the model are positive. Now, it is easy to conclude that parameter $a < 0$.

Similarly, for b given

$$b = \sum_{k,i=1}^n v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \beta_1^*}(0,0) \quad (82)$$

for $k = 1, 6, 8$. $v_1 = v_6 = v_8 = 0 \Rightarrow b_1 = b_6 = b_8 = 0$, we therefore compute the associated non-zero partial derivatives of F at the DFE for f_2, f_3, f_4, f_5, f_7 and f_9 where $\beta_5 = \beta_5^*$.

$$\frac{\partial^2 f_9}{\partial x_7 \partial \beta_5^*}(0,0) = 1 \quad (83)$$

Now, considering $v's, w's$ and the partial derivative above, one can also be able to show that (84) is equal to zero.

$$b_1 = b_2 = b_3 = b_4 = b_5 = b_6 = b_7 = b_8 = 0 \quad (84)$$

$$b_9 = v_9 w_7 \frac{\partial^2 f_2}{\partial x_7 \partial \beta_5^*}(0,0) \quad (85)$$

Substituting the values of $v's, w's$ and the partial derivative concerned, we have

$$b_9 = \frac{\beta_3 \beta_5 (1-\varepsilon) x_1^0}{(K_7 - \beta_4) T_h^0} \frac{\xi(K_7 - \beta_4)}{(\xi(K_7 - \beta_4) - \alpha_3 \beta_5)} \left(\frac{\alpha_1 \sigma}{\xi K_3} + \frac{\alpha_2 K_5 \sigma}{\xi \phi K_3} + \frac{\alpha_2 K_6 \phi \sigma}{\xi \phi K_3 K_4} \right) w_2 v_2. \quad (86)$$

Therefore

$$\begin{aligned} b &= b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8 + b_9 \\ &= \frac{\beta_3 \beta_5 (1-\varepsilon) x_1^0}{(K_7 - \beta_4) T_h^0} \frac{\xi(K_7 - \beta_4)}{(\xi(K_7 - \beta_4) - \alpha_3 \beta_5)} \left(\frac{\alpha_1 \sigma}{\xi K_3} + \frac{\alpha_2 K_5 \sigma}{\xi \phi K_3} + \frac{\alpha_2 K_6 \phi \sigma}{\xi \phi K_3 K_4} \right) w_2 v_2 > 0. \end{aligned} \quad (87)$$

Therefore, $a < 0$ and $b > 0$. So, by Theorem 2 of Castillo-Chavez and Song (2004) the following result is established. The EEP of the model (10) is LAS when $R_c > 1$.

3.5 Global stability of E^{**}

$$\text{Let } D_3 = \{(S_h, L_h, I_h, J_h, R_h, D_h, V, N_b, C_b) \in \Omega\} \quad (88)$$

be the stable manifold of the E^{**} .

Theorem 3: The endemic equilibrium of the model (10) is GAS in D_3 with $\delta_1 = \delta_2 = \rho = \gamma = 0$ whenever $R_c > 1$.

Proof: Suppose $R_c > 1$ and $\delta_1 = \delta_2 = \rho = \gamma = 0$ then the existence of the endemic equilibrium point is assured. Also, let the non-linear Lyapunov function (of the Goh-Volterra type) be

$$\begin{aligned} F &= \left(S_h - S_h^{**} - S_h^{**} \ln \frac{S_h}{S_h^{**}} \right) + \left(L_h - L_h^{**} - L_h^{**} \ln \frac{L_h}{L_h^{**}} \right) + \frac{(\sigma + \mu_h)}{\sigma} \left(I_h - I_h^{**} - I_h^{**} \ln \frac{I_h}{I_h^{**}} \right) \\ &+ \frac{(\sigma + \mu_h)(\phi + \mu_h)}{\sigma \phi} \left(J_h - J_h^{**} - J_h^{**} \ln \frac{J_h}{J_h^{**}} \right) + \frac{(\sigma + \mu_h)(\phi + \mu_h)(\tau + \mu_h)}{\sigma \phi \tau} \left(R_h - R_h^{**} - R_h^{**} \ln \frac{R_h}{R_h^{**}} \right) \\ &+ \left(N_b - N_b^{**} - N_b^{**} \ln \frac{N_b}{N_b^{**}} \right) + \left(C_b - C_b^{**} - C_b^{**} \ln \frac{C_b}{C_b^{**}} \right) \end{aligned} \quad (89)$$

Taking the time derivative of (89)

$$\begin{aligned} \dot{F} = & \left(\dot{S}_h - \frac{S_h^{**}}{S_h} \dot{S}_h \right) + \left(\dot{I}_h - \frac{I_h^{**}}{I_h} \dot{I}_h \right) + \frac{(\sigma + \mu_h)}{\sigma} \left(\dot{I}_h - \frac{I_h^{**}}{I_h} \dot{I}_h \right) + \frac{(\sigma + \mu_h)(\varphi + \mu_h)}{\sigma\varphi} \left(\dot{J}_h - \frac{J_h^{**}}{J_h} \dot{J}_h \right) \\ & + \frac{(\sigma + \mu_h)(\varphi + \mu_h)(\tau + \mu_h)}{\sigma\varphi\tau} \left(\dot{R}_h - \frac{R_h^{**}}{R_h} \dot{R}_h \right) + \left(\dot{N}_b - \frac{N_b^{**}}{N_b} \dot{N}_b \right) + \left(\dot{C}_b - \frac{C_b^{**}}{C_b} \dot{C}_b \right) \end{aligned} \quad (90)$$

with $T_h = \frac{\Lambda_h}{\mu_h}$, $\lambda_h = \frac{\beta_1(1-\epsilon)(I_h + \eta D_h)}{T_h} + \frac{\beta_2(1-\epsilon)V}{T_h} + \frac{\beta_3(1-\epsilon)C_b}{T_h}$

becomes

$$\begin{aligned} \tilde{\lambda}_h &= \beta_1(1-\epsilon) \frac{\Lambda_h}{\mu_h} (I_h + \eta D_h) + \beta_2(1-\epsilon) \frac{\Lambda_h}{\mu_h} V + \beta_3(1-\epsilon) \frac{\Lambda_h}{\mu_h} C_b \\ &= \tilde{\beta}_1(1-\epsilon)(I_h + \eta D_h) + \tilde{\beta}_2(1-\epsilon)V + \tilde{\beta}_3(1-\epsilon)C_b \end{aligned}$$

where $\tilde{\beta}_1 = \beta_1 \frac{\Lambda_h}{\mu_h}$, $\tilde{\beta}_2 = \beta_2 \frac{\Lambda_h}{\mu_h}$, $\tilde{\beta}_3 = \beta_3 \frac{\Lambda_h}{\mu_h}$

and

$$\tilde{\lambda}_b = \beta_4 \frac{\Lambda_b}{\mu_b} C_b + \beta_5 \frac{\Lambda_b}{\mu_b} V = \tilde{\beta}_4 C_b + \tilde{\beta}_5 C_b$$

where

$$\tilde{\beta}_4 = \beta_4 \frac{\Lambda_b}{\mu_b}, \quad \tilde{\beta}_5 = \beta_5 \frac{\Lambda_b}{\mu_b}$$

Substituting (10) in (49), we have

$$\begin{aligned} \dot{F} = & \Lambda_h - \tilde{\lambda}_h S_h - \mu_h S_h - \frac{S_h^{**}}{S_h} (\Lambda_h - \tilde{\lambda}_h S_h - \mu_h S_h) + \tilde{\lambda}_h S_h - (\sigma + \mu_h) L_h \\ & - \frac{L_h^{**}}{L_h} (\tilde{\lambda}_h S_h - (\sigma + \mu_h) L_h) + \frac{(\sigma + \mu_h)}{\sigma} (\sigma L_h - (\varphi + \mu_h) I_h) - \frac{(\sigma + \mu_h)}{\sigma} \frac{I_h^{**}}{I_h} (\sigma L_h - (\varphi + \mu_h) I_h) \\ & + \frac{(\sigma + \mu_h)(\varphi + \mu_h)}{\sigma\varphi} (\varphi I_h - (\tau + \mu_h) J_h) - \frac{(\sigma + \mu_h)(\varphi + \mu_h)}{\sigma\varphi} \frac{J_h^{**}}{J_h} (\varphi I_h - (\tau + \mu_h) J_h) \\ & + \frac{(\sigma + \mu_h)(\varphi + \mu_h)(\tau + \mu_h)}{\sigma\varphi\tau} (\tau J_h - \mu_h R_h) - \frac{(\sigma + \mu_h)(\varphi + \mu_h)(\tau + \mu_h)}{\sigma\varphi\tau} \frac{R_h^{**}}{R_h} (\tau J_h - \mu_h R_h) \\ & + \Lambda_b - \tilde{\lambda}_b N_b - (\delta_b + \mu_b) N_b - \frac{N_b^{**}}{N_b} (\Lambda_b - \tilde{\lambda}_b N_b - (\delta_b + \mu_b) N_b) \\ & + \tilde{\lambda}_b N_b - (\delta_b + \mu_b) C_b - \frac{C_b^{**}}{C_b} (\tilde{\lambda}_b N_b - (\delta_b + \mu_b) C_b) \end{aligned} \quad (91)$$

$$\Lambda_h = \tilde{\lambda}_h S_h^{**} + \mu_h S_h^{**} \quad (92)$$

$$\left. \begin{aligned} (\sigma + \mu_h)L_h^{**} &= \tilde{\lambda}_h S_h^{**} \\ (\varphi + \mu_h)I_h^{**} &= \sigma L_h^{**} \\ (\tau + \mu_h)J_h^{**} &= \varphi I_h^{**} \\ \mu_h R_h^{**} &= \tau J_h^{**} \end{aligned} \right\} \quad (93)$$

$$\tilde{\lambda}_b N_b = (\delta_b + \mu_b)C_b^{**} \quad (94)$$

$$\Lambda_b = \tilde{\lambda}_b N_b^{**} + (\delta_b + \mu_b)N_b^{**} \quad (95)$$

Substituting the relation in (92), (93), (94) and (95) into (91), we have

$$\begin{aligned} \dot{F} \leq & \mu_h S_h^{**} \left[2 - \frac{S_h}{S_h^{**}} - \frac{S_h^{**}}{S_h} \right] + (\sigma + \mu_h) L_h^{**} \left[6 - \frac{S_h^{**}}{S_h} - \frac{S_h L_h^{**}}{S_h^{**} L_h} - \frac{L_h I_h^{**}}{L_h^{**} I_h} - \frac{I_h J_h^{**}}{I_h^{**} J_h} - \frac{J_h R_h^{**}}{J_h^{**} R_h} - \frac{R_h}{R_h^{**}} \right] \\ & + (\delta_b + \mu_b) N_b^{**} \left[2 - \frac{N_b}{N_b^{**}} - \frac{N_b^{**}}{N_b} \right] + (\delta_b + \mu_b) C_b^{**} \left[3 - \frac{N_b^{**}}{N_b} - \frac{N_b C_b^{**}}{N_b^{**} C_b} - \frac{C_b}{C_b^{**}} \right] \end{aligned} \quad (96)$$

Since geometric mean is less than arithmetic mean, we have that

$$\left[2 - \frac{S_h}{S_h^{**}} - \frac{S_h^{**}}{S_h} \right] \leq 0, \quad \left[6 - \frac{S_h^{**}}{S_h} - \frac{S_h L_h^{**}}{S_h^{**} L_h} - \frac{L_h I_h^{**}}{L_h^{**} I_h} - \frac{I_h J_h^{**}}{I_h^{**} J_h} - \frac{J_h R_h^{**}}{J_h^{**} R_h} - \frac{R_h}{R_h^{**}} \right] \leq 0 \quad (97)$$

$$\left[2 - \frac{N_b}{N_b^{**}} - \frac{N_b^{**}}{N_b} \right] \leq 0, \quad \left[3 - \frac{N_b^{**}}{N_b} - \frac{N_b C_b^{**}}{N_b^{**} C_b} - \frac{C_b}{C_b^{**}} \right] \leq 0 \quad (98)$$

Thus, we have that $\dot{F} \leq 0$ for $\delta_1 = \delta_2 = \rho = \gamma = 0$, $R_c > 1$. Since the relevant variable in the equations for L_h, I_h, J_h, D_h, V and R_h are at endemic steady state, it follows that this can be substituted into the equations for L_h, I_h, J_h, D_h, V and R_h so that

$$\lim_{t \rightarrow \infty} (L_h(t), I_h(t), J_h(t), D_h(t), V(t), R_h(t)) \rightarrow (L_h^{**}, I_h^{**}, J_h^{**}, D_h^{**}, V^{**}, R_h^{**}). \quad (99)$$

Therefore, from Lasalle's invariant principle Lasalle, (1976), it implies that the endemic equilibrium E^{**} is globally asymptotically stable in Ω whenever $R_c > 1$. Hence, F is a Lyapunov function in Ω . The epidemiological implication of the result above is that, in the absence of vaccination and natural recovery $\rho = \gamma = 0$, Ebola virus cannot be eliminated from the society if the basic reproduction number is greater than unity.

4 Numerical Simulations

Simulations are performed in this section under maple software and MATLAB ODE 45 to know the sensitive parameters by numerical illustration and to detect the impact on the dynamics of EVD in the long-run. We will use the baseline parameters in Table (1) to simulate the system.

Table 1. Parameter values of system (1)

Parameters	Values (per day)	Sources
$\beta_1, \beta_2, \beta_3$	0.9, 0.7, 0.8	Abdulrahman, 2016
β_4, β_5	0.5, 0.6	Berge <i>et al.</i> , 2018
δ_1, δ_2	0.04227, 0.027855	Chowell <i>et al.</i> , (2004); Leung <i>et al.</i> , (2004); Safi and Gumel, 2011
δ_b	0.00014	Chowell <i>et al.</i> , (2004);
γ	0.03521	Leung <i>et al.</i> , (2004)
φ	(0,1)	variable
σ	0.1	Gumel <i>et al.</i> , 2014
τ	(0,1)	variable
η	0.25	Leung <i>et al.</i> , (2004)
ε	(0,1)	variable
$\alpha_1, \alpha_2, \alpha_3$	0.11, 0.21, 0.25	Assumed
ρ	(0,1)	variable
ϕ	(0,1)	variable
Λ_h	136	Safi and Gumel, 2011
Λ_b	10	Berge <i>et al.</i> , 2018
μ_h	0.0000351	Safi and Gumel, 2011
μ_b	0.0011	Berge <i>et al.</i> , 2018

Global asymptotic stability of equilibria

The global asymptotic stability of the disease-free equilibrium and endemic equilibrium of model (10) were analytically proved in Theorems 1 and 3, respectively. Hence, they are supported by Fig. 2 and Fig. 3 numerically, which are plotted for $\beta_1=0.9$, $\phi=0.04$ (Fig. 2) and $\beta_1=0.2$, $\phi=0.08$ (Fig.3). Specifically, Fig. 2 shows that when $R_c=0.4213$, EVD is wiped out after some days, while Fig.3 shows that for $R_c=3.2521$, EVD persists and becomes endemic in a society.

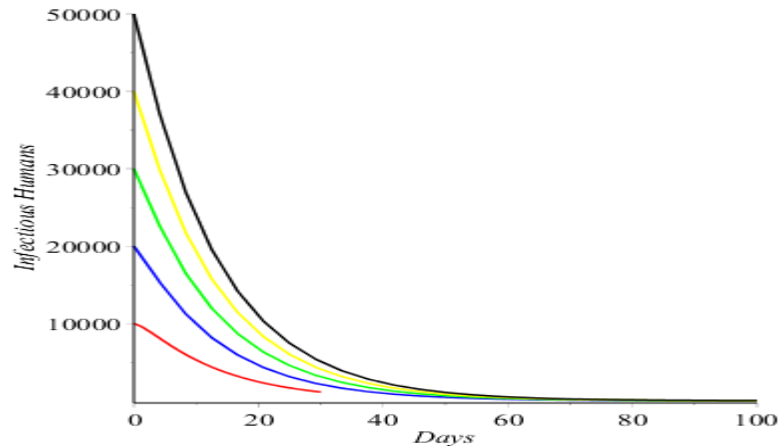


Figure 2. GAS of the disease-free equilibrium when $R_c=0.4213 < 1$. The solution curves are plotted with $\beta_1=0.9$, $\phi=0.04$, with different initial population of infectious humans and the remaining parameters are as in Table 1.

Figure 2 shows the GAS of the DFE, the solution curves are plotted with $R_c=0.4213 < 1$ and $\beta_1=0.9$, $\phi=0.04$. With different initial condition of Infectious human, no matter how large is the initial Infectious humans introduced into society whenever both the control reproduction numbers are less than unity the disease can be mitigated.

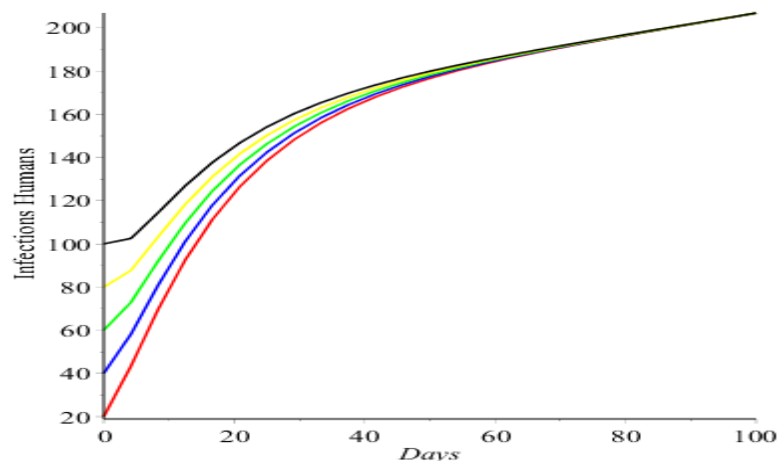


Figure 3. GAS of endemic equilibrium point when $R_c=3.2521 > 1$. The solution curves are plotted with $\beta_1=0.2$, $\phi=0.08$ and the remaining parameters are as in Table 1.

Figure 3 shows the global asymptotic stability of endemic equilibrium point, the solution curves are plotted with $\beta_1=0.2$, $\phi=0.08$ so that $R_c=3.2521 > 1$. With different initial condition of Infectious

human no matter how small is initial population of Infectious humans introduced into society whenever both the control reproduction numbers are greater than unity the disease cannot be mitigated in a society.

4.2 Assessing the impact of proper burial

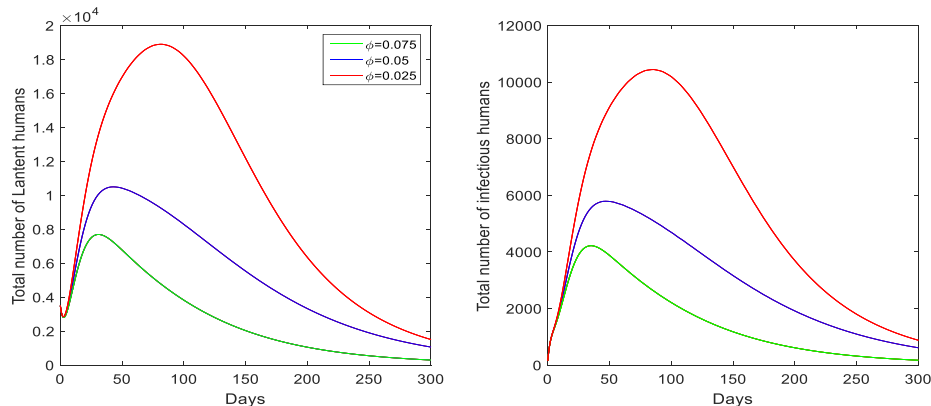


Figure 4. Assessing the impact of proper burial: The latent humans and infectious humans (non-hospitalized) as a function of proper burial rate.

Figure 4 shows that latent humans (E_h) and infectious humans (I_h) (non-hospitalized) are decreasing function of proper burial. Specifically, when the proper burial implementation is low, during the first forty days, the latent humans and infectious humans (non-hospitalized) increase very fast. After fifty days, the latent humans and infectious humans (non-hospitalized) decreases but the disease persist in the society (Note that $R_c = 1.0502$ in this case). If the proper burial is boosted to 75% ($\phi = 0.75$), then the disease drops down in short period of time but not totally wiped out: This shows that proper burial alone cannot mitigate the infection in the society.

4.3 Impact of personal hygiene

Figure 5 shows that latent humans (E_h) and infectious humans (I_h) (non-hospitalized) are decreasing function of personal hygiene. More precisely, when the personal hygiene implementation is low, during the first thirty days, the latent humans and infectious humans (non-hospitalized) increase very fast. After fifty days, the latent humans and infectious humans (non-hospitalized) numbers decreases but the disease continues in the society (Note that $R_c = 1.0502$ in this case). If the personal hygiene is boosted to 75% ($\varepsilon = 0.75$), then the disease diminishes in short period of time but not totally wiped out: This shows that personal hygiene alone cannot control the infection in the society but it can help.

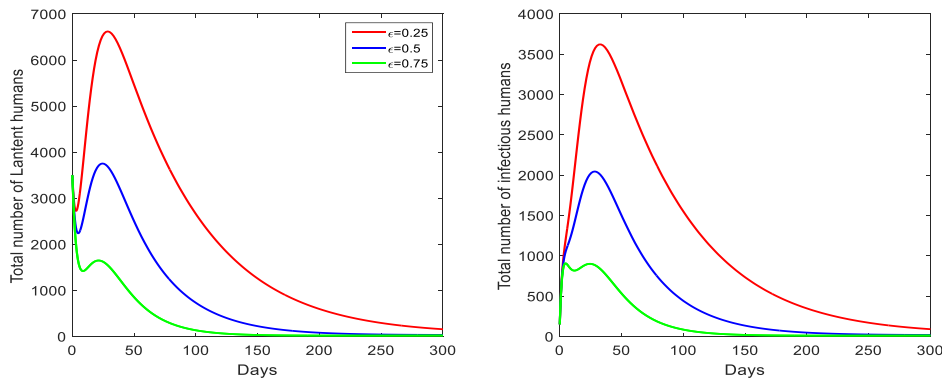


Figure 5. Impact of personal hygiene: The latent humans and infectious humans (non-hospitalized) as a function of personal hygiene.

4.4 Impact of isolation

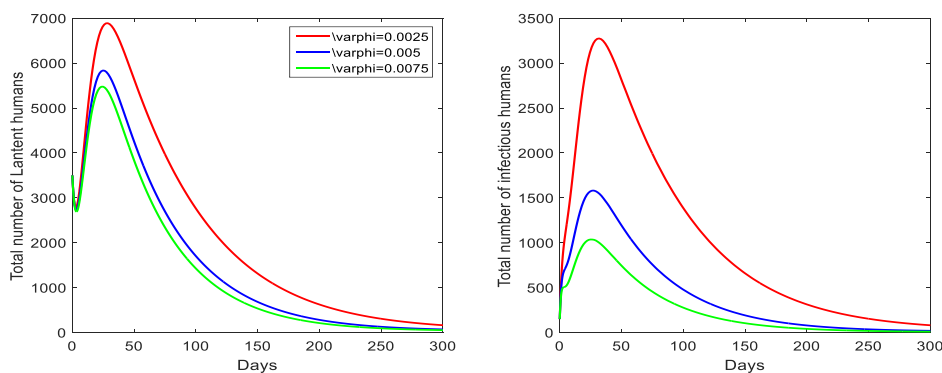


Figure 6. Impact of Isolation: The latent humans and infectious humans (non-hospitalized) as a function of Isolation rate.

Figure 6 show that isolation also can help in reducing the disease in a society but it cannot remove the disease completely from the society. The graph shows us that if the isolation rate is low the disease increases immensely in first fifty days but if it is high the disease decreases within short period of time.

4.5 Assessing the efficacy of vaccine

Without the loss of generality, we assumed the vaccination as parameter in this work. Figure 7 shows that the latent humans and infectious humans (non-hospitalized) are decreasing function of the vaccination parameter (ρ). To be specific, when the vaccination rate is minimal, the daily cases in both the two classes increases immensely in first twenty days. After fifty days, the number of infected ones decreases but EVD persists in the population (Note that $R_c = 1.0502$ in this case). If

the vaccination rate is boosted up to 0.75% of the population daily ($\rho = 0.0075$), then the disease will drop down and goes to extinction. So, a high rate of vaccination, that is vaccinating 0.75% of the population daily can mitigate the infection in the society.

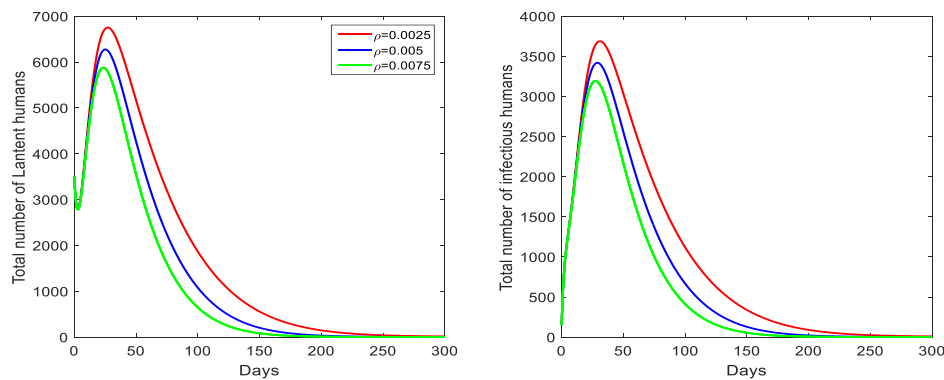


Figure 7. Assessing the efficacy of vaccine: The latent humans and infectious humans (non-hospitalized) as a function of vaccination rate.

4.6 Cumulative incidence varying different control parameters

In this sub-section we will consider variation of control parameter one after the other. Note that all the parameter values used in this sub-section are in Table 1 above with $R_c = 1.0502$.

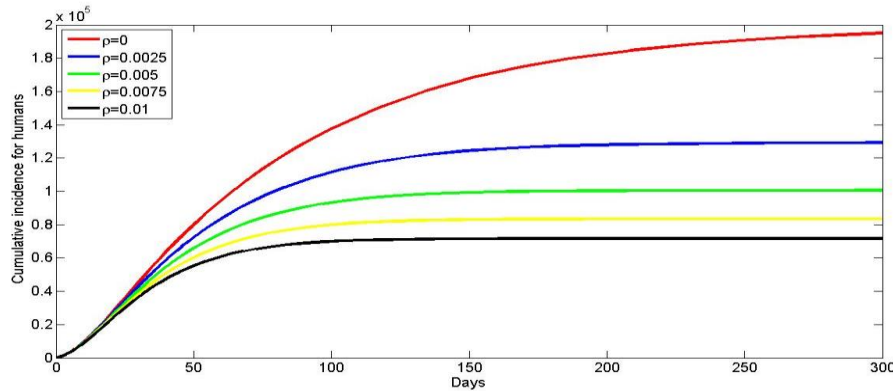


Figure 8. Cumulative incidence for humans varying vaccination rate.

Figure 8 is showing the cumulative incidence for humans varying vaccination rate, the graph reveals that when there is no vaccine the disease can escalate but if some certain percent of the population are vaccinated per day it will help to control the Ebola virus in the society.

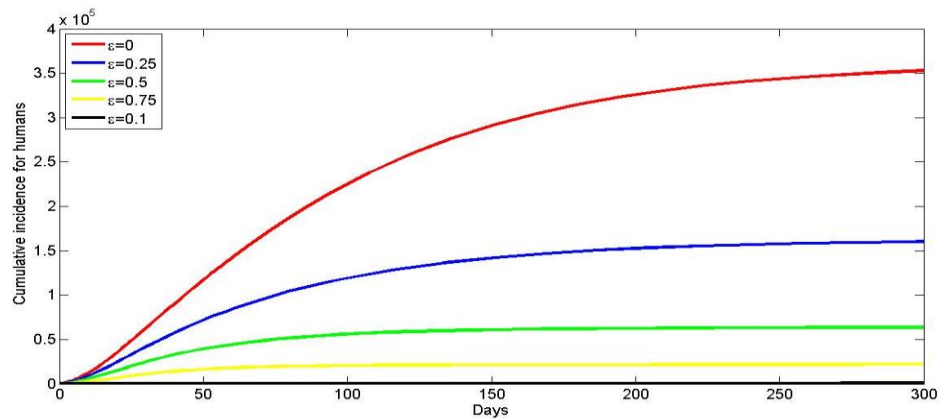


Figure 9. Cumulative incidence for humans varying personal hygiene.

Figure 9 is showing the cumulative incidence for humans varying personal hygiene rate, the graph reveals that when there is no personal hygiene control the disease can escalate within short period but if personal hygiene is adopted as a control strategy in the population, then the disease can be mitigated in a society.

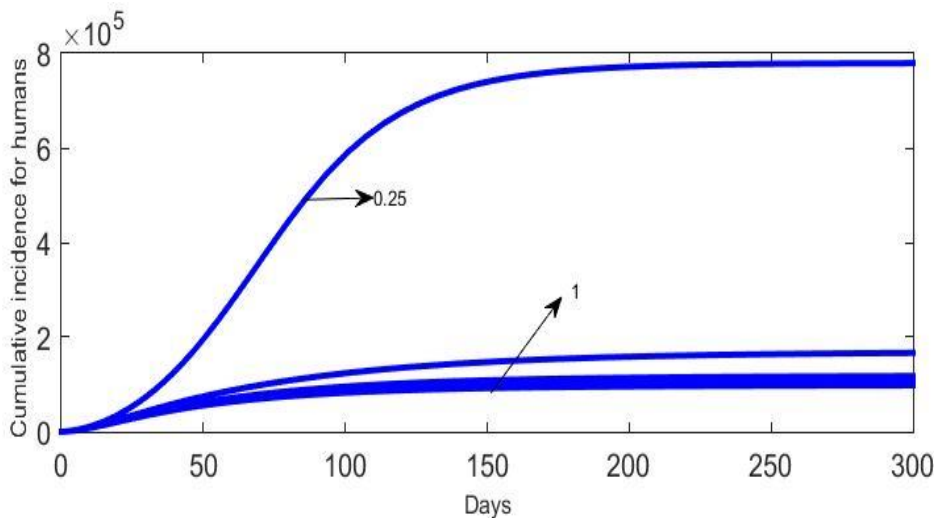


Figure 10. Cumulative incidence for humans varying isolation rate.

Figure 10 is showing the cumulative incidence for humans varying isolation rate, the graph reveals that when there is no isolation control in Infectious humans the disease can escalate within short period of time but if isolation is adopted as a control strategy in the population of infectious humans, then the disease can be mitigated in a society.

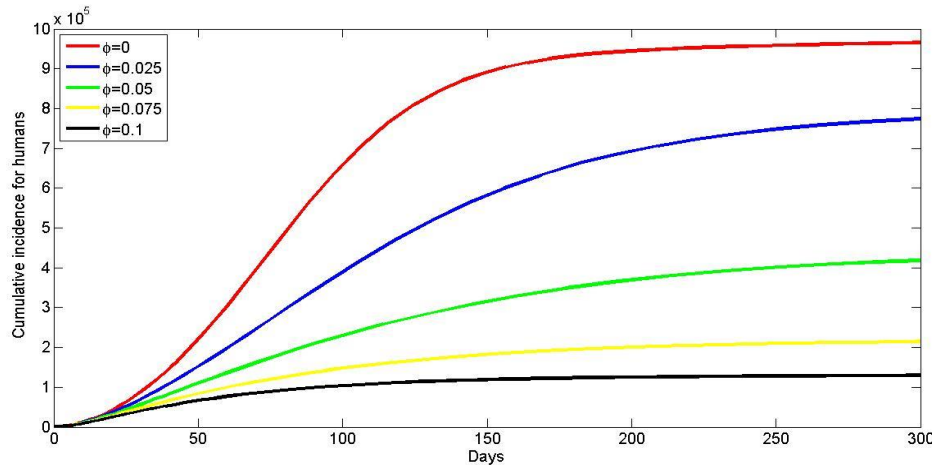


Figure 11. Cumulative incidence for humans varying proper burial rate.

Figure 11 is showing the cumulative incidence for humans varying proper burial rate, the graph reveals that when there is no proper burial control of deceased Infected humans the disease can escalate within short period of time but if proper burial is adopted as a control strategy in the population of humans, varying the proper burial rate shows significant changes and it also helps in controlling the disease in a society.

5 Summary and Conclusion

Over many years ago, series of EVD models were constructed and analyzed to investigate how the disease spread in human and between animals and humans, but only some few Authors considered vaccine and its influence on the EVD control. Again, only some few Authors considered bat to bat transmission. This article mainly concentrates on these strategies (which implies vaccination, personal hygiene, isolation and proper burial of EVD deceased humans), with in mind that hundred percent vaccination is not possible. In addition, (i) Some in latently infected humans L_h are assumed to recover naturally due to strong immune system, (ii) vaccinated are assumed to have permanent immunity from the Ebola virus, (iii) the efficacy of the vaccine is assumed to be almost hundred percent, (iv) the dead humans who are infected could transmit infection during funerals. In this case, the main results obtain are point out below.

1. However, this model considered threshold quantity very important, when the corresponding reproduction number is less than unity, the disease-free is a GAS. The meaning of this epidemiologically, no matter how few or large is the initial number of infectious individuals, if the reproduction number is less than unity, the disease will asymptotically drive to extinction. on the other hand, when the reproduction number is greater than unity, the DFE becomes unstable, the disease continuous and a GAS of EEP occurs.
2. Numerically, it was shown that: (1) the disease can be trash out if one percent of the population can be vaccinated daily. (2) the infected humans decrease in number as

vaccination rate increases. (3) Combining personal hygiene and proper burial leads to less infected individual.

The afore-mentioned imaginary and numerical results shows that: (a) vaccination alone is not capable of driving out the EVD to death. (b) Personal hygiene alone cannot be enough to drive out EV D in a society. Therefore, it is recommended to develop a model that will take into account the inherent to vaccine, though this will lead to delay differential equations, which can be more realistic but less mathematical tractable.

References

- [1] Abdulrahman, S. (2016). On the Modelling of Ebola: A Case Study Nigeria: *Abacus*, 43(2): 1-13.
- [2] Amenoghawan, C.O., Aboubakary, D. (2015). Modelling the transmission Dynamics of Ebola Virus. *Applied and Computational Mathematics*, 4(4): 313-320.
- [3] Anderson, R.M. and May, R.M. (1982). *Population Biology of Infectious Diseases*. New York, Springer-Verly.
- [4] Anderson, R.M. and May, R.M. (1991). *Infectious Diseases of Humans: Dynamics and Control*, Oxford, England: Oxford University Press.
- [5] Andrawus, J., Nwankwo, A. and Okuonghae, D. (2017). Bifurcation Analysis of the mathematical model for TB-Dengue co-infection. *Nigeria Research Journal of Engineering and Environmental Sciences*, 2(2): 390-407.
- [6] Azuaba, E., Akinwande, N.I., Abdulrahman, S. and Kuta, F.A. (2017). Existence of equilibrium points of the mathematical model of ebola virus disease dynamics incorporating infection age structure in the quarantined compartment with treatment. 13(4): 58-69.
- [7] Berge, T., Chapwanya, J., Lubuma, M.S. and Terefe, Y.A. (2018). A mathematical model for Ebola epidemic with self-protection measures. *Journal of Biological Systems*, 26(1): 107 - 131.
- [8] Birmingham, K. and Cooney, S. (2002), Ebola: Small but real progress (News feature)), *Nature med*, 8(4): 313.
- [9] Castillo-Chavez, C. x Espinoza, C., Moreno, V., Bichara. D. (2004) Assessing the efficiency of condom sanitive as a control strategy Ebola. *xiv:150:07415vi*.
- [10] Castillo-Chavez, C., Feng, Z., and Huang, W. (2004). On the computation of R_0 and its role on global stability. *Mathematical approaches for emerging and re-emerging infectious diseases. An Introduction. The IMA Volumes in Mathematics and its applications*. New York: Springer-Verlag 125: 229-250
- [11] Centres for Disease Control. (2019) West Africa Outbreak. Retrieved from <http://www.cdc.gov/vhf/Ebola/outbreak/2019-westafrica/previous-updates.html>
- [12] Chowell, G., Castillo-Chavez, C. , P. Fenimore, C. Kribs-Zaleta, L. Arriola, J. Hyman, (2004) Model parameters and outbreak control for SARS, *Emerg. Infect. Dis.*, 10:1258–1263
- [13] Eric Okyere, Johnson De-Graft Ankamah, Anthony Kodzo Hunkpe, and Dorcas Mensah, (2020). Deterministic Epidemic Models for Ebola Infection with Time-Dependent Controls, *Hindawi Discrete Dynamics in Nature and Society*. <https://doi.org/10.1155/2020/2823816>.
- [14] Espinoza, B., Moreno, V., Bichara, D. and Castillo-Chavez, C. (2015). Assessing the Efficiency of Condom Sanitive as a Control Strategy Ebola. *xiv:150:07415vi*.
- [15] Deepaa, O.S., Sravanthi Nallamallib, L. Nikitha, Singh Naikc, GopiVenkata and Sai Tejad,

- (2015., Mathematical Model for Transmission of Ebola, Science Direct, Elsevier. Procedia Computer Science, 48: 741–745.
- [16] Gumel, A.B. et al., (2014) Modelling strategies for controlling SARS outbreaks. Proc. R. Soc.Ser. B271: 2223–2232
- [17] Hayman, D.T.S. (2016). “Bats as Viral Reservoirs”, Annual Review of Virology, 3:77–99.
- [18] Herick L.K., Estomih S.M., Makinde D.O., and Immanuel, L.E. (2020). Modelling and optimal control of Ebola virus disease in the presence of treatment and quarantine of infectives, International Journal of Systems Science and Applied Mathematics, 5(4): 43 - 53.
- [19] Hethcot, H.W. (2000). The Mathematics of Infectious Diseases. Society for Industrial and Applied Mathematics Review. 42(4): 599-653.
- [20] Hussain H., Ali, A., Chaudary A., Jarral F., Yasmeen T., and Chaudary, F. (2017). Numerical solution for mathematical model of Ebola virus. M Int. J. Adv. Res., 5(3): 15321538.
- [21] Kalu, A.U., Agbonyim, A.N. and Agwu, A. (2016). A mathematical model for the control of the spread of Ebola virus disease in west Africa- a disease free equilibrium approach. British Journal of Mathematics and Computer Science, 12(5): 1-9.
- [22] La Salle, J. (1976). The stability of Dynamical systems. In Proceedings of the CBMS-NSF Regional Conference Series in Applied Mathematics 25, SIAM, Philadelphia, Pa, USA.
- [23] Lekone, P.E. and Finkenstadt, B.F. (2006). Statistical inference in stochastic epidemic seir model with control intervention: Ebola as a case study. Biometrics. 62: 1170-1177.
- [24] Leroy, E.M. B. Kumulungui, X. Pourrut, et al., (2005). Fruit bats as reservoirs of Ebola virus. Nature. 438(7068): 575-576.
- [25] Leung, G., et al. (2004). The epidemiology of severe acute respiratory syndrome in the 2003 Hong Kong epidemic: analysis of all 1755 patients. Ann. Intern. Med., 9: 662–673.
- [26] Murtaza, M., Yusof, I.M., Kassim, M., Jeffree, M. S., Ellzam, E.M. and Shariffa, A.M. (2016). Ebola virus disease, management and prevention. 15(4): 142-148.
- [27] Nourridine S., Bradford G., and Fefferman N.H. (2020). Mathematical model of the role of asymptomatic infection in outbreaks of some. emerging pathogens. Tropical Medicine and Infectious Disease. Trop. Med. Infect. Dis. 71: 185.
- [28] Okuonghae, D. and Ikhimwin, B.O. (2016). Dynamics of mathematical model for tuberculosis with variability in susceptibility and disease progressions due to difference in awareness level. Front. Micro. Biol., 6:1530-1538.
- [29] Okuonghae, D. and Omosigho, S.E. (2011). Analysis of mathematical model for tuberculosis: what could be done to increase case detection. Journal of Theoretical Biology, 269:31-45.
- [30] Pourrut, X., Souris, M., Towner, J.S. et al. (2009). Large serological survey showing cocirculation of Ebola and Marburg viruses in Gabonese bat populations, and a high seroprevalence of both viruses in Rousettus aegyptiacus. BMC Infectious Disease 9: 1659.
- [31] Rachah, A. and Torres, F.M. (2017). Analysis simulation and optimal control of a SEIR model for Ebola virus with demographic effects. Comm. Fac. Uni. Ank. Ser. A1 Maths. Stat. xiv:1705.01079v1[q-bioPE]
- [32] Safi, M. and Gumel A.B. (2011). Mathematical analysis of a disease transmission model with quarantine, isolation and an imperfect vaccine. Computer and Mathematics Application, 61:3044–3070.
- [33] Tae Sug Do and Young S.L. (2016). Modeling the spread of Ebola. Osong Public Health Res Perspect., 71:43-48.
- [34] Tsanou, B., Samuel, B., Jean, L. and Martins, L.M.M. (2018). Modelling Ebola virus disease transmissions in a complex virus life ecology. Mathematical Biosciences and Engineering,

- 15(5): 21-56.
- [35] Ustun, C. and Ozguller O. (2015). Ebola: As an infectious disease and as a potential bioterrorism agent. *Turk J Med Sci. and Applications*, 35: 1-4.
- [36] van den Driessche, P., and Watmough, J. (2002). Reproduction number and sub threshold Endemic Equilibrium for compartmental models of disease transmission. *Mathematical Biosciences*, 180:29-48.
- [37] World Health Organization (WHO) (2014). International Travel on Health. Ebola Virus Disease outbreak in West Africa. Travel and transport Risk Assessments: Recommendations for Public Health Authorities and Transport Sector. Available at: <http://www.who.int/ith/update/2014042/en/>.
- [38] World Health Organization (WHO) (2019). Fact sheet general information on Ebola virus disease. Controlling the Infection WHO response. <https://www.who.int/ebola>.
- [39] World Health Organization. (2019). Facts Sheet General Information on Ebola Virus, Geneva World Health Organization. Geneva. <http://www.who.int/watersanitationhealth/publication/technotes/en/>.
- [40] Zineb E.L., Rhoubari, Hajar B., Khalid H., and Noura Y. (2018). Mathematical modeling of Ebola virus disease in bat population. *Hindawi Discrete Dynamics in Nature and Society*, Article ID 5104524.