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The spread of covid-19 disease and the health of Nigeria

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Abstract

COVID-19 is a major cause of death till date, claiming many lives in Nigeria and around the world. Nigeria was classified as one of the countries in Africa that had COVID-19 disease in 2020. February 27,2020, was when the first confirmed case of the novel coronavirus was reported in Nigeria. This was another terrible time when the health of Nigerians was under another serious attack apart from corruption and other economic challenges facing the country. In this study, Mathematical models are the major tools used for studying the transmission dynamics of covid-19. In this journal, we are considering a model on the spread of covid-19 disease and the health of Nigeria and analysed. the equilibrium states of the model which are the disease-free equilibrium states and the endemic equilibrium states were obtained and based on the Routh-Hurwitz criterion, the eigenvalues model equations have negative real parts, that is if and only if all the eigenvalues are negative, then the condition for stability is satisfied.

Keywords: COVID-19; spread; disease; health; Nigeria

1 Introduction

COVID-19 is an infectious disease caused by the SARS-CoV-2 virus. People who are infected with COVID-19 will experience some symptoms and recover without special treatment attention depending on their level of immunity. People with mild symptoms who are healthy can manage their symptoms at home [11].

However, some who become seriously ill require some medical treatment. The virus can spread from an infected person's mouth or nose in small liquid particles when they cough, sneeze, speak, sing or breathe. These particles range from larger respiratory droplets to smaller aerosols. You can be infected by breathing in the virus if you are near someone who has COVID-19, or by touching a contaminated surface and then your eyes, nose or mouth. The virus spreads more easily indoors and in crowded settings [14].

Nigeria was classified as one of the countries in Africa that had Covid-19 disease in 2020 [8]. Meanwhile, February 27, 2020, was when the first confirmed case of the novel coronavirus was reported in Nigeria. It was the case of a 44-yearold Italian citizen who arrived the Murtala Muhammed

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international Airport, Lagos at 10pm on January 24, 2020 aboard Turkish airline from Milan, Italy and proceeded to his company on February 25, 2020 and was presented at the staff clinic in Ogun state on February 26, 2020 and to Lagos because he was suspected with high index by the Managing Physician who later referred and him to the IDH Lagos on 27, 2020 where he was confirmed to have the disease [6]. Since then, the Federal Government has put in place several measures through the Presidential Task Force (PTF-COVID-19) working with the Federal Ministry of Health to curtail the spread and protect the health of Nigerians [8]. From 25th -29th June 2022, 332 new confirmed cases were recorded in Nigeria. Till date, 257290 cases have been confirmed, 250229 cases have been discharged and 3,144 deaths have been recorded in 36 states and the Federal Capital Territory. The 332 new cases are reported from 6 States- Lagos (251), Rivers (50), FCT (21), Kano (7), Delta (2) and Oyo (1) [9]. From 10th to 12th September 2022, 167 new confirmed cases and 1 death was recorded in Nigeria. To date, 264,617 cases have been confirmed, 257,880 cases have been discharged and 3,155 deaths have been recorded in 36 states and the Federal Capital Territory [9]. The 167 new cases are reported from 10 States-Edo (84), Lagos (55), Rivers (11), Kano (5), Cross River (4), Adamawa (3), Delta (2), Ekiti (1), FCT (1) and Kaduna (1). A multi-sectoral national emergency operations center (EOC), activated at Level 2, continues to coordinate the national response activities [9].

Researchers and Scientists have laboured tirelessly to bring some solutions and controls which include the use of facemask, lockdown and other measures employed to curb the COVID-19 rapid spread in Nigeria and around the world.

Mathematical models which help to gain a better insight into real life situations like this COVID-19, have been developed to study the transmission and control of the disease both in Nigeria and around the world by some researchers [4]. Some researchers who have done some major work on the transmission dynamics of COVID-9 Disease to mention but a few among them: Calistus et al. (2020), in their study, "Mathematical assessment of the impact of non-pharmaceutical interventions on curtailing the 2019 novel Coronavirus emphasized the important of social-distancing in curtailing the burden of COVID-19. Also, they found out that an increase in the adherence level of social-distancing protocols, which resulted in dramatic reduction of the burden of the pandemic, and the timely implementation of social-distancing measures in numerous states of the US, which may have averted a catastrophic outcome with respect to the burden of COVID-19. Using face-masks in public (including the low efficacy cloth masks) is very useful in minimizing community transmission and burden of COVID-19, provided their coverage level is high. The masks coverage needed to eliminate COVID-19 decreases if the masks-based intervention is combined with the strict social-distancing strategy [5]. Abba et al (2021) in their journal "A primer on using mathematical COVID-19 dynamics: Modeling, analysis and simulations for understanding the dynamics of COVID-19 [2].

The pandemic has a high mortality rate and currently endemic around the world, having claimed thousands of lives here in Nigeria and other countries [13]. Manuel (2020) in his study, "Modeling behavioural change and COVID-19 containment in Mexico: A trade-off between lockdown and compliance" fitted his model to the available data. The initial phase of the epidemic, from February 17th to March 23rd, 2020, is used to estimate the contact rates, infectious periods and mortality rate using both confirmed cases (by date of symptoms initiation), and daily mortality. Data on deaths after March 23rd, 2020 is used to estimate the mortality rate after the mitigation measures are implemented [7]. Adesoye and Ilyas, on "mathematical model of COVID-19 in Nigeria with optimal control" says that PTF, NCDC, and FMOH can use facemask, hand sanitizers along with social distancing to control the disease [12].

2 Model formulation

A mathematical modelling for the transmission dynamics of COVID-19 disease and the health of Nigeria is considered and formulated in this work. The population is divided into five (5) compartments, namely: Susceptible S(t), Latent L(t), Infected I(t), Quarantined $\mathrm{Q(t)}$ Recovered R(t).

The Total population is

$$\mathrm{N(t) = S(t) + L(t) + I(t) + Q(t) + R(t)} \tag{2.1}$$

This is the schematic diagram describing the model (3.1) - (3.5) below:

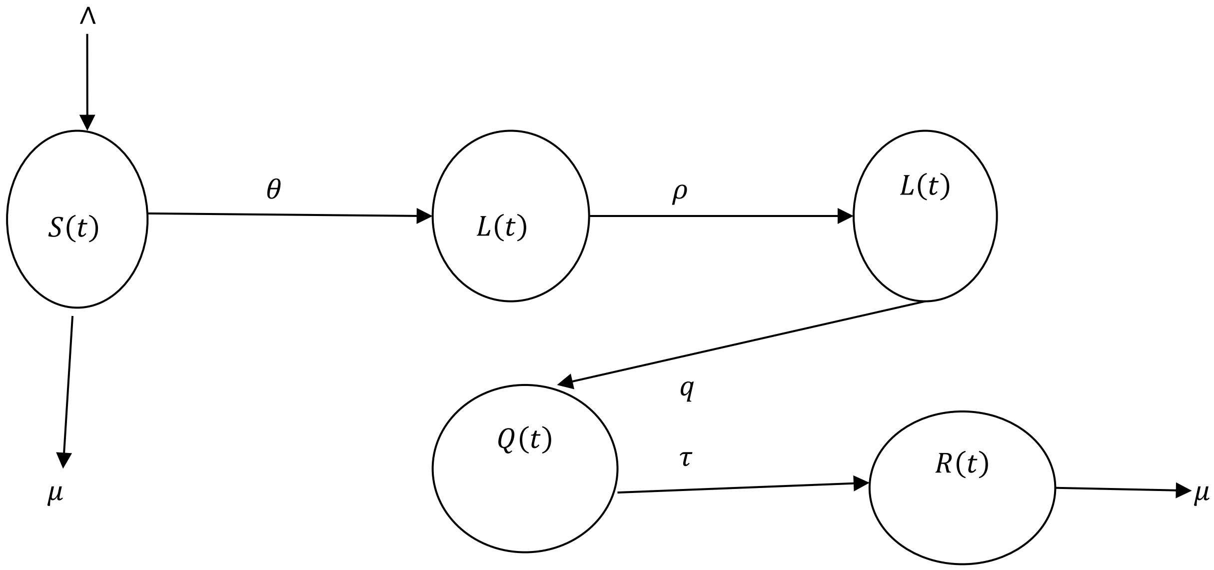

Figure 2.1: Schematic diagram of Covid-19 transmission dynamics model.

The following assumptions were made to formulate the model:

- The mixing of people is homogeneous, meaning that all individuals have equal chance of getting infected if they come in adequate contact with infectious individuals.
- Those in $\mathrm{S(t)}$ get infected through contact with $\mathrm{I(t)}$.
- $\mathrm{L(t)}$ are infected but not yet infectious, since they get infectious, only when they are symptomatic.
- The isolation of $\mathrm{I(t)}$ into $\mathrm{Q(t)}$ cause the spread of covid-19 to be very low at a treatment rate τ.
- $\delta < \sigma$ due to the treatment of $\mathrm{Q(t)}$ at the rate τ .

2.1 Variables and parameter of the model

The variables are defined as follows:

Table 1: Variables

S/N	VARIABLE	DESCRIPTION
1.	$S(t)$	Susceptible class at time t
2.	. $L(t)$	Exposed class at time t
3.	$I(t)$	Infectious class at time t
4.	$Q(t)$	Quarantined class at time t
5.	$R(t)$	Recovered class at time t

The Parameters are defined as follows:

TABLE 2: Parameters

S/N	Parameter	Definition
1	Λ	birth rate
2	μ	death rate
3	δ	disease induced death rate of $I(t)$
4	σ	disease induced death rate of $Q(t)$
5	$\theta(1-f)$	effective contact rate between $I(t)$ and $S(t)$
6	p	progression rate from $L(t)$ to $I(t)$
7	q	rate of quarantine
8	τ	treatment rate
9	f	rate of facemask prevention for covid-19

3 Model equations

From the schematic diagram 2.1, the model equations (3.1) – (3.5) are given below

$$\frac{dS(t)}{dt} = \Lambda - \frac{\theta I(t)S(t)(1-f)}{N} - \mu S(t) + fS(t) \tag{3.1}$$

$$\frac{dL(t)}{dt} = \frac{\theta I(t)S(t)(1-f)}{N} - (\rho + \mu - f)L(t) \tag{3.2}$$

$$\frac{dI(t)}{dt} = \rho L(t) - (q + \mu + \delta)I(t) \tag{3.3}$$

$$\frac{dQ(t)}{dt} = qI(t) - (\tau + \mu - f + \sigma)Q(t) \tag{3.4}$$

$$\frac{dR(t)}{dt} = (\tau + f)Q(t) - \mu R(t) \tag{3.5}$$

$$\frac{dN(t)}{dt} = (f + \mu)S(t) + (f - \mu)L(t) - (\mu + \delta)I - (\sigma + \mu)Q(t) + \mu R(t) \tag{3.6}$$

3.1 Positivity of solutions

The model (3.1) to (3.6) is epidemiologically and mathematically well posed in the domain Ω with the initial conditions.

Let $\Omega = (S, \quad L, \; I, \quad Q, \quad R) \in R^5$ (3.7)

$$S + L + I + Q + R \leq N$$

3.1.1 Properties of the Model

To analyze the model (3.1) to (3.6) we consider the following theorem

Theorem 1: The solutions of the model equations (3.1) to (3.6) are positive for all time $t \geq 0$ provided that the initial conditions are positive.

Proof:
Employing the given assumptions, that all initial conditions are positive, i.e.

$$S(0) > 0, \quad L(0) > 0, \quad I(0) > 0, \quad Q(0) > 0, \quad R(0) > 0. \tag{3.8}$$

We have by contradiction that the solutions of (3.1) to (3.5) are positive if we assume for a contradiction that there exists first time,

$$t_1 : S(t_1) = 0$$
and $S(t) > 0, \quad L(t) > 0, \quad I(t) > 0, \quad Q(t) > 0, \quad R(t) > 0$ (3.9)
$$0 < t < t_1$$

or there exists

$$t_2 : L(t_2) = 0$$
and$S(t) > 0, \quad L(t) > 0, \quad I(t) > 0, \quad Q(t) > 0, \quad R(t) > 0$ (3.10)
$$0 < t < t_2$$

or there exists

$$t_3 : I(t_3) = 0$$
and$S(t) > 0, \quad L(t) > 0, \quad I(t) > 0, \quad Q(t) > 0, \quad R(t) > 0$ (3.11)
$$0 < t < t_3$$

or there exists

$$t_4 : Q(t_4) = 0$$
and$S(t) > 0, \quad L(t) > 0, \quad I1(t) > 0, \quad Q(t) > 0, \quad R(t) > 0, \quad I2(t) > 0$ (3.12)
$$0 < t < t_4$$

or there exists

$$t_5 : R(t_5) = 0$$
and $S(t) > 0, \quad L(t) > 0, \quad I(t) > 0, \quad Q(t) > 0, \quad R(t) > 0$ (3.13)
$$0 < t < t_5$$

or there exists

$$t_6 : I(t_6) = 0$$
and $S(t) > 0, \quad L(t) > 0, \quad I(t) > 0, \quad Q(t) > 0, \quad R(t) > 0$ (3.14)
$$0 < t < t_6$$

Now, in the case where

$S(t) = 0$ (3.15)

gives,

$$\frac{dS(t_1)}{dt} = \lim_{t=t_1} \frac{S(t_1)-S(t)}{t_1-t} < 0 \qquad (3.16)$$

and similarly, gives

$$\frac{dL(t_2)}{dt} < 0, \quad \frac{dI(t_3)}{dt} < 0, \quad \frac{dQ(t_4)}{dt} < 0, \quad \frac{dR(t_5)}{dt} < 0 \qquad (3.17)$$

However, from equations (3.1) to (3.5) gives,

$$\frac{dS(t_1)}{dt} = \Lambda - \frac{\theta I(t)S(t)(1-f)}{N} - \mu S(t) > 0 \qquad (3.18)$$

i.e.

$$S^i(t_1) = \Lambda > 0 \qquad (3.19)$$

which contradicts (3.17). Therefore, $S(t_1) \neq 0$, and Swill remain positive for all t.

Similarly, for the remaining variables gives

$$L^i(t_2) = \frac{\theta I(t)S(t)(1-f)}{N} + \beta L(t) > 0 \qquad (3.20)$$

$$I^i(t_3) = \rho L > 0 \qquad (3.21)$$

$$Q^i(t_4) = qI > 0 \qquad (3.22)$$

$$R^i(t_5) = (\tau + f)Q > 0 \qquad (3.23)$$

$$I^i = \rho L(t) - (q + \mu + \delta)I(t) \quad > 0. \qquad (3.24)$$

These are contradictions of what was supposed for each of the variables, meaning that $L(t_2) \neq 0$, , $Q(t_4) \neq 0$, $R(t_5) \neq 0$, and $I(t_6) \neq 0$. Hence, S, L, I, Q, R and remain positive for all t . By this, it is showed that all the solutions of (3.1) to (3.6) are inR^5, provided that the initial conditions are positive. Thus the feasible region is positively- invariant. It is hence, sufficient to consider the dynamics of the model (3.1) to (3.6) in the region Ω.

3.2 Boundedness of solution

The model (3.1) to (3.8) is epidemiologically and mathematically well posed in the domain Ω with the initial conditions.

Let $\quad \Omega = (S, \quad L, \quad I, \quad Q, \quad R, \quad D) \in R^5$

$S \geq 0, \quad L \geq 0, \quad I \geq 0, \quad Q \geq 0, \quad R \geq 0, \quad D \geq 0,$
$S + L + I + Q + R \leq N.$

Theorem 2: All solutions$(S(t), L(t), I(t), R(t), Q(t)$ of Model are bounded

Proof: Model 1
From the five populations we obtain

$$.\frac{d(S+L+I+Q+R)}{dt} = \Lambda - (f+\mu)S(t) + (f-\mu)L(t) - (\mu+\delta)I - (\sigma+\mu)Q(t) + (f+\mu)R(t)$$

$.\frac{d(S+L+I+Q+R)}{dt} = \wedge - (S+L+R)f + (S+L+I+R)\mu - \delta I - \sigma Q(t)$
$\leq \wedge - \mu(S+L+I+R).$

Then $\lim_{t\to\infty}(S+L+I+R) \leq \frac{\wedge}{\mu}$.

3.3 Existence of Equilibrium States E

Let $E(S(t), L(t), I(t), R(t), Q(t))$ be the equilibrium states of the model system.
At equilibrium state, the rate of change of each variable is equal to zero.
i.e. $\frac{dS}{dt} = \frac{dL}{dt} = \frac{dI1}{dt} = \frac{dQ}{dt} = \frac{dR}{dt} = 0.$ (3.25)

Let

$$\big(S(t), L(t), I\,(t), Q(t), R(t)\big) = (v, w, x, y, z), \quad (3.26)$$
$$N(t) = n \quad (3.27)$$
where
$$n = v + w + x + y + z. \quad (3.28)$$

Hence, equations (3.1) to (3.8) become

$$\wedge - \frac{\theta vx(1-f)}{n} - \mu v + fv = 0\,, \quad (3.29)$$

$$\frac{\theta vx(1-f)}{n} - (\rho + \mu - f)w = 0, \quad (3.30)$$

$$\rho w - (q + \mu + \delta)x = 0, \quad (3.31)$$

$$qx - (\tau + \mu + \sigma - f)y = 0\,, \quad (3.32)$$

$$(\tau + f)y - \mu z = 0. \quad (3.33)$$

The equilibrium states are then obtained by solving equations (3.30) to (3.36).

3.3.1 Disease-Free Equilibrium E_0

At the disease free equilibrium state E_0, there is absence of infection, thus the infected classes are zero. The whole population comprise of the susceptible class and so we have

$$v = \frac{\Lambda}{\mu} \quad (3.32)$$

Adding equations (3.7) and (3.8) gives

$$w = \frac{\Lambda}{\rho - \mu}. \quad (3.33)$$

Hence all other variables, x, y and z, are given below as:

$$x = \frac{\Lambda\rho}{(\rho-\mu)(q+\mu+\delta)}, \tag{3.34}$$
(3.34)

$$y = \frac{q\,\Lambda\rho}{(\rho-\mu)(q+\mu+\sigma)(\tau+\mu+\sigma)}, \tag{3.35}$$

$$z = \frac{\tau q\,\Lambda\rho}{\mu(\rho-\mu)(q+\mu+\sigma)(\tau+\mu+\sigma}. \tag{3.36}$$

Therefore, the disease-free equilibrium (DFE) state is given by:

$$E_0 = (v, w, x, y, z) = \left(\frac{\Lambda}{\mu}, 0,0,0,0\right). \tag{3.37}$$

3.3.2 Endemic Equilibrium State

From equations (3.32) to (3.36), we have the endemic equilibrium states of the model.

The endemic Equilibrium is given by

$$E_1 = (v, w, x, y, z) = \left(\frac{\Lambda}{\mu}, \frac{\Lambda}{\rho-\mu}, \frac{\Lambda\rho}{(\rho-\mu)(q+\mu+\delta)}, \frac{q\,\Lambda\rho}{(\rho-\mu)(q+\mu+\sigma)(\tau+\mu+\sigma)}, \frac{\tau q\,\Lambda\rho}{\mu(\rho-\mu)(q+\mu+\sigma)(\tau+\mu+\sigma}\right) \tag{3.38}$$

3.3.3 Stability Analysis

From (3.1) to (3.5) we obtained the Jacobian matrix given by,

$$\mathbf{J} = \begin{bmatrix} -\mu - \frac{\theta x(1-f)}{n} & 0 & \frac{\theta v(1-f)}{n} & 0 & 0 \\ \frac{\theta x(1-f)}{n} & -(\rho-\mu) & \frac{\theta v(1-f)}{n} & 0 & 0 \\ 0 & \rho & -(q+\mu+\sigma) & 0 & 0 \\ 0 & 0 & q & -(\tau+\mu+\sigma) & 0 \\ 0 & 0 & 0 & \tau & -\mu \end{bmatrix} = 0 \tag{3.39}$$

3.3.2 Local stability of Disease-Free Equilibrium $\mathbf{E_0}$

We used the Jacobian stability technique of determining the local stability of the system. Consider the Jacobian matrix of (3.39). At disease free equilibrium, E_0 is given by:

$$J = \begin{bmatrix} -\mu & 0 & \frac{\theta v(1-f)}{n} & 0 & 0 \\ \frac{\theta x(1-f)}{n} & -(\rho-\mu) & \frac{\theta v(1-f)}{n} & 0 & 0 \\ 0 & \rho & -(q+\mu+\sigma) & 0 & 0 \\ 0 & 0 & q & -(\tau+\mu+\sigma) & 0 \\ 0 & 0 & 0 & \tau & -\mu \end{bmatrix} = 0 \quad (3.39)$$

Thus from (3.39), the eigenvalues are given by

$$\lambda = -\mu < 0 \quad (3.40)$$
$$\lambda = -(\rho-\mu) < 0 \quad (3.42)$$
$$\lambda = -(q+\mu+\sigma) < 0 \quad (3.43)$$
$$\lambda = -(\tau+\mu+\sigma) < 0 \quad (3.44)$$
$$\lambda = -\mu < 0. \quad (3.45)$$

4 Result

The condition for stability using the Routh Hauwitz criteria is that λ_i where $i = 1,2,3,4,5$ negative real parts. Therefore from (3.41) to (3.45),

λ_1 is negative if $\lambda_1 = -\mu < 0$
λ_2 is negative if $\lambda_2 = -(\rho-\mu) < 0$ implies that $\mu < \rho$
λ_3 is negative since $\lambda_3 - (q+\mu+\sigma) < 0$ which implies that $\mu+\sigma < q$
λ_4 is negative since $\lambda_4 = -(\tau+\mu+\sigma) < 0$ which implies that $\tau > \mu+\sigma$
λ_5 is negative since $\lambda_5 = -\mu < 0$

4.2 Discussion of result

$\tau > \mu + \sigma$ implies that the inequality (3.44) holds and so the condition for stability of the disease free equilibrium state is locally asymptotically stable, which means that the treatment rate τ, is greater than the natural death rate μ and the disease induced death rate [1].

4.3 Conclusion

Covid-19 epidemic has shown that in the absence of a strong public health care delivery system, millions of lives even a rare disease can risk the lives of millions of people. The crux of this epidemic is that a large scale and coordinated international response is the need of the hour to support Nigerians and all who are at-risk, the need for the nation to intensify her response activities and strengthen national capacities.

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