International Journal of Mathematical Analysis and Modelling

(Formerly Journal of the Nigerian Society for Mathematical Biology)

Volume 5, Issue 4 (Nov.), 2022

ISSN (Print): 2682 - 5694
ISSN (Online): 2682 - 5708

International Journal of Mathematical Analysis and Modelling
Volume 5, Issue 4, November 2022, Pages 38 – 49

Mathematical modelling of skilling fishery management for sustainable development of an economy

C. Nkuturum[*] and I. George[†]

Abstract

This study considered a non-linear mathematical model of skilling fishery management to understand the dynamics of lives in an aquatic system which involves two zones of unrestricted and restricted fishing zones. The model analysis showcased the existence of equilibrium points in line with the biological and bionomic, dynamical behaviour of equilibrium points and also derived the conditions for stability and instability of the system. The behaviour of this dynamical model of fishery management were illustrated using numerical simulations to establish the analytical results. The findings of this study showed that inadequate knowledge of the resource and ecosystem, lack of interaction between the government and stakeholders, inaccessibility of financial resources to meet specific needs, weak implementation of agricultural policies and programmes, inadequate practical advisory/extension services and lack of monitoring and evaluation of programme/project team led the system to be unstable which implies that the eigen values were greater than zero. This paper identified that management of fisheries is affected by climate change, human actions and population, shortage of food, poverty and lack of policy executions. Amongst all, the study suggested that there should be introduction of environmental impact assessment for fisheries policies to be implemented, skilled or trained fisheries administrators that will monitor and evaluate the policy/programme consistency, against specified goals need to be given more financial resources, and properly staffed and equipped to address effectively all aspects of sustainable fisheries development and management. should be adopted and their efficacy monitored and evaluated against specified goals. There should be better protection to environmental hazards, control of waste disposal and pollution. Finally, government at all levels should recognize its benefit and work towards integrating skilling fisheries management activities, including sharing of expertise and resources for education, research, technology, monitoring, control and surveillance activities and development of the relevant legislative framework.

Keywords: skilling; fishery; zones; management; sustainable development; economy; model

[*] Corresponding Author. Department of Mathematics, Rivers State University, Port Harcourt, Nigeria; Email: christienkuturum245@gmail.com

[†] Department of Mathematics/Statistics, Ignatius Ajuru University of Education, Port Harcourt, Nigeria

1 Introduction

The sustainable fisheries development and management is an integration of bio-ecological, technological, environmental, economic and social scopes to bring about and sustainably improve the welfare of all the people engaged directly or indirectly in the fisheries sector as well as the natural productive system. Thus, sustainable development of the fisheries sector in any nation has become of greatest importance to ensure adequate food at affordable prices and as the main source of economic and social progress for the rural poor and the world at large. Sustainable development emphasizes the need to organize and control the dynamics and the complex interactions between man, production activities, and natural resources in order to promote their co-existence and their common evolution. However, this development must integrate the triple nethermost line framework captured in FAO (1995). In Nigeria, there are fisheries laws and policies that brought about in the interim increased fish production. Nevertheless, policy shifts and unsustainable implementation have led to decline of the fisheries sector over the years leaving the target rural economies more vulnerable than ever. Sustainable fisheries development can only be achieved through regulated fishing, which considers fishery management objectives and proper enforcement of regulations.

Nigeria is reasonably endowed with large rivers, small water bodies and some natural springs. It also has an extensive coastline of approximately 900km and an exclusive economic zone of about 217,313 km² (Sea Around Us, 2007). Nigeria has a high demand for fish of about 1.5 million metric tons thus, resulting in a demand-supply gap of about a million metric tons and a per capital consumption of 7.5-8.5kg annually (FDF, 2005). Nkuturum and George (2021) in their work on ecological growth model in marketing dynamics: the role of human actions stated that human action increases the demand-supply of goods in the market.

Thus, fisheries are crucial to the Nigerian economy in contributing about 5.4% of the gross domestic product (FAO, 1995). Although contribution of the fisheries sector is essential to agricultural development in Nigeria to ensure adequate food at affordable prices and as the main source of economic and social progress for the rural poor (Berezansky, et al. 2011). Also, more than half of policies and programmes in agriculture development focus on forest matters while less emphasis is placed on food and animal production requires immediate attention. For fisheries, a useful definition would be the one agreed at FAO in 1988 which states that sustainable development has been defined by the World Commission on Environment and Development (1987) as development that meets the needs of the present generation without compromising the ability of future generations to meet.

Fisheries Management systems in a nation comprises of three methods as the traditional systems: which focuses on operation by the administration of traditional authorities that enforce regulations to control fishing. This pin points on practices such as ban on fishing at certain time of the year or during fish breeding period. The mixed systems: involve the participation of both the traditional and modern government administrations. Finally, the modern systems: includes those operated by the administrations of the central government where fisheries regulations are enforced by agriculture officers of the fisheries departments (Biswas, et al. 2017). Thus, the issues confronting the management of the fisheries are: environmental Change which goes to climatic patterns and man's activities, exogenous factors address human population, poverty and food demand and weak policies and policy implementations.

Efficient skilling and sustainable fisheries management of natural resources, its control measure depends mainly on the essential understanding the mechanisms of their evolution over time where mathematical modelling in terms of non-linear differential equations (ODEs) can play a significant role. Hence, mathematical modelling is broadly used to discuss different types of real phenomena which leads to design better prediction, prevention, management and control strategies. The epitome of this study are the empirical studies done by the following scientists as in Biswas *et al.* (2017) who studied on the potential impacts of global climate change on fish production in Bangladesh. Followed Nkuturum (2021) studied on distortion of ecological systems by human actions: a mathematical model of two coexisting species in a community.

Also, Clark (2016) developed a mathematical model to describe the optimal management of renewable resources. Das, et al. (2009) showed the bioeconomic harvesting of a prey-predator fishery. De Lara and Doyen (2008) discussed on the sustainable management of natural resources: mathematical modelling and methods. Also, Dubey, et al. (2003) carried out a work on a model for fishery resources with reserve area for more knowledge on the applications of mathematical modelling.

Previously, many researchers presented their mathematical models as to optimize the skilling of fish production and management. Mathematical modelling is the use of equations, expressions and mathematics language to translate and proffer solutions to real life situation problems from an application area into traceable mathematical formulation with analytical, theoretical and numerical analysis. Thus, La Salle and Lefschetz, (1961) showed the stability by Lyapunov's direct method with applications. Also, Leung and Wang (1976) presented an analysis of models for commercial fishing covering mathematical and economic aspects. Mesterton-Gibbons, (1996) and Kitabatake, (1982) studied the effects on the optimal policy for combined harvesting of predator-prey and a dynamic predator-prey model for fishery resources in lake Kasumigaura. One of the findings showed that there is irreversibility of capital investment upon optimal exploitation policies for renewable resources stocks. Zhang et al. (2000) proposed and analysed a model to study the stage structured predator-prey model and optimal harvesting policy of the organisms. The study derived necessary and sufficient condition for the coexistence and extinction of species. Song and Chen (2001) carried out a study on the optimal harvesting policy and stability for a two-species competitive system with stage structure, and obtained conditions for the existence of a globally asymptotically stable positive equilibrium and a threshold of harvesting for the mature population and finally proposed a dynamic model for a single-species fishery which depends partially on a logistically growing resource.

The problems identified in skilling fishery management for sustainable development of an economy are as follows: derisory knowledge of the resource and ecosystem, lack of interaction between the government and stakeholders, inaccessibility of financial resources to meet specific needs, weak implementation of agricultural policies and programmes, inadequate practical advisory/extension services and lack of monitoring and evaluation of programme/project team. Based on the above elucidated literatures and challenges enumerated, the researcher deemed it necessary to proposed a mathematical model to optimize the production of fish using system of nonlinear differential equations, discussed the equilibrium points, dynamical behaviour of the points and obtained the conditions of stability and instability of the two zones, ascertained the impact of human effort to both fishing zones and finally, discussed the numerical simulations with graphical representations.

2 Model formulation

This paper considered a fishery resource consisting of two zones: a free fishing zone and a reserved area zone. We study a prey-predator system in a two-patch environment, one accessible to both prey and predators and the other one being a refuge for the prey. Each patch is supposed to be homogenous. The prey refuge constitutes a reserve area of prey and no fishing is permitted in the reserved zone while the unreserved zone area is an open-access fishery zone. It is assumed that the prey migrates between the two patches randomly. It is assumed that, in each zone growth of fish population follows logistic model. Keeping these in view, the model becomes:

$$\frac{dU}{dt} = \beta_1 U\left(1 - \frac{U}{C}\right) - M_1 U + M_2 R - qEU \qquad 1$$

$$\frac{dR}{dt} = \beta_2 R\left(1 - \frac{R}{K}\right) + M_1 U - M_2 R \qquad 2$$

In the equations above, $U(t)$ is the biomass densities of the same population inside the unrestricted area and R(t) is the biomass densities of the same population inside the restricted area respectively at time t. E is the human effort for harvesting the fish population in the unreserved zone and the value of qE is called fishing mortality. Again, a and b are the intrinsic growth rate of fish subpopulation inside the unreserved and reserved area respectively. Let the fish subpopulation of the unreserved area at a rate migrate into reserved area at a rate M_1 and the fish subpopulation of the reserved and migrate into unreserved area at a rate M_2. The carrying capacities of fish species in unreserved and reserved area C and K respectively, q is the catch ability coefficient of fish species in the unreserved area. If there Is no migration of fish population from reserved area to unreserved area (i.e. M_2= 0) and $a - M_1 - qE < 0$ then $\frac{\partial U}{\partial t} < 0.$ similarly, if there I no migration of fish population from unreserved area to restricted area (i.e. $M_1 = 0.$) and $b - M_2 < 0$, then $\frac{dR}{\partial t} < 0.$

3 Model analysis

The non-linear system in equations (1) - (2) has qualitatively analysed in this section so as to find the equilibrium points and the dynamical behaviour of that points as well as test the stability at positivity analysis equilibrium points.

Now we will prove that all the variable in the system model equations (1) and (2) are positive.

Lemma 1. If $U(t) \geq 0,\ R(t) \geq 0,$ then the solution of $U(t)$ and $R(t)$ of the model system of equations (1) and (2) are positive.

Proof: To prove this lemma, we get from the equation (1), $\frac{dU}{dt} = aU\left(1 - \frac{U}{C}\right) - M_1 U + M_2 R - qEU$

Now, $\frac{dU}{dt} + M_1 U + qEU \geq 0 \Rightarrow \frac{dU}{dt} + (M_1 + qE)U \geq 0$ 3

Putting $M_1 - qE = \emptyset$ and the integrating factor, *IF* = $\int e^{\emptyset dt} = e^{\emptyset t}$ 4

Multiplying by *IF* on both sides, we get,

$$U' e^{\emptyset t} + \emptyset U e^{\emptyset t} \geq 0 \Rightarrow (U' + \emptyset U)e^{\emptyset t} \geq 0 \qquad 5$$

$$\frac{dU}{dt} e^{\emptyset t} U \geq 0 \Rightarrow e^{\emptyset t} U \geq A \qquad 6$$

Therefore, $U \geq Ae^{-(M_1 + qE)t}$ 7

where, A is an integrating constant. For the value of A, under the initial conditions, when *t* = 0, then $U(t) = U(0)$, hence $U(0) \geq$ A. 8

Now putting the value of A in equation (8) gives,

$U(t) \geq U(0)e^{-(\lambda_1 + qE)t}$ 9

Since $U(t) \geq 0$ and $M_1 - qE \geq 0$, hence $U(t) \geq 0$ for all $t \geq 0$.

Similarly, from equation (2) yields,

$$\frac{dR}{dt} = \beta_2 R\left(1 - \frac{R}{K}\right) + M_1 U - M_2 R \geq 0 \Rightarrow \frac{dR}{dt} + M_2 R \geq 0 \qquad 10$$

Using integrating factor $IF = \int e^{M_2 dt} = e^{M_2 t}$ 11

Multiply both sides of equation (10) by equation (11) gives,

$$e^{M_2 t}\frac{dR}{dt} + M_2 R e^{M_2 t} \geq 0 \Rightarrow \frac{d}{dt} e^{M_2 t} R \geq 0 \Rightarrow e^{M_2 t} R \geq B. \qquad 12$$

Thus, $R \geq Be^{-M_2 t}$. 13

where, B is an integrating constant. For the value of B, under the initial conditions, when $t = 0$, then $R(t) = R(0)$, hence $R(0) \geq B$. 14

Now putting the value of B in equation (14) gives,

$R(t) \geq R(0)e^{-M_2 t}$. 15

Since $R(t) \geq 0$ and $M_2 \geq 0$, hence $U(t) \geq 0$ for all $t \geq 0$. Therefore, $U(t)$ and $R(t)$ are positive for all time, $t \geq 0$.

3.1 Existence of equilibrium

The steady states of the two system of equations were obtained by equating the derivatives to zero and resolving it equations analytically. The two possible steady states are given as: S(0, 0) and $S(U^*, R^*)$ respectively. Now, (U^*, R^*) are the positive solutions of equations (1-2).

$$0 = \beta_1 U\left(1 - \frac{U}{C}\right) - M_1 U + M_2 R - qEU \qquad 16$$

$$0 = \beta_2 R\left(1 - \frac{R}{K}\right) + M_1 U - M_2 R \qquad 17$$

Eliminating R from equation (16) yields,

$$R = \frac{\beta_1 U^2 - \beta_1 UC + M_1 CU + CqEU}{CM_2}. \qquad 18$$

Substitute the value of R into equation (18)

$$\beta_2\left(\frac{\beta_1 U^2 - \beta_1 UC + M_1 CU + CqEU}{CM_2}\right)\left(1 - \frac{\beta_1 U^2 - \beta_1 UC + M_1 CU + CqEU}{\frac{CM_2}{K}}\right) + M_1 U - M_2\left(\frac{\beta_1 U^2 - \beta_1 UC + M_1 CU + CqEU}{CM_2}\right) = 0 \qquad 19$$

$$= -\frac{\beta_2 \beta_1^2 U^4}{KC^2 M_2^2} + \frac{\beta_2 \beta_1^2 CU^3}{KC^2 M_2^2} + 2\beta_1\beta_2(\beta_1 - M_1 - qE)U^2 - \left\{\frac{\beta_2(\beta_1 - M_1 - qE)^2}{CM_2^2} - \frac{(\beta_2 - M_2)\beta_1}{CM_2}\right\}U + \frac{(\beta_2 - M_2)}{M_2}(\beta_1 - M_1 - qE) - M_1 = 0 \qquad 20$$

$= aU^4 + bU^3 + cU^2 + dU + y = 0,$ 21

where, $a = -\frac{\beta_2\beta_1^2}{KC^2M_2^2}$, $b = \frac{\beta_2\beta_1^2}{KC^2M_2^2}$, $c = 2\beta_1\beta_2(\beta_1 - M_1 - qE)$, $d = -\left\{\frac{\beta_2(\beta_1-M_1-qE)^2}{CM_2^2} - \frac{(\beta_2-M_2)\beta_1}{CM_2}\right\}$
and $y = \frac{(\beta_2-M_2)}{M_2}(\beta_1 - M_1 - qE) - M_1.$

22

The equation (21) has a unique positive solution $U = U^*$ by satisfying the inequalities below:
$\frac{\beta_2(\beta_1-M_1-qE)^2}{CM_2^2} < \frac{(\beta_2-M_2)\beta_1}{C}, \frac{(\beta_2-M_2)}{M_2}(\beta_1 - M_1 - qE) < M_1M_2.$ 23

Putting the values of U^*, R^* into (16) and for positivity, $U^* > \left(\frac{C}{\beta_1}\right)(\beta_1 - M_1 - qE)$ 24

3.2 Dynamical behaviour of equilibria

The dynamical behaviour of equilibria points can be studied by computing vibrational matrices corresponding to each equilibrium. In equation (1), considering that the trivial equilibrium point p_0 is unstable. Using the Routh-Hurwitz criteria makes it easy to check that all eigenvalues of the vibrational matrix corresponding to p^* have negative real parts. Therefore, $p^*$is locally asymptotically stable in the two-dimensional plane. This indicates that a small circle with centre p^* can be determined such that any solution $U(t), R(t)$ of equation (2) which is inside the circle at time $t = t_1$ will remain inside the circle for all $t \geq t_1$ and will tend to (U^*, R^*) as $t \to \infty$.

Lemma 2. *Let* $Q = \left\{(U, R) \in R_2^2 : p = U + R \leq \frac{\mu}{\emptyset}\right\}$ *be a region of interest for all solutions initiating in the interior of the positive quadrant for* $\emptyset$ *is a positive constant and* $\mu = \frac{C}{4\beta_1}(\beta_1 - \emptyset - eq)^2 + \frac{K}{4\beta_2}(\beta_2 + \emptyset)^2$. This lemma will show that all solutions of model (2) are positive and uniformly bounded.

Proof: Let, $P(t) = U(t) + R(t)$, and $\emptyset > 0$ be a constant. Then,
$\frac{dP}{dt} = -\frac{\beta_1U^2}{C} + (\beta_1 - \emptyset - Eq)^2U^2 - \frac{\beta_2R^2}{K} + (\beta_2 + \emptyset) \leq \frac{K}{4\beta_1}(\beta_1 - \emptyset - Eq)^2 + \frac{K}{4\beta_2}(\beta_2 + \emptyset)^2 = \mu$ 25
Using the theory of differential inequality in Birkhoff and Rota, (1882),
$0 < P(t), U(t), R(t) \leq \frac{\mu}{\emptyset}\left(1 - e^{-\emptyset t}\right) + P(0), U(0), R(0), e^{-\emptyset t},$ 26
when $t \to \infty$ and $0 < P < \frac{\mu}{\emptyset}$

Global stability of equilibrium point ($\boldsymbol{p}^*$).

Theorem 1. The equilibrium point p^* *is globally asymptotically stable.*
Proof: Let us consider the following definite function about p^*
$Q = \left(U - U^*In\frac{U}{U^*}\right) + A\left(R - R^* - R^*In\frac{R}{R^*}\right)$ 27
Upon differentiating Q with respect to time *t*, gives,
$\frac{dQ}{dt} = -\frac{\beta_1}{C}(U - U^*)^2 - \frac{R^*}{U^*}\frac{M_2\beta_2}{M_1\beta_1}(R - R^*)^2 - \frac{M_2}{U^*UR}(UR^* - RU^*) < 0$ 28

This implies that $\frac{dQ}{dt}$ is negative by Liapunov's (1961). Hence, it follows that the positive equilibrium is globally asymptotically stable. Stability analysis reveals that the trivial equilibrium is an unstable system while, the feasible equilibrium is a point whereby both species are existing in the system for the competition, at this point they are stable but the population decrease with time.

3.3 Stability of steady states

Let the two functions W$= W(U,R) = \frac{dU}{dt}$ and V$= V(U,R) = \frac{dR}{dt}$ from the steady state $(U^*, R^*) = (0,0)$ for equations (1) and (2). Thus, the Jacobian matrix gives:

$$J(U,R) = \begin{pmatrix} \frac{\partial W}{\partial U} & \frac{\partial W}{\partial R} \\ \frac{\partial V}{\partial U} & \frac{\partial V}{\partial R} \end{pmatrix} = \begin{pmatrix} \beta_1 - \frac{2\beta_1 U}{C} - M_1 - qE & M_2 \\ M_1 & \beta_2 - \frac{2\beta_2 R}{K} - M_2 \end{pmatrix} \qquad 29$$

Thus, evaluating at the steady state of (0, 0) the Jacobian matrix J is

$$J(0,0) = \begin{pmatrix} \beta_1 - M_1 - qE & M_2 \\ M_1 & \beta_2 - M_2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \beta_1 - M_1 - qE - \lambda & 0 \\ 0 & \beta_2 - M_2 - \lambda \end{pmatrix} \qquad 30$$

$\beta_1 - M_1 - qE - \lambda_1 = 0$ and $\beta_2 - M_2 - \lambda_2 = 0$

Thus, the Eigen values $\boldsymbol{\lambda}$ are:

$\lambda_1 = \beta_1 - M_1 - qE$ and $\lambda_2 = \beta_2 - M_2$

For $\lambda_1 > \lambda_2 > 0$. Hence, the system is unstable

Here, the eigenvalues $\boldsymbol{\lambda}$ of the matrix "*J*" determine the stability of the states. Depending on $\boldsymbol{\lambda}$, the stability conditions are: 1. If the eigenvalue $\boldsymbol{\lambda}$ <0, then the steady state is stable, 2. If the eigenvalue $\boldsymbol{\lambda}$ >0, then the system is unstable. The findings depicts that the consumption rate of fishes is very high to the extent that at some time, t there should be no matured fish in the zones which in turn causes recurrent skilling on its sustainability. This implies that the biomass density of the two zones or areas are not the same, so also exchange rate in the economy of nations are unstable. This results also showcased that the knowledge of experts about the resource and ecosystem in line with the agricultural policies are not properly implemented. These results conform to Nkuturum (2016, 2017) on the study mathematical model of predator-prey relationship with human disturbance which showed that at time, t one of the species would go extinction while the other recuperate making the system unstable.

4 Numerical simulations and discussion of findings

This section showcases the numerically solved proposed model equations (1-2) and discussion of findings. Already, the existence of equilibrium, positivity analysis and stability of steady states have been discussed above. The model is simulated using the parameter values the total effort applied for harvesting the fish population in the reserve area E = 0.2. The carrying capacities of fish species in unreserved and reserved area respectively C = 110 and K = 200. The intrinsic growth rates of fish sub-population are respectively $\beta_1 = 0.03$ and $\beta_2 = 0.04$. The catch ability of fish coefficient q = 0.001. The fish sub-population of unreserved and reserved area are respectively $M_1 = M_2 = 0.05$ and qE denote the fishing mortality rate of the model (1). Then the result is displayed in the Fig. 1. The displayed graph depicts the biomass densities of reserved area unreserved zones or areas where the population increased and decreased within 90 days. Firstly, the Fig. 1 showed that the population of unreserved area and reserved area are same on the 9 days. In 90 days, the population of biomass densities of unreserved area increases above 100. At the same time the population of biomass densities of reserved area increases above 100. Once more, Fig. 2 the population of reserved area increases above 900. At the same time the population of unreserved area increases above 400 and unreserved area are same at 10 days. If the input value of the total effort applied for harvesting the fish population and fish catch ability coefficient increases, then the population decreases. If the population of the total effort applied for harvesting the fish population and fish catch ability coefficient decreases so the population decreases means that human beings have vital roles to play in skilling fisheries management for sustainable development of an economy. Similarly, by changing the values of parameter the shape of graph changed. The Fig. 3 and Fig. 4 are changing the parameter values too, altered the graph in shape. When the parameter values of intrinsic growth rate of unreserved and reserved area are changed by increasing it respectively, then the population are also increasing.

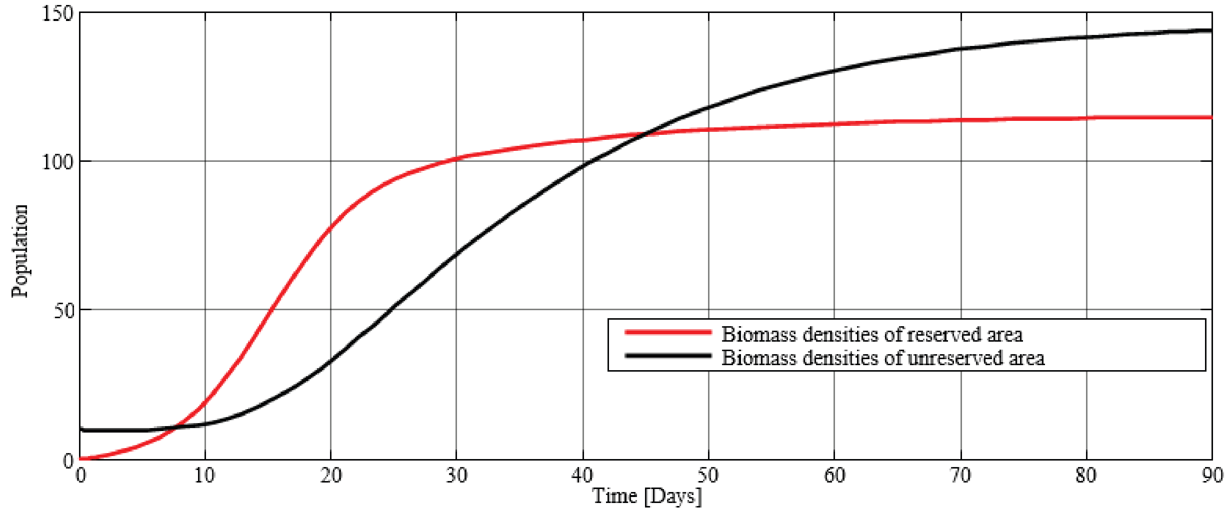

Figure 1. Showing the graph of fishery model of two zones population with time (Days)

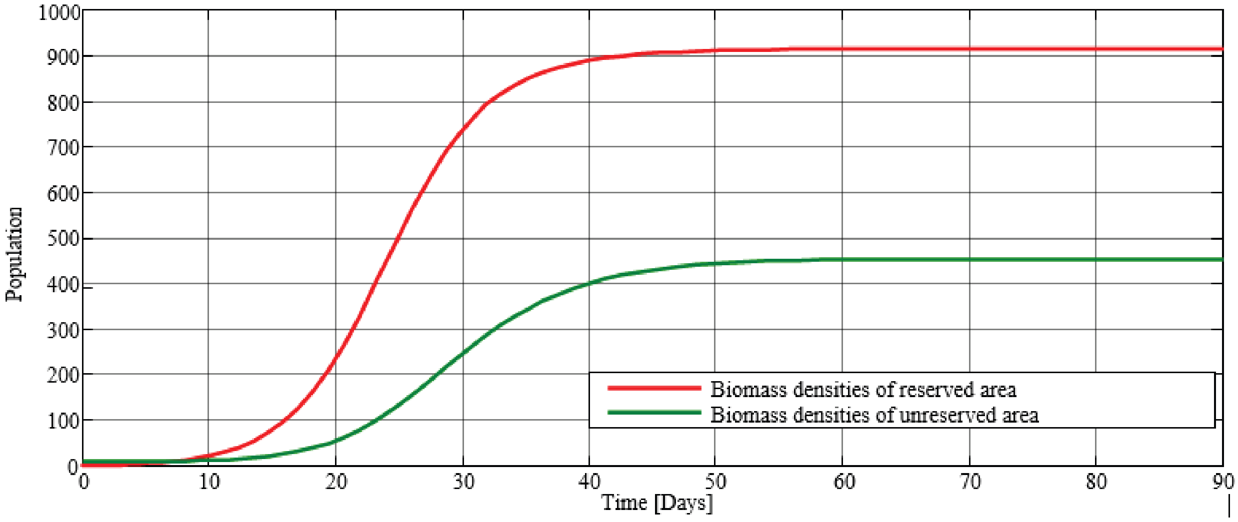

Figure 2. Showing variation of population with time (Days) for E = 0.002, C = 1000, K = 200, $\boldsymbol{\beta_1} = \mathbf{0.03}$, $\boldsymbol{\beta_2} = \mathbf{0.04}$ and q = 0.001

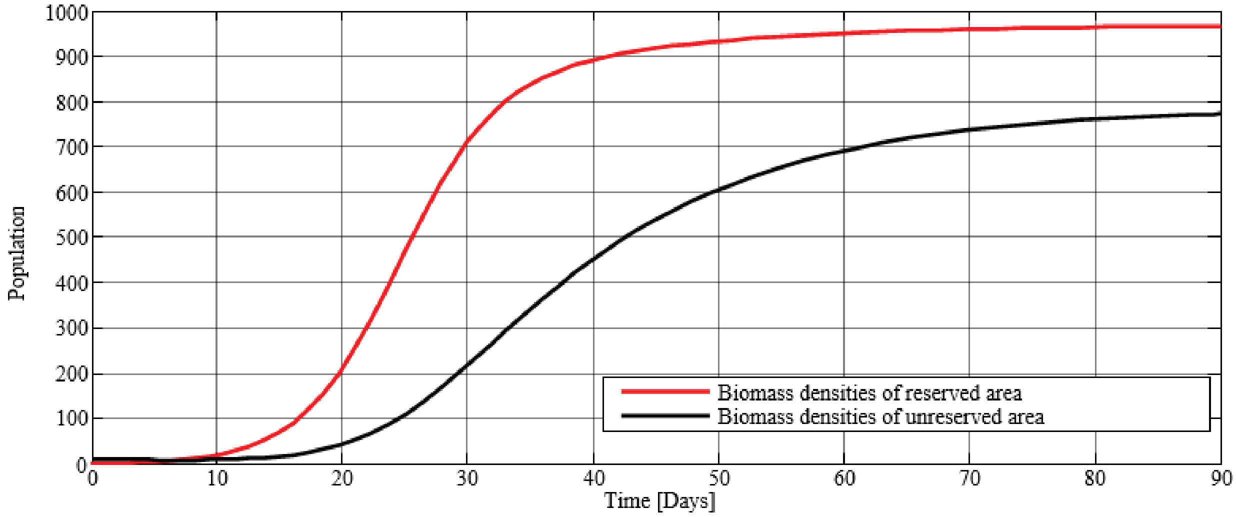

Figure 3. Variation of population with time (Days) for E = 0.002, C = 1000, K = 200, $\boldsymbol{\beta_1 = 0.299}$, $\boldsymbol{\beta_2 = 0.004}$ and q = 0.00001

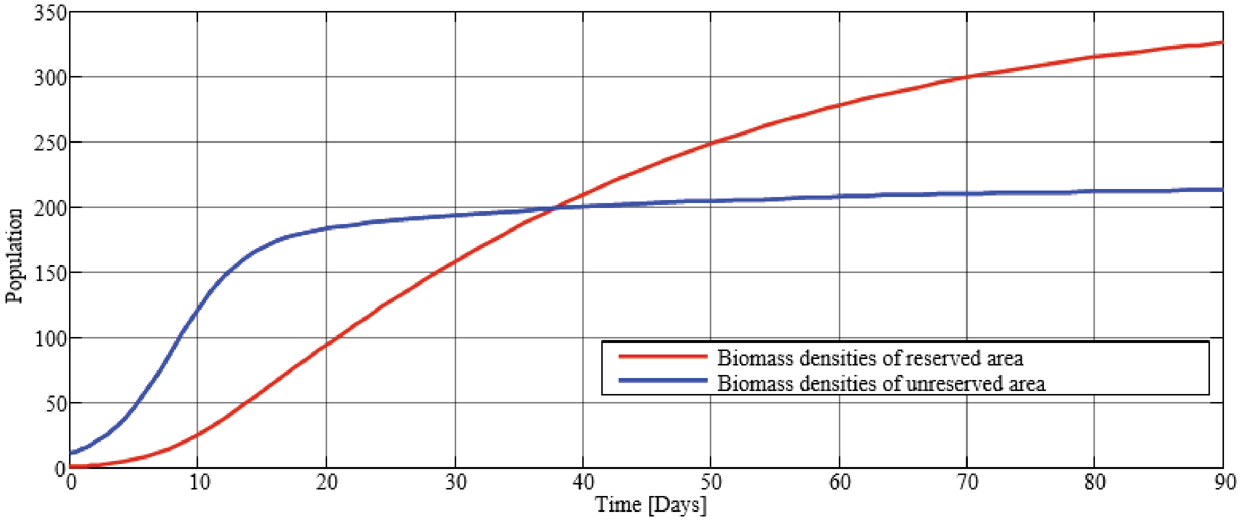

Figure 4. Variation of population with time (Days) for E = 0.002, C = 1000, K = 200, $\boldsymbol{\beta_1 = 0.03}$, $\boldsymbol{\beta_2 = 0.4}$ and q = 0.00001

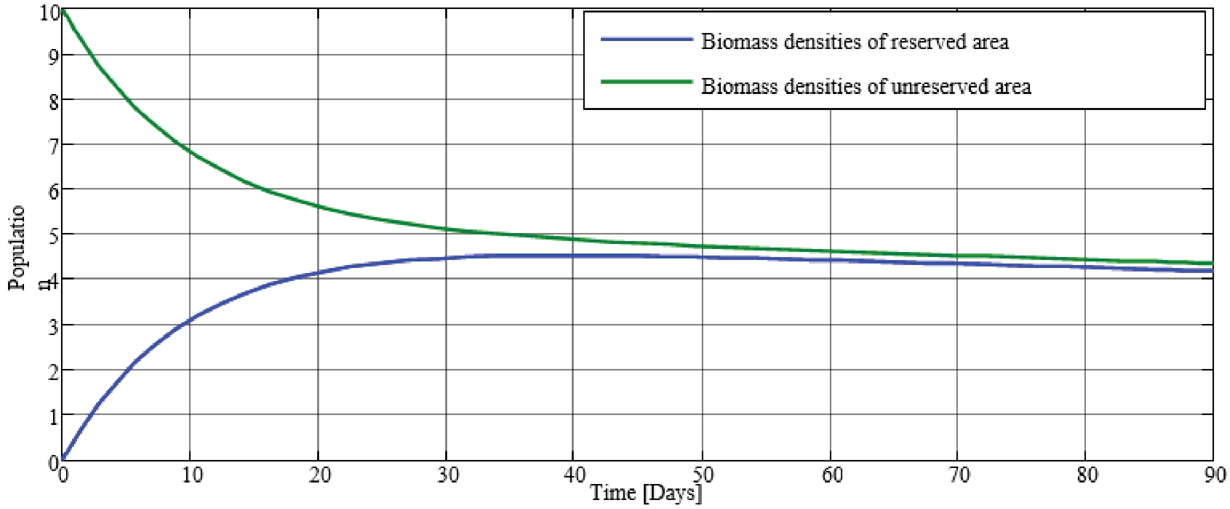

Figure 5. Variation of population with time (Days) for E = 4, C = 1000, K = 200, $\boldsymbol{\beta_1 = 0.03}$, $\boldsymbol{\beta_2 = 0.004}$ and q = 0.1

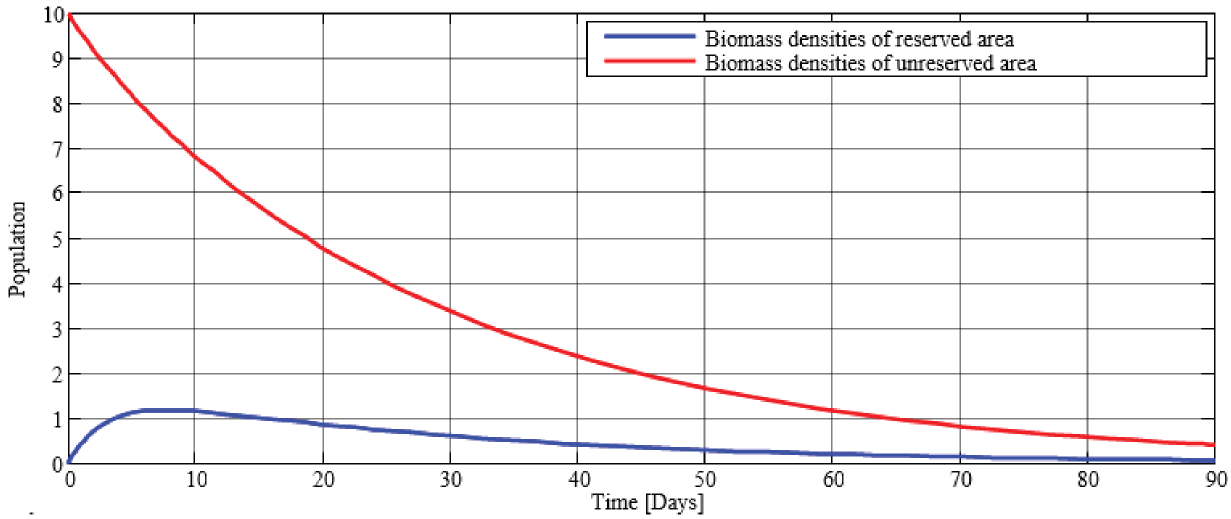

Figure 6. Variation of population with time (Days) for E = 2.8, C = 1000, K = 200, $\boldsymbol{\beta_1 = 0.3}$, $\boldsymbol{\beta_2 = 0.006}$ and q = 0.2

Hence, if the parameter values of carrying capacities of fish spices in the unreserved and reserved area are used for fish production respectively C and K area increasing, then the graph is increasing and by changing the parameter values C and K are increasing then, the fish production is increasing. Likewise, the Fig. 5 and Fig. 6 are going below the X axis showing that the population decreases with time. The maximum fish production depends on carrying capacities intrinsic growth rate of the unreserved and reserved area and harvesting rate or fishing mortality. In this Figures, it is detected that the harvesting rate of fishing mortality increases as the graph is decreases. Again, the fishing mortality decreases as the graph increases. In these figures, by changing the values of parameter the characteristics figure is changed. So, the model is good for skilling fisheries management for sustainable development of an economy that could lead to minimum and maximum fish production in a reserved and unreserved areas respectively.

The model contains two fixed points: the trivial equilibrium $(U^*, R^*) = (0,0)$ where both populations disappear; when evaluated gives the eigen values as $\lambda_1 = \beta_1 - M_1 - qE$ and $\lambda_2 = \beta_2 - M_2$ depicting unstable system and the feasible equilibrium implies coexistence of both populations. Interestingly, the feasible equilibrium showed abundance of the biomass density in the zones is independent of their own respective growth or death rates but instead it is set by the productivity of the other species in the reserved zone. However, the traditional management system is the most effective especially at the rural community level whereby the village heads have responsibilities of enforcing and regulating the management Systems in their various domains. The mixed system is most prevalent and governments generally lack the logistic support in terms of personnel, funds, field vehicles and so on to enforce fisheries laws and regulations. In general, fisheries management systems in Nigeria can be described as variable and hindered by poor financial support for policy implementation in which the findings of this study rated in percentage gives as mixed system is most common (60%), followed by the traditional type 30% while modern type accounts for only 10%.

5 Conclusion

In this mathematical modelling, the system has been assumed that the two ecosystems; one free fishing zone and the other reserved zone. In this paper, we analysed the catfish production by using many mathematical models. It has been found that the total user's cost of harvest per unit of effort equals to the discounted value of the future marginal profit of the effort at the steady-state level. It has also been noted that if the discounted rate increases, then the economic rent decreases and even may tend to zero if the discounted rate tend to infinity. The possibility that the predator species in this model be itself harvested for commercial purposes, or that its non-use value be an important element in the overall economic value of the marine ecosystem have not been considered in this paper. Their inclusion in the model would surely lead to characterize another set of trade-offs which are bound to be at the centre of marine reserve policies. We have derived existence of equilibrium and globally asymptotically stable and unstable. Finally, skilling fisheries management for sustainable development can be achieved through regulated fishing which considers fishery management objectives that address the status of the resource, human activities, the health of the environment, post-harvest technology and trade, as well as other economic concerns, social benefits, legal and administrative support as observed from graphical representation with different parameter values.

This study can be extended to add noise term, diffusion term and feeding rate of the fishes. This study suggests that there should be introduction of environmental impact assessment for fisheries policies to be implemented, skilled or trained fisheries administrators that will monitor and evaluate the policy/programme consistency, against specified goals need to be given more financial resources, and properly staffed and equipped to address effectively all aspects of sustainable fisheries development and management. should be adopted and their efficacy monitored and evaluated against specified goals. There should be better protection to environmental hazards, control of waste disposal and pollution.

Finally, government at all levels should recognize its benefit and work towards integrating skilling fisheries management activities, including sharing of expertise and resources for education, research, technology, monitoring, control and surveillance activities and development of the relevant legislative framework due to its economic gains.

References

[1] Berezansky, L; Idels, L. and Kipnis, M. (2011). Mathematical model of marine protected areas. IMA J. Appl. Math. 76(1):312-325.
[2] Birkhoff, G. and Rota, G.S. (1882). Ordinary differential equations. Ginn, Boston.
[3] Biswas, Md. H. A.; Hossain, Md. R. and Mondal, M. K. (2017). Mathematical modelling applied to sustainable management of marine resources. ELSEVIER Procedia Engineering, 194(1):337-344
[4] Clark, C. W. (1976). Mathematical Bioeconomics: The optimal management of renewable resources. Wiley, New York.
[5]Das, T. Mukherjee, R.N., and Chaudhuri,K. S. (2009). Bioeconomic harvesting of a prey-Predator fishery. J. Bio. Dyn. 3(1):447-462.
[6] De Lara, M. and Doyen, L. (2008). Sustainable management of natural resources: mathematical models and methods. Springer, Verlag Berlin.
[7] Dubey, B., Chandra, P., and Sinha, P. (2003). A model for fishery resource with reserve area. Nonl. Ana.: Real World Appl., 4(1):625-637.
[8] FAO (1995). The code of conduct for responsible fisheries. FAO: 41
[9] FDF (Federal Department of Fisheries) (2005). Report of presidential committee on fisheries and aquaculture development: Consolidated Report., Vol. 1
[10] Kitabatake, Y. (1982). A dynamic predator-prey model for fishery resources: a case of lake Kasumigaura. Environ. Plan. A, 14(1):225-235.
[11] La Salle, J. and Lefschetz, S. (1961). Stability by Liapunovs direct method with applications. Academic Press, New York.
[12] Leung, A. and Wang, A. (1976). Analysis of models for commercial fishing: mathematical and economical aspects. Econometrica, 44(1):295-303.
[13] Mesterton-Gibbons, M. (1996). On the optimal policy for combined harvesting of predator prey. Nat. Res. Model., 3(1):235-244.
[14] Nkuturum, C. and George, I. (2021). Ecological growth model in marketing dynamics: the role of human actions. Nigeria society of Mathematical Biology (NSMB 5th conference proceedings).
[15] Nkuturum, C. (2021). Distortion of ecological systems by human actions: a mathematical model of two coexisting species in a community. Nigeria society of Mathematical Biology (NSMB 5th conference Proceedings).
[16] Nkuturum, C. (2017). Mathematical model of predator-prey relationship with human disturbance. Department Of Mathematics, University of Nigeria, Nsukka. https://www.repository.unn.edu.ng
[17] Nkuturum, C. (2016). Mathematical model of predator-prey relationship and human disturbance. International Journal of Education Development, 6:183-196.
[18] Sea Around Us (2007). A global database on marine fisheries and ecosystems. www.seaaroundus.org.
[19] Song, X. and Chen, L. (2001). Optimal harvesting and stability for a two-species competitive System with stage structure. Math. Biosci. 170(1):173-186.
[20] Zhang, X., Chen, L. and Neumann, A.U. (2000). The stage structured predator-prey model and optimal harvesting policy. Math. Biosci. 168(1):201-210.