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A series solution to asset valuation and pricing of linear evolution equations with stochastic partial derivatives

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Abstract

The achievement of any investment depends mainly on the value of asset which impels the entire financial power of every company and differential equations are well known prevailing mathematical tool used for the prediction of stock prices. Therefore, this paper, considered four investment equations with stochastic and stock return rates parameters in the model. Detailed conditions are achieved which govern asset price return rates through multiplicative effects, additive invers effects, multiplicative inverse effects, stock volatility and asset growth-rates of assets parameter respectively. Consequently, the impacts on the value of asset prices of investors in capital market were analyzed; and graphical results of stock variables and the effect of relevant parameters were discussed for the purpose of investment plans.

Keywords: stock price; asset pricing; series solution; stochastic analysis; return rates

1 Introduction

Making short or long term business plans needs financial assets which is geared towards bringing returns to the investors over time. Financial assets are the basis with through which individual or group of persons generate wealth to become comfortable in life. In any time-varying investment, cash flows acquired by the investor as cash dividends over the period as the rate of return is expected to cover-up unforeseen risks of the investments. More so, an investor must take into account on par the safety and investment rates of returns to avoid event risks; like market crashes, flood disasters and political crisis which may directly or indirectly impact on investments. However, without capital investment human needs may be difficult to actualize but, even the most secure stocks can encounter losses for investors or other stakeholders, therefore an investor may need to rebalance his portfolio of investment or diversify his asset allocation in order to reduce overall portfolio risk and improve expected returns to cover-up loop-holes in the investment.

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It is worthy of note that, in modeling the value of the rate of return on stocks and other relevant stochastic parameters involved in making investment decisions that accounts for certain level of unpredicted randomness, an accurate analytical method of solution is to be adopted based on the specifics of the problem in order to stimulate realistic stock quantities or variables; that is why differential equation models with stochastic parameters are most frequently adopted to capture other environmental effects.

A differential equation [11] is an important tool for harnessing different components into a single system and analyzing the inter-relationships that exists between these components which otherwise would continue to be independent of each other. Differential equations are one of the most frequently used tools for mathematical modeling in engineering and sciences [8-9]. Thus, differential equations have been used in different ways and independent results obtained. For instance,[10] considered the unstable nature of stock market forces using proposed differential equation model. [13] studied the variation in capital market changes where they developed a differential equation with stochastic parameter in the model obtaining an analytical solution which determined the equilibrium prices.

However, [16] considered a set of simultaneous differential equations of stock prices of the share in both A and H stock markets. They applied iteration method and results were obtained. [2] Studied the application of differential equation on stock market prices where they modeled two competing growth rates and there carrying capacity on stock market prices which was chosen based on minimum variance criteria. Similarly, [11] investigated a problem using differential equation in stock market analysis; the two competing growth rates and it's carrying capacity were considered. In [12], ordinary differential equations with variable coefficients problem were determined; the Elzaki transform was applied and results obtained. See for example [3],[4],[6],[7],[14],[15], and [18] for more research works on stock market price modeling.

This research work considers four systems of second order ordinary differential equations in time varying investments with stochastic parameter whose rate of returns is assumed to follow: multiplicative effects, additive inverse effects, multiplicative inverse effects; all having quadratic functions over time. These investment equations were solved analytically by adopting series solution of Ferobenius method and four solutions were obtained. To verify the validity of the analytical solutions, the value of assets and relevant parameters in the model were assed using detailed conditions of predicting stock returns and asset valuations; and the effects of rate of returns, asset valuation and stock volatility displayed graphically. This paper extends the works of [5] by solving the problem in second order ordinary differential equation domain.

This paper is organized as follows: Section 2.1 presents the Mathematical framework, problem formulation is seen in Subsection 2.1.1, method of solution is seen in Subsection 2.1.2, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2 Mathematical framework

Thus, let $S(t)$ be the price of some risky asset at time t ,and μ , an expected rate of returns on the stock and dt as a relative change during the trading days such that the stock follows a random walk which is govern by a stochastic differential equation

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \quad (1)$$

where, μ is drift and σ the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega, \xi, \wp)$, ξ is a $\sigma-\text{algebra}$ generated by $W_t, t \geq 0$.

Theorem 1: (Ito's formula) Let $(\Omega, \beta, \mu, \mathrm{F}(\beta)$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta)$ possessing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t) \quad (2)$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \square))$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau + f\frac{\partial u}{\partial x}dW(t) \quad (3)$$

The solution to stochastic differential equation (1) can be obtained easily using Theorem 1 and is given by:

$$S(t) = S_0 \exp\left\{\sigma dW(t) + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\}, \forall t \in [0,1].$$

From (1) Black-Scholes Partial Differential Equation (PDE) is given as:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0. \quad (4)$$

Following the method of [14]

$$dS(t) = \hat{\alpha}S(t)dt + \sigma S(t)dW(t), \hat{\alpha} = \alpha + \lambda \quad (5)$$

where λ is the market price of risk. The equation governing stock option is backward Black-Scholes partial differential equation given (in one variable) as Osu (2008), Osu, et al. (2009).

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (\alpha - \lambda)S\frac{\partial V}{\partial S} - ru = 0. \quad (6)$$

Assume $\lambda = 0$, (6) gives

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \alpha S\frac{\partial V}{\partial S} - rV = -\frac{\partial V}{\partial t}. \quad (7)$$

From (7) $V \neq V(t)$ hence V is a function of S alone and we arrived at the following 2nd order nonhomogeneous differential equation by also defining the index price of the form: $-\varphi - GR_t$ which replaces RHS of (7) below.

$$\frac{1}{2}\sigma^2 S^2 \frac{d^2V}{dS^2} + \alpha S \frac{dV}{dS} - rV = -\varphi - GR_t \tag{8}$$

where φ represents constant and GR_t is growth of the underlying asset.

Suppose the interest rate $r = 1$ in the left hand side of (8) becomes:

$$\frac{1}{2}\sigma^2 S^2 \frac{d^2V}{dS^2} + \alpha S \frac{dV}{dS} - V = -\varphi - GR_t\,. \tag{9}$$

Dividing homogeneous part of (9) by $\frac{\sigma^2 S}{2}$ gives

$$S\frac{d^2V}{dS^2} + \frac{2\alpha}{\sigma}\frac{dV}{dS} - \frac{2rV}{S\sigma^2} = 0\,. \tag{10}$$

Suppose the trading of shares or adjustment of portfolio is allowed to take place. We therefore adjust the stock variables following the method of [13] by setting:

$\frac{2\alpha}{\sigma} = 1,\ \frac{2r}{S\sigma^2} = S\sigma^2 V,$ Since the rate of change of investment output value depends on the investment output at present time t ,[7] (10) becomes

$$S\frac{d^2V}{dS^2} + \frac{dV}{dS} - S\sigma^2 V = 0 \tag{11}$$

The last term of LHS of (11) is the aggregate intrinsic value of stock, [7] hence it comprises the volatility of the underlying asset throughout the trading days. Secondly, risk factors S describe any kind of risk and uncertainty present in the financial market such as stock prices, interest rate, strike price etc. So, combining (11) and (LHS) of (10) gives a complete 2nd non-homogenous ordinary differential equation below:

$$S\frac{d^2V}{dS^2} + \frac{dV}{dS} - S\sigma^2 V = -\varphi - GR_t\,. \tag{12}$$

2.1 Problem Formulation

Now, we consider capital investment plans where investors will be faced with some decisions on how to value assets and its return rates. So, investors at all seasons receive its wages at different time intervals. So, considering four different investments selected stocks (companies) to be represented as $S_1.S_2, S_3$ and S_4 be defined in the system of variable coefficient differential equations, the factors underlying price changes are very uncertain and are described in probability terms. Hence the rate of returns is defined following the method of [3] and [5] respectively:

$$R_t := (\lambda_1\lambda_2)^2, \ldots \text{where } t = 1, 2\ldots \tag{13}$$

$$R_t := (\lambda_1 + \lambda_2)^2, \ldots, \text{where } t = 1, 2\ldots \tag{14}$$

$$R_t := \left\{(\lambda_1 + \lambda_2)^{-1}\right\}^2, \ldots, \text{where } t = 1, 2\ldots \tag{15}$$

$$R_t := \left\{(\lambda_1\lambda_2)^{-1}\right\}^2, \ldots, \text{where } t = 1, 2\ldots \tag{16}$$

Using (13)-(16) in (12) independently gives the following investment equations for four different stocks (companies):

$$S_1 \frac{d^2V_{1(t)}}{dS_1{}^2} + \frac{dV_{1(t)}}{dS_1} - (\lambda_1\lambda_2)^2 S_1\sigma^2 V_{1(t)} = 0 \tag{17}$$

$$S_2 \frac{d^2V_{2(t)}}{dS_2{}^2} + \frac{dV_{2(t)}}{dS_2} - \left\{(\lambda_1 + \lambda_2)^{-1}\right\}^2 S_2\sigma^2 V_{2(t)} = 0 \tag{18}$$

$$S_3 \frac{d^2V_{3(t)}}{dS_3{}^2} + \frac{dV_{3(t)}}{dS_3} - \left\{(\lambda_1 + \lambda_2)^{-1}\right\}^2 S_3\sigma^2 V_{3(t)} = -\varphi - GR_t V_{4(t)} \tag{19}$$

$$S_4 \frac{d^2V_{4(t)}}{dS_4{}^2} + \frac{dV_{4(t)}}{dS_4} - \left\{(\lambda_1\lambda_2)^{-1}\right\}^2 S_4\sigma^2 V_{4(t)} = 0\,. \tag{20}$$

With the following boundary conditions

$$\left.\begin{array}{l} V_{1(t)} = V_1\alpha,\ V_{2(t)} = V_2\alpha,\ V_{3(t)} = 0, V_{4(t)} = V_4\alpha \\ \dfrac{dV_{1(t)}}{dS_1} = \dfrac{dV_{2(t)}}{dS_2} = \dfrac{dV_{3(t)}}{dS_3} = \dfrac{dV_{4(t)}}{dS_4} = 0 \end{array}\right\} \tag{21}$$

2.2 Method of Solution

The propose model (17)-(20) consist of a system of variable coefficient differential equations whose solutions are not trivial. We adopts the method of Ferobenius in solving for $V_{1(t)}(S), V_{2(t)}(S)$,$V_{3(t)}(S)$ and $V_{4(t)}(S)$, Firstly, we solve equation (17) and (18), then solve (20) and substituting the result in equation (19) to obtain following stock quantities. To tackle this problems adequately we note that $\mathrm{V}_{1(t)}(\mathrm{S}),\ \mathrm{V}_{2(t)}(\mathrm{S}),\ \mathrm{V}_{3(t)}(\mathrm{S}), \mathrm{V}_{4(t)}(\mathrm{S}) < \infty$ for all $S \in [0,1)$.

Solving Investment Equation when stock returns follow multiplicative effects:

From the homogenous part of (17)

$$\text{Let } V_{1(t)} = \sum_{m=0}^{\infty} a_m S_1^{m+c} = S_1^{c} \sum_{m=0}^{\infty} a_m S_1^{m} \tag{22}$$

$$V'_{1(t)} = \sum_{m=0}^{\infty} a_m (m+c) S_1^{m+c-1} = S_1^{c=1} \sum_{n=0}^{\infty} a_m (m+c) S_1^{m} \tag{23}$$

$$V''_{1(t)} = S_1^{c-2} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_1^{m} = S_1^{c-1} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_1^{m} \tag{24}$$

substituting (18)-(20) into (17) after long algebraic simplifications gives

$$V_{1(t)}(S) = v_1 = \alpha_1 \left\{ \begin{array}{l} 1 + \dfrac{(\lambda_1\lambda_2)^2 \sigma^2 S^2}{2^2} + \dfrac{(\lambda_1\lambda_2)^4 \sigma^4 S^4}{2^2 \times 4^2} + \dfrac{(\lambda_1\lambda_2)^6 \sigma^6 S^6}{2^2 \times 4^2 \times 6^2} + \dfrac{(\lambda_1\lambda_2)^8 \sigma^8 S^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \dfrac{(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{array} \right\} \tag{25}$$

Another is given by $v_2 = \dfrac{dV_{1(t)}}{dc}$

$$\frac{dV_{1(t)}}{dc} = a_0 S^c lnr \left\{ 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S^2}{(c+2)^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S^4}{(c+2)^2 (c+4)^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S^6}{(c+2)^2 (c+4)^2 (c+6)^2} \right.$$

$$\left. + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S^8}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} .. \right\}$$

$$+ a_0 S^c \frac{d}{dc} \left\{ 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S^2}{(c+2)^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S^4}{(c+2)^2 (c+4)^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S^6}{(c+2)^2 (c+4)^2 (c+6)^2} \right.$$

$$\left. + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S^8}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} + \dots \right\}$$

$$\frac{dV_{1(t)}}{dc} = a_0 S^c \ln S \left\{ 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S^2}{(c+2)^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S^4}{(c+2)^2 (c+4)^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S^6}{(c+2)^2 (c+4)^2 (c+6)^2} \right.$$

$$\left. + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2} + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^2 (c+4)^2 (c+6)^2 (c+8)^2 (c+10)^2} + \dots \right\}$$

$$+ a_0 r^c \left\{ 0 - \frac{2(\lambda_1\lambda_2)^2 \sigma^2 S^2}{(c+2)^3} - \frac{4(\lambda_1\lambda_2)^4 \sigma^4 S^4 (c+3)}{(c+2)^3 (c+4)^3} - \frac{8(\lambda_1\lambda_2)^6 \sigma^6 S^6}{(c+2)^3 (c+4)^3 (c+6)^3} + \frac{16(\lambda_1\lambda_2)^8 \sigma^8 S^8}{(c+2)^3 (c+4)^3 (c+6)^3 (c+8)^3} \right.$$

$$\left. - \frac{32(\lambda_1\lambda_2)^{10} \sigma^{10} S^{10}}{(c+2)^3 (c+4)^3 (c+6)^3 (c+8)^3} + \dots \right\}$$

$$V_{2(t)}(S_2) = v_1 = \phi_1 \left\{ \begin{array}{l} 1 + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2 \sigma^2 {S_2}^2}{2^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4 \sigma^4 {S_2}^4}{2^2 \times 4^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6 \sigma^6 {S_2}^6}{2^2 \times 4^2 \times 6^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8 \sigma^8 {S_2}^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10} \sigma^{10} {S_2}^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \dots \end{array} \right\}$$

$$V_{1(t)} = v_2 = \alpha_2 \left\{ \begin{array}{l} \ln S_1 \left(\begin{array}{l} 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^{\ 2}}{2^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^{\ 4}}{2^2 \times 4^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^{\ 6}}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^{\ 8}}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S_1^{\ 10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{array} \right) \\ + a_0 S_1^{\ c} \left(\begin{array}{l} - \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^{\ 2}}{2^2} - \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^{\ 4}}{2^3 \times 4^2} - \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^{\ 6}}{4^3 \times 6^3} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^{\ 8}}{2^5 \times 6^3 \times 8^3} \\ - \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S_1^{\ 10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \ldots \end{array} \right) \end{array} \right\} \tag{26}$$

A linear combination (25) and (26) gives the complete solution

$$\begin{aligned} V_{1(t)}(S_1) = {} & \alpha_1 \left\{ \begin{array}{l} 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^{\ 2}}{2^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^{\ 4}}{2^2 \times 4^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^{\ 6}}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^{\ 8}}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S_1^{\ 10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{array} \right\} \\ & + \alpha_2 \left\{ \begin{array}{l} \ln S_1 \left(\begin{array}{l} 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^{\ 2}}{2^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^{\ 4}}{2^2 \times 4^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^{\ 6}}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^{\ 8}}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} S_1^{\ 10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{array} \right) \\ - \frac{(\lambda_1\lambda_2)^2 \sigma^2 S_1^{\ 2}}{2^2} - \frac{(\lambda_1\lambda_2)^4 \sigma^4 S_1^{\ 4}}{2^3 \times 4^2} - \frac{(\lambda_1\lambda_2)^6 \sigma^6 S_1^{\ 6}}{4^3 \times 6^3} + \frac{(\lambda_1\lambda_2)^8 \sigma^8 S_1^{\ 8}}{2^5 \times 6^3 \times 8^3} - \frac{(\lambda_1\lambda_2)^2 \sigma^{10} S_1^{\ 10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \ldots \end{array} \right\} \end{aligned} \tag{27}$$

Applying the boundary conditions in (21) and setting $\alpha_2 = 0$ gives $\alpha_1 = \frac{V_1 \alpha}{y_1(1)}$

$$y_1(\alpha) = \left(\begin{array}{l} 1 + \frac{(\lambda_1\lambda_2)^2 \sigma^2 \alpha^2}{2^2} + \frac{(\lambda_1\lambda_2)^4 \sigma^4 \alpha^4}{2^2 \times 4^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^6 \alpha^6}{2^2 \times 4^2 \times 6^2} + \frac{(\lambda_1\lambda_2)^6 \sigma^8 \alpha^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ \qquad + \frac{(\lambda_1\lambda_2)^{10} \sigma^{10} \alpha^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{array} \right)$$

$$V_{1(t)}(S_1) = \frac{V_1\alpha}{y_1(\alpha)}\left\{\begin{array}{l} 1+\dfrac{(\lambda_1\lambda_2)^2\sigma^2 S_1{}^2}{2^2}+\dfrac{(\lambda_1\lambda_2)^4\sigma^4 S_1{}^4}{2^2\times 4^2}+\dfrac{(\lambda_1\lambda_2)^6\sigma^6 S_1{}^6}{2^2\times 4^2\times 6^2}+\dfrac{(\lambda_1\lambda_2)^6\sigma^8 S_1{}^8}{2^2\times 4^2\times 6^2\times 8^2} \\ +\dfrac{(\lambda_1\lambda_2)^{10}\sigma^{10} S_1{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}+\ldots \end{array}\right\}$$

(28)

Solving Investment Equation when stock returns follow additive effects:

Solving (18) from left hand side using Frobenius method

$$\text{Let } V_{2(t)} = \sum_{m=0}^{\infty} a_m S_2{}^{m+c} = S_2{}^c \sum_{m=0}^{\infty} a_m S_2{}^m \tag{29}$$

$$V'_{2(t)} = \sum_{m=0}^{\infty} a_m (m+c) S_2{}^{m+c-1} = S_2{}^{c=1} \sum_{m=0}^{\infty} a_m (m+c) S_2{}^m \tag{30}$$

$$V''_{2(t)} = S_2{}^{c-2} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_2{}^m = S_2{}^{c-1} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_2{}^m \tag{31}$$

substituting (29)-(31) into (18) and after long algebraic expressions gives

$$V_{2(t)} = v_2 = \phi_2\left\{\begin{array}{l} \ln S_2\left(\begin{array}{l} 1+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2\sigma^2 S_2{}^2}{2^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4\sigma^4 S_2{}^4}{2^2\times 4^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6\sigma^6 S_2{}^6}{2^2\times 4^2\times 6^2} \\ +\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8\sigma^8 S_2{}^8}{2^2\times 4^2\times 6^2\times 8^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10}\sigma^{10} S_2{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2} \end{array}\right) \\ +a_0 S_2{}^c\left(\begin{array}{l} -\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2\sigma^2 S_2{}^2}{2^2}-\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4\sigma^4 S_2{}^4}{2^3\times 4^2}-\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6\sigma^6 S_2{}^6}{4^3\times 6^3} \\ +\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8\sigma^8 S_2{}^8}{2^5\times 6^3\times 8^3}-\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10}\sigma^{10} S_2{}^{10}}{4^2\times 6^3\times 8^3\times 10^3}+\ldots \end{array}\right) \end{array}\right\} \tag{32}$$

A linear combination (32) and (33) then applying the boundary conditions (21) and setting $\alpha_2 = 0$ gives the complete solution

$$V_{2(t)}(S_2)=\phi_1\left\{\begin{array}{l}1+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^2\sigma^2 S_2{}^2}{2^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^4\sigma^4 S_2{}^4}{2^2\times 4^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^6\sigma^6 S_2{}^6}{2^2\times 4^2\times 6^2}\\ +\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^8\sigma^8 S_2{}^8}{2^2\times 4^2\times 6^2\times 8^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^{10}\sigma^{10} S_2{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}+\ldots\end{array}\right\}$$

$$+\phi_2\left\{\begin{array}{l}\ln S_2\left(\begin{array}{l}1+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^2\sigma^2 S_2{}^2}{2^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^4\sigma^4 S_2{}^4}{2^2\times 4^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^6\sigma^6 S_2{}^6}{2^2\times 4^2\times 6^2}\\ +\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^8\sigma^8 S_2{}^8}{2^2\times 4^2\times 6^2\times 8^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^{10}\sigma^{10} S_2{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}\end{array}\right)\\ -\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^2\sigma^2 S_2{}^2}{2^2}-\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^4\sigma^4 S_2{}^4}{2^3\times 4^2}-\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^6\sigma^6 S_2{}^6}{4^3\times 6^3}\\ +\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^8\sigma^8 S_2{}^8}{2^5\times 6^3\times 8^3}-\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^{10}\sigma^{10} S_2{}^{10}}{4^2\times 6^3\times 8^3\times 10^3}+\ldots\end{array}\right\} \tag{33}$$

$$\phi_1=\frac{V_2\alpha}{y_2(\alpha)},$$

where

$$y_2(\alpha)=\left\{\begin{array}{l}1+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^2\sigma^2\alpha^2}{2^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^4\sigma^4\alpha^4}{2^2\times 4^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^6\sigma^6\alpha^6}{2^2\times 4^2\times 6^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^8\sigma^8\alpha^8}{2^2\times 4^2\times 6^2\times 8^2}\\ +\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^{10}\sigma^{10}\alpha^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}+\ldots\end{array}\right\}$$

$$V_{2(t)}(S_2)=\frac{V_2\alpha}{y_2(\alpha)}\left\{\begin{array}{l}1+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^2\sigma^2 S_2{}^2}{2^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^4\sigma^4 S_2{}^4}{2^2\times 4^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^6\sigma^6 S_2{}^6}{2^2\times 4^2\times 6^2}\\ +\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^6\sigma^8 S_2{}^8}{2^2\times 4^2\times 6^2\times 8^2}+\frac{\left\{\left(\lambda_1+\lambda_2\right)^{-1}\right\}^{10}\sigma^{10} S_2{}^{10}}{2^2\times 4^2\times 6^2\times 8^2\times 10^2}+\ldots\end{array}\right\} \tag{34}$$

Solving investment equation when stock returns follows additive inverse effects :

$$\text{Let } V_{3(t)} = \sum_{m=0}^{\infty} a_m S_3^{\,m+c} = S_3^{\,c} \sum_{m=0}^{\infty} a_m S_3^{\,m} \tag{35}$$

$$V'_{3(t)} = \sum_{m=0}^{\infty} a_m \left(m+c\right) S_3^{\,m+c-1} = S_3^{\,c=1} \sum_{n=0}^{\infty} a_m \left(m+c\right) S_3^{\,m} \tag{36}$$

$$V''_{3(t)} = S_3^{\,c-2} \sum_{m=0}^{\infty} a_m \left(m+c\right)\left(m+c-1\right) S_3^{\,m} = S_3^{\,c-1} \sum_{m=0}^{\infty} a_m \left(m+c\right)\left(m+c-1\right) S_3^{\,m}, \tag{37}$$

substituting (35)-(37) into (19) and long algebraic simplifications gives

$$V_{3(t)}(S_3) = v_1 = \alpha_1 \left\{ \begin{array}{l} 1 + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2 \sigma^2 S_3^{\,2}}{2^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4 \sigma^4 S_3^{\,4}}{2^2 \times 4^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6 \sigma^6 S_3^{\,6}}{2^2 \times 4^2 \times 6^2} \\ + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8 \sigma^8 S_3^{\,8}}{2^2 \times 4^2 \times 6^2 \times 8^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10} \sigma^{10} S_3^{\,10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{array} \right\} \tag{38}$$

$$V_{3(t)} = v_2 = \alpha_2 \left\{ \begin{array}{l} \ln S_3 \left(\begin{array}{l} 1 + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2 \sigma^2 S_3^{\,2}}{2^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4 \sigma^4 S_3^{\,4}}{2^2 \times 4^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6 \sigma^6 S_3^{\,6}}{2^2 \times 4^2 \times 6^2} \\ + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8 \sigma^8 S_3^{\,8}}{2^2 \times 4^2 \times 6^2 \times 8^2} + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10} \sigma^{10} S_3^{\,10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{array} \right) \\ + a_0 S_3^{\,c} \left(\begin{array}{l} - \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2 \sigma^2 S_3^{\,2}}{2^2} - \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4 \sigma^4 S_3^{\,4}}{2^3 \times 4^2} - \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6 \sigma^6 S_3^{\,6}}{4^3 \times 6^3} \\ + \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8 \sigma^8 S_3^{\,8}}{2^5 \times 6^3 \times 8^3} - \dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10} \sigma^{10} S_3^{\,10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \ldots \end{array} \right) \end{array} \right\} \tag{39}$$

A linear combination (38) and (39) then applying the boundary conditions (21) and setting $\alpha_2 = 0$ gives the complete solution

$$V_{3(t)}(S_3) = \begin{Bmatrix} 1 + \frac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^2 \sigma^2 S_3^{\ 2}}{2^2} + \frac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^4 \sigma^4 S_3^{\ 4}}{2^2 \times 4^2} + \frac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^6 \sigma^6 S_3^{\ 6}}{2^2 \times 4^2 \times 6^2} \\ + \frac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^8 \sigma^8 S_3^{\ 8}}{2^2 \times 4^2 \times 6^2 \times 8^2} + \frac{\left\{(\lambda_1+\lambda_2)^{-1}\right\}^{10} \sigma^{10} v^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{Bmatrix} \quad (40)$$

Solving investment equation when stock returns follow multiplicative inverse effects:

From (20)

$$\text{Let } V_{4(t)} = \sum_{m=0}^{\infty} a_m S_4^{\ m+c} = S_4^{\ c} \sum_{m=0}^{\infty} a_m S_4^{\ m} \quad (41)$$

$$V'_{4(t)} = \sum_{m=0}^{\infty} a_m (m+c) S_4^{\ m+c-1} = S_4^{\ c=1} \sum_{n=0}^{\infty} a_m (m+c) S_4^{\ m} \quad (42)$$

$$V''_{4(t)} = S_4^{\ c-2} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_4^{\ m} = S_4^{\ c-1} \sum_{m=0}^{\infty} a_m (m+c)(m+c-1) S_4^{\ m}, \quad (43)$$

substituting (41)-(43) into (21) and after long algebraic simplificatins gives

$$V_{4(t)}(S_4) = v_1 = \alpha_1 \begin{Bmatrix} 1 + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^2 \sigma^2 S_4^{\ 2}}{2^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^4 \sigma^4 S_4^{\ 4}}{2^2 \times 4^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6 \sigma^6 S_4^{\ 6}}{2^2 \times 4^2 \times 6^2} \\ + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^8 \sigma^8 S_4^{\ 8}}{2^2 \times 4^2 \times 6^2 \times 8^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^{10} \sigma^{10} S_4^{\ 10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \ldots \end{Bmatrix} \quad (44)$$

$$V_{4(t)} = v_2 = \alpha_2 \left\{ \begin{array}{l} \ln S_4 \left(\begin{array}{l} 1 + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^2 \sigma^2 S_4{}^2}{2^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^4 \sigma^4 S_4{}^4}{2^2 \times 4^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6 \sigma^6 S_4{}^6}{2^2 \times 4^2 \times 6^2} \\ + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^8 \sigma^8 S_4{}^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^{10} \sigma^{10} S_4{}^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} \end{array} \right) \\ + a_0 S_4{}^c \left(\begin{array}{l} -\frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^2 \sigma^2 S_4{}^2}{2^2} - \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^4 \sigma^4 S_4{}^4}{2^3 \times 4^2} - \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6 \sigma^6 S_4{}^6}{4^3 \times 6^3} \\ + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^8 \sigma^8 S_4{}^8}{2^5 \times 6^3 \times 8^3} - \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^{10} \sigma^{10} S_4{}^{10}}{4^2 \times 6^3 \times 8^3 \times 10^3} + \dots \end{array} \right) \end{array} \right\} \quad (45)$$

A linear combination (44) and (45) then applying the boundary conditions in(21) and setting $\alpha_2 = 0$ gives the complete solution $\alpha_1 = \frac{V_4\alpha}{y_3(\alpha)}$

where

$$y_3(\alpha) = \left\{ \begin{array}{l} 1 + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^2 \sigma^2 \alpha^2}{2^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^4 \sigma^4 \alpha^4}{2^2 \times 4^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6 \sigma^6 \alpha^6}{2^2 \times 4^2 \times 6^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^8 \sigma^8 \alpha^8}{2^2 \times 4^2 \times 6^2 \times 8^2} \\ + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^{10} \sigma^{10} \alpha^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \dots \end{array} \right\}$$

$$V_{4(t)}(S_4) = \frac{V_4\alpha}{y_3(1)} \left\{ \begin{array}{l} 1 + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^2 \sigma^2 S_4{}^2}{2^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^4 \sigma^4 S_4{}^4}{2^2 \times 4^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6 \sigma^6 S_4{}^6}{2^2 \times 4^2 \times 6^2} \\ + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6 \sigma^8 S_4{}^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^{10} \sigma^{10} S_4{}^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \dots \end{array} \right\} \quad (46)$$

Non-homogenous part of investment equation when stock returns follow additive inverse effects

To get the particular solution of (19) which is the non-homogenous part of the problem, we solve using the method of undetermined coefficients.

$$V_{4(t)}(S_4) = \alpha_1 \left\{ \begin{array}{l} 1 + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^2 \sigma^2 S_4{}^2}{2^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^4 \sigma^4 S_4{}^4}{2^2 \times 4^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6 \sigma^6 S_4{}^6}{2^2 \times 4^2 \times 6^2} \\ + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^6 \sigma^8 S_4{}^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \frac{\left\{(\lambda_1\lambda_2)^{-1}\right\}^{10} \sigma^{10} S_4{}^{10}}{2^2 \times 4^2 \times 6^2 \times 8^2 \times 10^2} + \dots \end{array} \right\}$$

From the complementary function for (19)

$$V_{3(t)c}\left(S_3\right)=C_c\left[\begin{array}{l}1+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_3)\right\}^2}{2^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_3)\right\}^4}{\left(2\times4\right)^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_3)\right\}^6}{\left(2\times4\times6\right)^2}\\+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_3)\right\}^8}{\left(2\times4\times6\times8\right)^2}+\dfrac{\left\{(\lambda_1+\lambda_2)^{-1}\sigma S_3)\right\}^{10}}{\left(2\times4\times6\times8\times10\right)^2}+\ldots\end{array}\right] \quad (47)$$

Consider the particular solution

$$V_p\left(S\right)=A_0+A_1S^2+A_2S^4+A_3S^6+A_4S^8+A_5S^{10} \quad (48)$$

$$V_p'\left(S\right)=2A_1S+4A_2S^3+6A_3S^5+8A_4S^7+10A_5S^9 \quad (49)$$

$$V_p''\left(S\right)=2A_1+12A_2S^2+30A_3S^4+56A_4S^6+90A_5S^8 \quad (50)$$

Putting (48)-(50) into (19) gives the following:

$$\begin{array}{l}\Rightarrow 4A_1+16A_2S^2+36A_3S^4+64A_4S^6+100A_5S^8-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_0-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_1S^2\\-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_2S^4-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_3S^6-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_4S^8-\left\{(\lambda_1+\lambda_2)^{-1}\sigma\right\}^2A_5S^{10}\equiv\\-\varphi-GR_tV_{4(2)}\left(\begin{array}{l}1+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\sigma S\right\}^2}{2^2}+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\sigma S\right\}^{24}}{(2\times4)^2}+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\sigma S\right\}^6}{(2\times4\times6)^2}+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\sigma S\right\}^8}{(2\times4\times6\times8)^2}\\+\dfrac{\left\{(\lambda_1\lambda_2)^{-1}\sigma S\right\}^{10}}{(2\times4\times6\times8\times10)^2}\end{array}\right)\end{array} \quad (51)$$

Taking like terms and simplifying after some algebraic expressions gives

$$\begin{array}{l}V_3\left(S_3\right)=c+A_0+\left(\dfrac{c((\lambda_1+\lambda_2)^{-1}\sigma)^2}{4}+A_1\right)S_3^{\ 2}+\left(\dfrac{c((\lambda_1+\lambda_2)^{-1}\sigma)^4}{\left(2\times4\right)^2}+A_2\right)S_3^{\ 4}\\+\left(\dfrac{c((\lambda_1+\lambda_2)^{-1}\sigma)^6}{\left(2\times4\times6\right)^2}+A_3\right)S_3^{\ 6}+\left(\dfrac{c((\lambda_1+\lambda_2)^{-1}\sigma)^8}{\left(2\times4\times6\times8\right)^2}+A_4\right)S_3^{\ 8}+\left(\dfrac{c((\lambda_1+\lambda_2)^{-1}\sigma)^{10}}{\left(2\times4\times6\times8\times10\right)^2}+A_5\right)S_3^{\ 10}\end{array} \quad (52)$$

where $A_0, A_1, A_2, A_3 A_4$ *and* A_5 are constants which is seen in appendix 1

3 Results and Discussion

In this Section, we present the computational results for the problem formulated in (17)-(21) whose solution is in (28), (34), (46) and (52). These solutions are implemented in a mathematica programming language. However, the following parameter values were explicitly used in the simulation study: $c = 1.0, \lambda_1 = 180, \lambda_2 = 120, \varphi = 1.0, V_{2\alpha} = 0.1$ and $V_\alpha = 0.1$. The non-linear 2nd order differential equations governing the problem are solved analytically for four different investment equations.

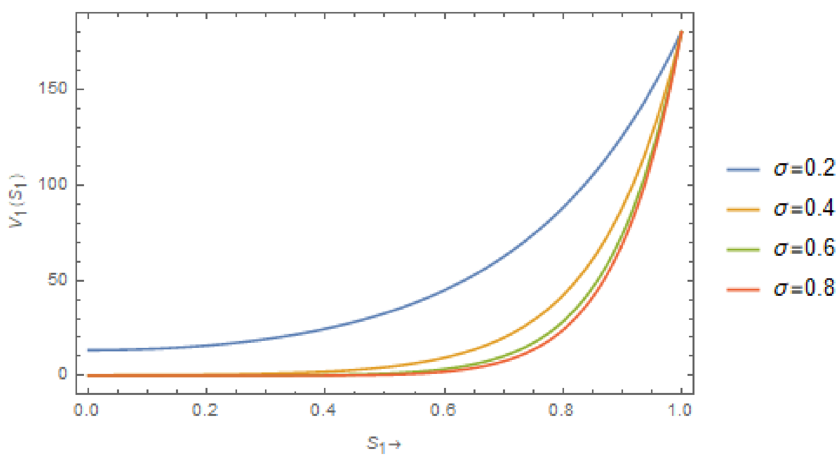

Figure 1: Stock (1) (S_1) stock return profiles against different values S of volatility(σ)

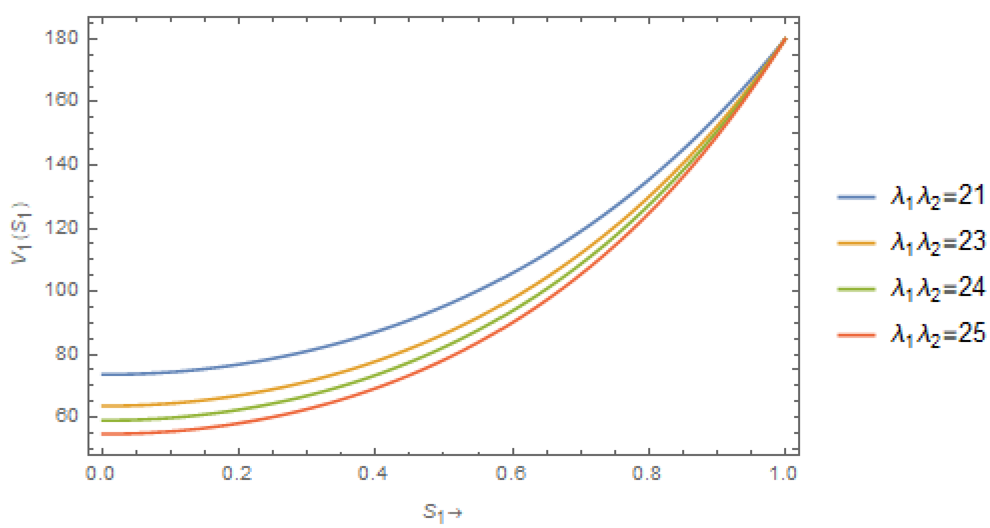

Figure 2: Plot of Multiplicative effects stock returns for assessing the value of assets for company (1)

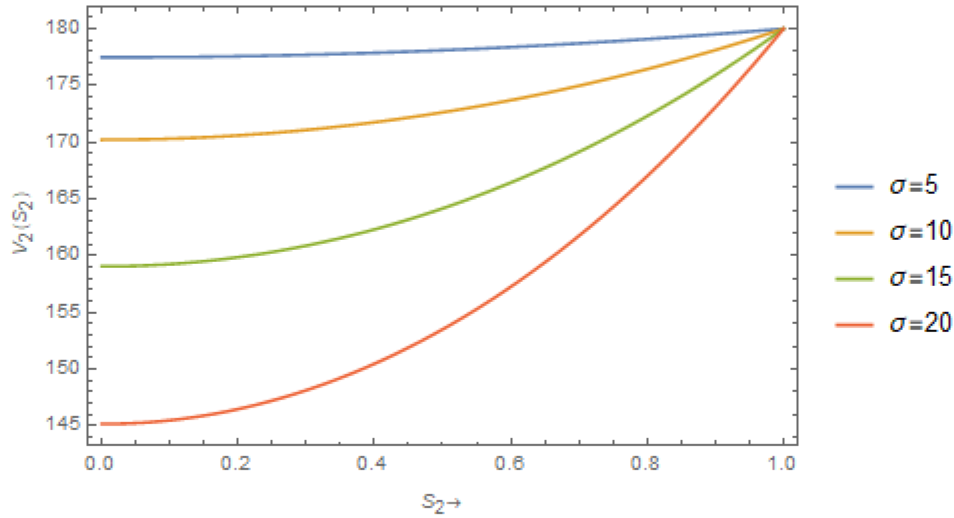

Figure 3: Stock (2) (S_2) stock return profiles against different values S of volatility(σ)

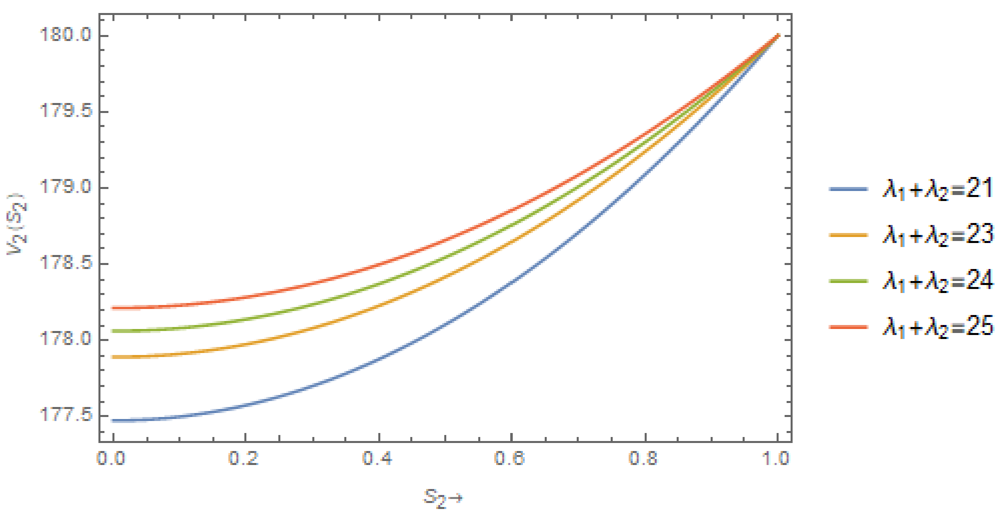

Figure 4: Plot of Additive inverse stock returns for assessing the value of assets for company (2)

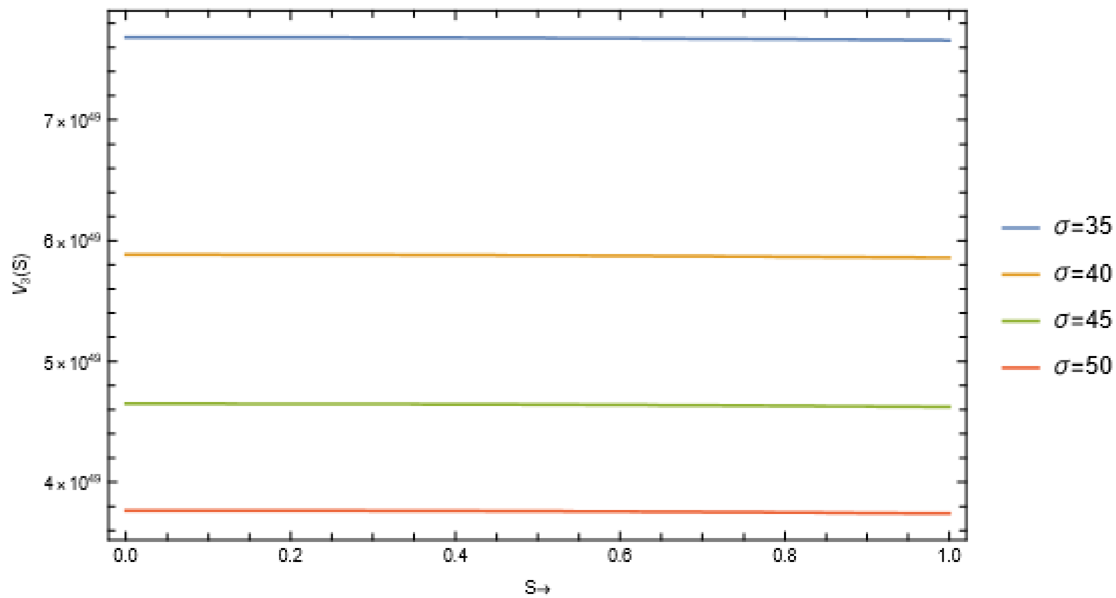

Figure 5: Stock (3) (S_3) stock return profiles against S with different values of volatility(σ)

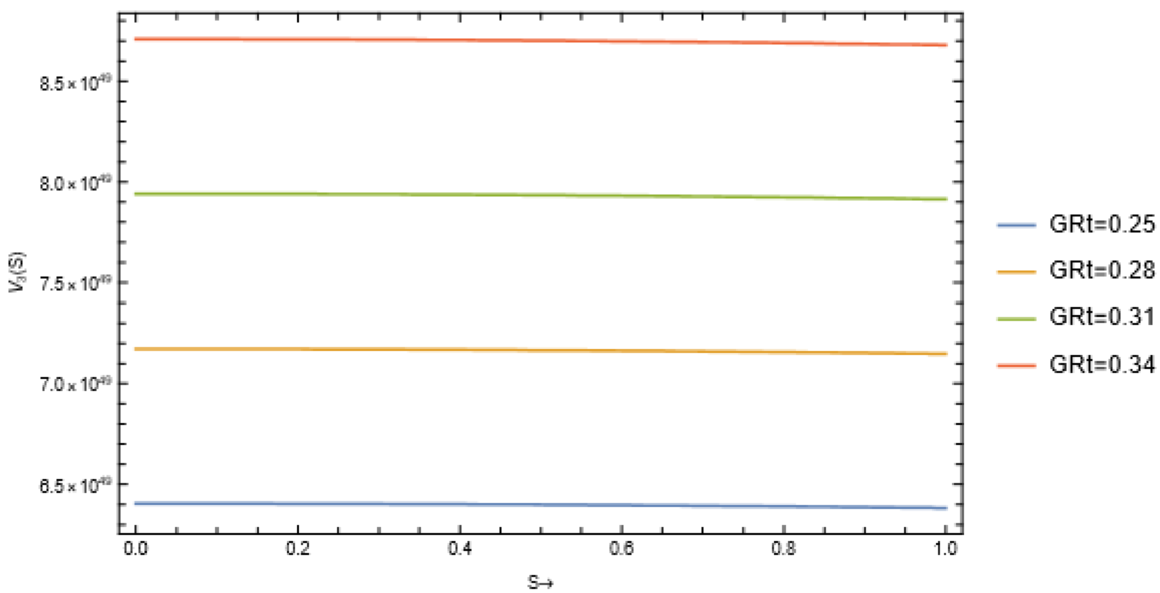

Figure 6: Stock (3) (S_3) stock return profiles against S with different values of volatility(GR_t)

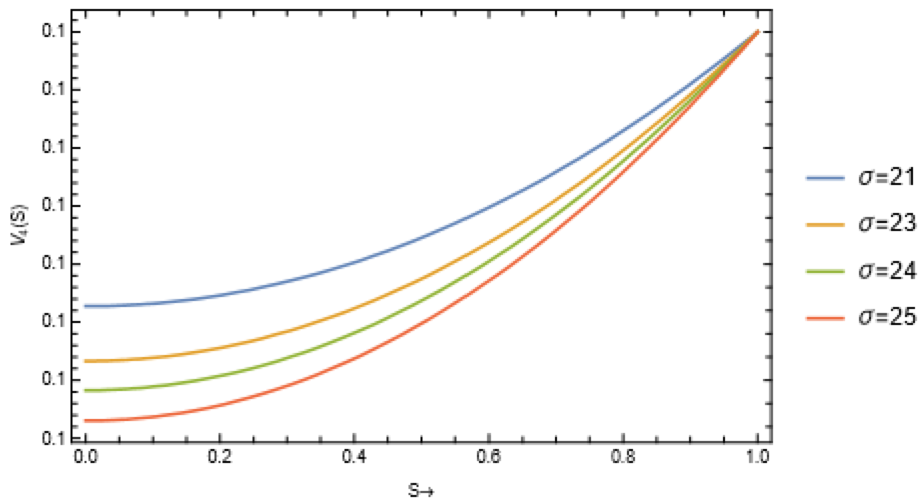

Figure 7: Stock (4) (S_4) stock return profiles against S with different values of volatility

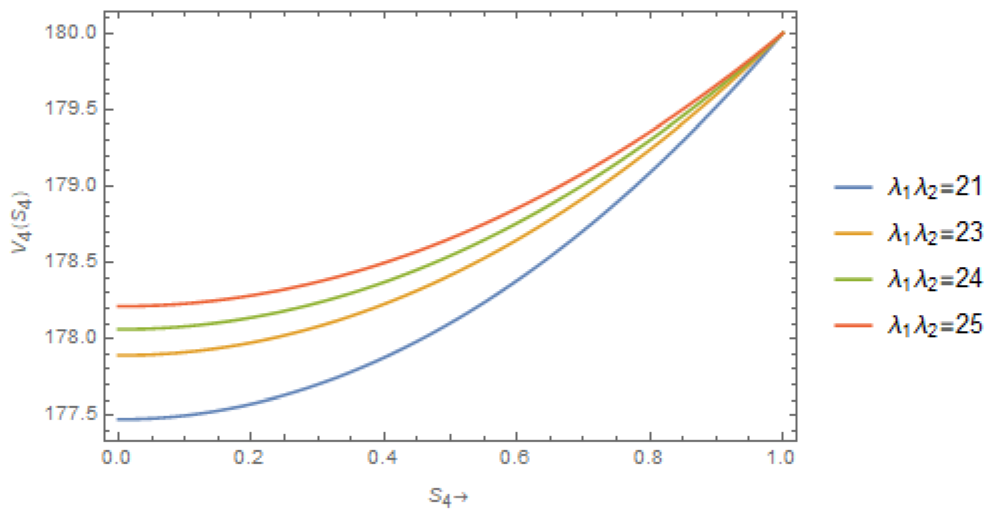

Figure 8: Plot of Multiplicative inverse stock returns for assessing the value of assets for company (4)

Figure 1 shows the variation of Stock (1) rate of return with volatility parameter. It can be seen that Stock (1) return reduces with increasing stock volatility. This is absolutely correct, because when there's high volatility in the market price, investors find it difficult to invest; thereby causing low rate of return in an investment. There will be no investor who will like to maximize loss in an investment; causing a kind of panic buying, tensions everywhere in stock trading. Visual inspection of the plot shows high levels of price changes in the capital investment.

Figure 2 shows the variation of Stock (1) rate of return which follows multiplicative effects. In this situation the results indicate increase in return rates decreases the gain of asset. This remark shows that, there are some stochastic bubbles in financial market which affected the investments. For instance, selling an auction price due to products expiring dates etc, one will be compelled to dispose the commodities on any amount in order to avoid expired products. So, in that case the levels of profit cannot be compared to the original amount of the investments. Also, the issue of over- staff in managerial cadre of the investments may causes the rate of return to decrease. This could also mean there is a serious crash worth of millions in any of the currency for capital investment. A crash simply implies a substantial droplet in the overall value of a market, creating unnecessary situation where lots of investors will not invest in capital market as a result of massive losses. The disadvantage of the scenario is that it results to panic buying, selling and deteriorating of markets which of course affects the entire masses.

Figure 3 shows the variation of Stock (2) rate of return with volatility parameter. A little increase in stock volatility parameter reduces stock return rates. That is, to say that volatility affects trading business over time in the investment which might be indexed with millions of naira. This situation informs investors on the activities of volatility in the capital investment hence it measures the price changes.

Figure 4 shows the variation of Stock (2) rate of return parameter. It is clear that increasing return rates when stock return rates follow additive inverse effect also increases the value of asset pricing throughout the stipulated trading days. It implies that additive inverse effect is profit maximizing in terms of long- and short-term investment plans.

Figure 5 shows the variation of Stock (3) rate of return with volatility parameter. It can be seen that Stock (3) rate of returns is pararrel; hence an increase in stock volatility decreases rate of returns. Therefore, profit making may not be possible due to the uniqueness of horizontal trend lines.
Figure 6 shows the variation of Stock (3) return with growth rate parameter. It is observed that a little increase in the growth-rate parameter increases the return rate of the capital investment over the trading days. This situation is quite obvious because when an investor increases its turn-over, the financial strength of the business will rise to a certain level.

Figure 7 shows the variation of Stock (4) return with volatility parameter. It is observed that increase in stock volatility reduces the rate of return of an investment. This is correct because when there's a high volatility in price formation of a commodity; investors will not be willing to invest, even up to final consumers will not also be eager to buy from the investors thereby causing low rate of return in the investment. In another scenario, the plot has an upward trend which moves separately and converges to a point during the trading days. This remark shows that rate of return in multiplicative inverse has a limit of how it can yield profit to an investor; this also describes the formative of random walk-in stochastic process.

In Figure 8, increasing return rate when stock returns follow multiplicative inverse effect also increases the value of asset pricing which is beneficial to every investor in terms of profit making. These remarks showing such pattern of trend implies high levels of progress during the period of an investments.

4 Conclusion

This paper, considered four investment equations with stochastic and stock return rates parameters in the model. Detailed conditions are achieved which govern asset price return rates through multiplicative effects, additive invers effects , multiplicative inverse effects ,stock volatility and asset growth-rates of assets parameter respectively .Therefore, the impacts on the value of asset prices of investors in capital market were analyzed; which shows: (i) Stock (1) return reduces with increasing stock volatility(ii) increasing growth rates in an investment automatically grows the financial strength of the investment over time (iii) A little increase in stock volatility parameter reduces stock return rates.(iv) increasing return rates when stock return rates follows additive inverse effect also increases the value of asset pricing throughout the stipulated trading days.(v) an increase in stock volatility decreases rate of returns(vi) a little increase in the growth-rate parameter increases the return rate of the capital investment over the trading days.

APPENDIX 1

Constants of coupled investment equations

$$\Rightarrow A_5 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[\frac{\left((\lambda_1\lambda_2)^{-1}\sigma\right)^{10} GR_tV_{4(t)}}{(2\times4\times6\times8\times10)^2}\right], \Rightarrow A_4 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[100A_5+\frac{\left((\lambda_1\lambda_2)^{-1}\sigma\right)^{8} GR_tV_{4(t)}}{(2\times4\times6\times8)^2}\right]$$

$$\Rightarrow A_3 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[64A_4-\frac{\left((\lambda_1\lambda_2)^{-1}\sigma\right)^{6} GR_tV_{4(t)}}{(2\times4\times6)^2}\right], \Rightarrow A_2 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[36A_3-\frac{\left((\lambda_1\lambda_2)^{-1}\sigma\right)^{4} GR_tV_{4(t)}}{(2\times4)^2}\right]$$

$$\Rightarrow A_1 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[16A_2-\frac{\left((\lambda_1\lambda_2)^{-1}\sigma\right)^{2} GR_tV_{4(t)}}{4}\right], \Rightarrow A_0 = -\frac{1}{\left((\lambda_1+\lambda_2)^{-1}\sigma\right)^2}\left[4A_1+\varphi-GR_tV_{4(t)}\right]$$

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