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The approximate solution of advanced stochastic time-delay differential equations using block Adams Moulton methods and its application in claims settlement for policyholders satisfaction in the Nigerian insurance industry

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Abstract

In this paper, Block Adams Moulton Methods (BAMM) is implemented for the approximate solution of Advanced Stochastic Time-Delay Differential Equations (ASTDDEs) to determine the level of policyholders' satisfaction in settling their claims in Nigerian Insurance Industry. With the help of continuous development of multistep collocation method by matrix inversion techniques, the discrete schemes of the proposed method (BAMM) were derived. The analysis of basic properties of the method such as order and error constant, consistency, zero stability, convergence and region of absolute stability were carried-out and proved satisfactory. The numerical solutions of the method were obtained, computed and presented graphically by solving some advanced stochastic time-delay differential equations using the derived discrete schemes which revealed the effect of advanced stochastic time-delay on the policyholders' claims settlement. The accuracy of the method was examined by comparing the results obtained with other existing methods and was found to give better approximation.

Keywords: advanced stochastic time-delay differential equations; Block Adams Moulton methods; policyholder; claims settlement; insurance industry; Nigeria

1 Introduction

According to [3, 18, 14], claim is an official request made by a policyholder to an insurance industry demanding for a payment based on the terms of the insurance policy. [13, 7, 8], defined claims settlement as corporate policies and industry practices that insurance companies use to validate policyholder payment or reimbursement requests. It has been noticed many-a-times that the claim settlement process for policyholders in Nigerian Insurance Industries takes longer time than expected. This can happen for various reasons such as policyholders not submitting their claims on time, loss of insurance policy

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papers/documents, not following the right pre-authorization claims process for cashless medical treatments and advanced time-delay in exchange of information or documents between organizations, hospitals and insurance companies. In reality, the above stated reasons always result to advanced random/stochastic time-delays between the time the claims occur and the time of settlement of such claims. Advanced stochastic time-delay differential equation (ASTDDE) is a stochastic differential equation where the increment of the process depends not only on current state but also on the advanced/future part of the system being modeled which contains the random/stochastic noise term.

To reveal the applications of SDDEs in applied sciences, economics and engineering, authors such as [6, 19, 2, 17, 9, 4, 1] applied Euler-Maruyama scheme and interpolation techniques such as Hermite, Nordsieck, Newton divided difference and Neville's interpolation to construct continuous split-step schemes of SDDE on a continuous interval $t_0 \le t \le t_a$ and evaluation of the delay terms for the numerical solutions of SDDEs but encountered some obstacles as studied by [12]. In order to overcome the advanced random/stochastic time-delays experienced by the policyholders in settling their claims by the insurance companies and the obstacles encountered in the use of interpolation techniques for the evaluation of the delay terms, we applied Block Adams Moulton Methods (BAMM) as a linear multistep collocation method to discretize ASTDDEs on a discrete interval $[t_0, t_a)$ in order to obtain its discrete schemes for its numerical solution.

From [10], advanced stochastic time-delay differential equation (ASTDDE) can be express as

$$dy(t) = \mu(y(t), y(t+\tau), t)dt + \varepsilon(y(t), y(t+\tau), t)d\beta(t) \quad \text{for } t > t_0, \tau > 0$$
$$y(t) = \phi(t), \text{ for } t \le t_0 \tag{1}$$

where $\varphi(t)$ is the initial function, $y(t)$ is the stochastic process of the current state, t is the time, τ is called the delay, $(t+\tau)$ is the advanced/future time-delay term and $y(t+\tau)$ is the solution of the advanced time-delay term on the drift part of (1). $\beta(t)$ is the Standard Brownian Motion with its differential equivalence as $d\beta(t)$ as the volatility noise term or Wiener process together with solution of the advanced time-delay term as $y(t+\tau)ds(t)$ on the volatility or diffusion part of (1). The drift part of the equation (1) $dy(t) = \mu(y(t), y(t+\tau), t)dt$ is deterministic and takes care of the time the claims occur and the time such claims are settled without any risk involved. The volatility or diffusion part $dy(t) = \varepsilon(y(t), y(t+\tau), t)ds(t)$ is stochastic which takes care of the advanced random/stochastic time-delays between the time the claims occur and the time of settlement of such claims.

2 Derivation of the method

By matrix inversion techniques of the continuous construction of the *k*-step multistep collocation method formulated by [15], the discrete schemes of Block Adams Moulton Methods (BAMM) for step numbers k = 2 and 3 are obtained and presented as;

For $k=2$ of BAMM

$$y_w = y_{w+1} - \frac{5}{12} a f_w - \frac{8}{12} a f_{w+1} + \frac{1}{12} a f_{w+2}$$
$$y_{w+2} = y_{w+1} - \frac{1}{12} af_w + \frac{8}{12} af_{w+1} + \frac{5}{12} a f_{w+2} \tag{2}$$

For $k=3$ of BAMM

$$y_w = y_{w+2} - \frac{1}{3} a f_w - \frac{4}{3} a f_{w+1} - \frac{1}{3} a f_{w+2}$$

$$y_{w+1} = y_{w+2} + \frac{1}{24} a f_w - \frac{13}{24} a f_{w+1} - \frac{13}{24} a f_{w+2} + \frac{1}{24} a f_{w+3}$$

$$y_{w+3} = y_{w+2} + \frac{1}{24} a f_w - \frac{5}{24} a f_{w+1} + \frac{19}{24} a f_{w+2} + \frac{9}{24} a f_{w+3} \tag{3}$$

3 Analysis of basic properties of the method

Using the conditions proposed by [11] and [5], the order and error constant, consistency, zero stability, convergence and region of absolute stability of (2) and (3) were analyzed.

3.1 Order and error constant

According to [11], the Linear Multistep Method is said to be of order m if $c_0 = c_1 = 0, \ldots, c_m = 0$ but $c_{m+1} \neq 0$ and c_{m+1} is called the error constant.

The order and error constants for (2) are obtained as follows

$$c_0 = c_1 = c_2 = c_3 = \begin{pmatrix} 0 & 0 \end{pmatrix}^T$$

$$c_4 = \begin{pmatrix} -\frac{1}{24} & -\frac{1}{24} \end{pmatrix}^T$$

Hence, (2) has an order $m=3$ and error constant $\begin{pmatrix} -\frac{1}{24} & -\frac{1}{24} \end{pmatrix}^T$

Using the same techniques, (3) can be presented as follows

$$c_0 = C_1 = C_2 = C_3 = C_4 = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$$

$$c_5 = \begin{pmatrix} \frac{1}{90} & -\frac{11}{720} & -\frac{19}{720} \end{pmatrix}^T.$$

Therefore, (3) has order $m=4$ and error constant, $\begin{pmatrix} \frac{1}{90} & -\frac{11}{720} & -\frac{19}{720} \end{pmatrix}^T$.

3.2 Consistency

According to [11], a computational method is said to be consistent if the order e is greater than 1 i.e., $e \geq 1$. Since the order of our proposed method BAMM as analyzed for the discrete schemes (2) and (3) is greater than 1 i.e. $m \geq 1$, hence, the method is consistent.

3.3 Zero stability analysis

In [5], a computational method is said to be zero stable if the roots $r_s, s = 1, 2, 3, \ldots, n$ of the first characteristic polynomial $\varphi(e)$ expressed as $\varphi(e) = \det(eZ_2^{(1)} - Z_1^{(1)})$ satisfies $|e_s| \leq 1$ and the roots $|e_s|$ is simple or distinct.
The zero stability for (2) is examined as follows

$$\begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} y_{w+1} \\ y_{w+2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} y_{w-1} \\ y_w \end{pmatrix}$$

$$+a\begin{pmatrix} -\frac{2}{3} & \frac{1}{12} \\ \frac{2}{3} & \frac{5}{12} \end{pmatrix}\begin{pmatrix} f_{w+1} \\ f_{w+2} \end{pmatrix} + a\begin{pmatrix} 0 & -\frac{5}{12} \\ 0 & -\frac{1}{12} \end{pmatrix}\begin{pmatrix} f_{w-1} \\ f_w \end{pmatrix},$$

where

$$Z_2^{(1)} = \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}, Z_1^{(1)} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, V_2^{(1)} = \begin{pmatrix} -\frac{2}{3} & \frac{1}{12} \\ \frac{2}{3} & \frac{5}{12} \end{pmatrix} \text{ and } U_2^{(1)} = \begin{pmatrix} 0 & -\frac{5}{12} \\ 0 & -\frac{1}{12} \end{pmatrix}$$

$$\begin{aligned} \varphi(e) &= \det(eZ_2^{(1)} - Z_1^{(1)}) \\ &= \left|eZ_2^{(1)} - Z_1^{(1)}\right| = 0 \end{aligned} \tag{4}$$

Now we have,

$$\varphi(e) = \left| e\begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right| = \begin{pmatrix} -e & 0 \\ -e & e \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \varphi(e) = \begin{pmatrix} -e & 1 \\ -e & e \end{pmatrix}$$

Using Maple (18) software, we obtain

$\varphi(e) = e^2 + e$

$\Rightarrow e^2 + e = 0$

$\Rightarrow e_1 = 1, e_2 = 0$. Since $|e_i| \leq 1,\ i = 1, 2$, the discrete schemes in (2) is zero stable.

Applying the same approach for (3), we have

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} y_{w+1} \\ y_{w+2} \\ y_{w+3} \end{pmatrix}=\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} y_{w-2} \\ y_{w-1} \\ y_{w} \end{pmatrix}$$

$$+a\begin{pmatrix} -\frac{4}{3} & \frac{1}{3} & 0 \\ -\frac{13}{24} & -\frac{13}{24} & \frac{1}{24} \\ -\frac{5}{24} & \frac{19}{24} & \frac{3}{8} \end{pmatrix}\begin{pmatrix} f_{w+1} \\ f_{w+2} \\ f_{w+3} \end{pmatrix}+a\begin{pmatrix} 0 & 0 & -\frac{1}{3} \\ 0 & 0 & \frac{1}{24} \\ 0 & 0 & \frac{1}{24} \end{pmatrix}\begin{pmatrix} f_{w-2} \\ f_{w-1} \\ f_{w} \end{pmatrix}$$

where

$$Z_2^{(2)}=\begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix}, Z_1^{(2)}=\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, V_2^{(2)}=\begin{pmatrix} -\frac{4}{3} & \frac{1}{3} & 0 \\ -\frac{13}{24} & -\frac{13}{24} & \frac{1}{24} \\ -\frac{5}{24} & \frac{19}{24} & \frac{3}{8} \end{pmatrix} \text{ and } U_2^{(2)}=\begin{pmatrix} 0 & 0 & -\frac{1}{3} \\ 0 & 0 & \frac{1}{24} \\ 0 & 0 & \frac{1}{24} \end{pmatrix}$$

$$\begin{aligned}\varphi(e) &= \det(eZ_2^{(2)}-Z_1^{(2)}) \\ &= \left|eZ_2^{(2)}-Z_1^{(2)}\right| = 0\end{aligned} \qquad (5)$$

Now we have,

$$\varphi(e)=\left|e\begin{pmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix}-\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right|=\left|\begin{pmatrix} 0 & -e & 0 \\ e & -e & 0 \\ 0 & -e & e \end{pmatrix}-\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right|$$

$$\Rightarrow \varphi(e)=\begin{pmatrix} 0 & -e & 1 \\ e & -e & 0 \\ 0 & -e & e \end{pmatrix}$$

The following are obtained using Maple (18) software,

$$\begin{aligned}&\varphi(e)=e^3-e^2 \\ &\Rightarrow e^3+e^2=0 \\ &\Rightarrow e_1=1, e_2=0, e_3=0.\end{aligned}$$

Since $|e_i|\le 1,\ i=1,2,3$, the discrete schemes in (3) is zero stable.

3.4 Convergence

Theorem 1: The necessary and sufficient condition for a linear multistep method to be convergent as stated by [5] is that it must be consistent and zero stable. Since the discrete schemes (2) and (3) are both consistent and zero stable, therefore the method is convergent.

3.5 Region of absolute stability

The regions of absolute stability of the numerical methods for ASTDDEs are considered. We considered finding the P - and Q -stability by applying (2) and (3) to the ASTDDEs of this form

$$dy(t) = \mu(y(t) + y(t+\tau))dt + \varepsilon(y(t) + y(t+\tau))d\beta(t) \quad \text{for } t > t_0, \tau > 0$$
$$y(t) = \phi(t), \text{ for } t \leq t_0 \tag{6}$$

where $\phi(t)$ is the initial function, μ, ε are complex coefficients, $\tau = \eta a, \eta \in \sum^{+}$, a is the step size and $\eta = \frac{\tau}{a}, \eta$ is a positive integer. Let $H_1 = a\mu$ and $H_2 = a\varepsilon$, then the P -and Q -stability of (2) and (3) for $\eta = 1$ are investigated, plotted using Maple 18 and MATLAB and presented in figure 1 to 4 below.

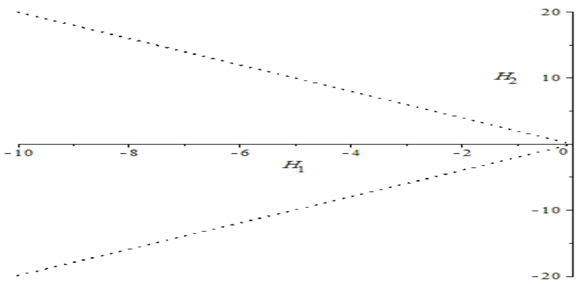

Figure 1. Region of P-stability (BAMM) in (2)

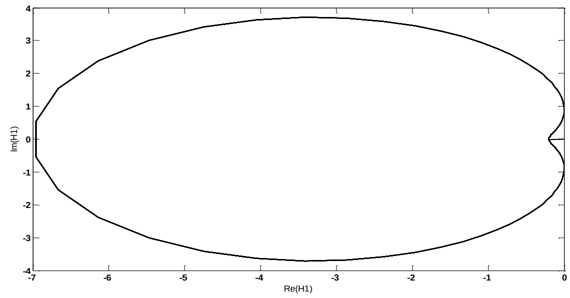

Figure.3. Region of Q-stability (BAMM) in (2)

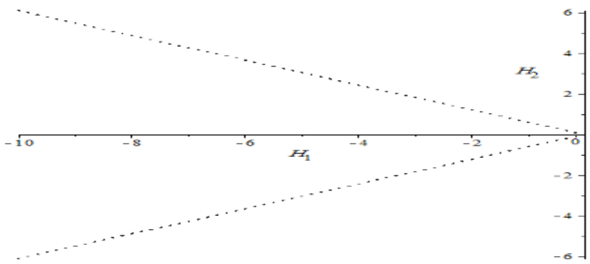

Figure 2. Region of P-stability (BAMM) in (3)

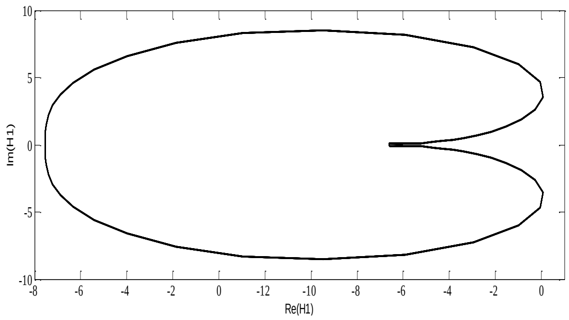

Figure 4. Region of Q-stability (BAMM) in (3)

The P-stability regions in Figs 1 to 2 lie inside the open-ended region while the Q-stability regions in Figs 3 to 4 lie inside the enclosed region.

4 Evaluations of Delay Terms

Here, we shall formulate two accurate and efficient mathematical expressions for the evaluation of the advanced time-delay terms on the drift and the volatility noise term on the diffusion part of the stochastic time-delay differential equations. The advanced time-delay term $(t+\tau)$ shall be evaluated with the accurate and efficient formula of this form

$$\delta_{w+j}(t) = \frac{w}{d}\left(\left(dc + \left(w + j - \eta - 1\right)a\right)\right), d \neq 0 \quad (7)$$

Using normalized Brownian Motion, we formulated an expression to evaluate the volatility noise term $d\beta(t)$ such that the distribution is Gaussian with $N(0,1)$ whose mean μ is 0 and the standard deviation σ is 1. The random process is expressed as

$$\beta(t)=\frac{1}{\sqrt{((w+j-\eta-1)a)\pi}}e^{-t^2/(w+j-\eta-1)a}, \text{ for } t\geq 0 \tag{8}$$

Then by differentiating (8), it gives

$$d\beta(t)=\frac{-2t}{(w+j-\eta-1)a\sqrt{((w+j-\eta-1)a)\pi}}e^{-t^2/(w+j-\eta-1)a}, \text{ for } t\geq 0 \tag{9}$$

where $j\in(-k,k),k$ is a step number, $\eta=\frac{\tau}{a}\in Z,\tau=\eta a;\tau$ is the delay term, $w=0,1,2,...,W-1$ and W is the number of solutions in the giving interval which is implemented to evaluate the volatility noise term $d\beta(t)$.

5 Numerical Computations

In this section, the evaluated advanced time-delay term and the volatility noise term using the two expressions (7) and (9) developed above shall be incorporated into some advanced stochastic time-delay differential equations before its evaluation with the discrete schemes (2) and (3) at constant step size a = 0.01 to obtain its computational solutions of $dy(t)$.

5.1 Numerical problems

Problem 1

$dy(t)=\cos(t)((y(y(t)+2))dt+(y(y(t)+2))d\beta(t),\ 0\leq t\leq 3$

$\alpha(t)=1, t\geq 0$

Exact solution $\alpha(t)=1+\sin(t)$.

Problem 2

$dy(t)=-(y(t-1+e^{-t})+\sin(t-1+e^{-t})+\cos(t))dt+(y(t-1+e^{-t})+\sin(t-1+e^{-t})+\cos(t))d\beta(t),$

$0\leq t\leq 3$

$\alpha(t)=\sin(t), t\leq 0$

Exact Solution $\alpha(t)=\sin(t)$.

6 Results, graphical presentations and discussions

The above problems were solved using the discrete schemes (2) and (3) derived by Block Adams Moulton Methods (BAMM) and the results of the absolute random errors obtained are presented in tables 1 to 2

Table 1: Absolute Random Errors of BAMM for $k = 2$ and 3 using Problem 1

t	K = 2 Absolute Random Error	K = 3 Absolute Random Error
1	0.845719238	0.845719236
2	0.917929971	0.917929971
3	0.154361373	0.154271898
4	0.73881897	0.738732318
5	0.935633477	0.935805002
6	0.250690119	0.251117461
7	0.692001936	0.691574594
8	1.030780292	1.03035295
9	0.460997752	0.459635207
10	0.487584536	0.488442522
11	0.934908259	0.936260796
12	0.462769879	0.464605564
13	0.503583055	0.501747369
14	1.083678941	1.081843256
15	0.75340265	0.751219726
16	0.17474429	0.177026758
17	0.838660557	0.840598537
18	0.618718851	0.620290407
19	0.290043944	0.288472389
20	1.060916364	1.059344809
21	0.989485244	0.990763859
22	0.148704384	0.14812801
23	0.68801341	0.686514681
24	0.746870479	0.743291542
25	0.022305383	0.02588432
26	0.913027428	0.916606365
27	1.098762672	1.106099222
28	0.405121759	0.411231154
29	0.539433381	0.532758559
30	0.873856852	0.866656324

CPU time of BAMM for k = 2 is 0.03s and k = 3 is 0.10s

Table 2: Absolute Random Errors of BAMM for *k* = 2 and 3 using Problem 2

t	K = 2 Absolute Random Error	K = 3 Absolute Random Error
1	0.837046383	0.836665921
2	0.895889047	0.895889047
3	0.123316526	0.119881874
4	0.783541932	0.782755906
5	0.989876475	0.99335545
6	0.319076026	0.321521156
7	0.613683492	0.610862708
8	0.93800642	0.93556129
9	0.358412665	0.351507123
10	0.604326121	0.606307176
11	1.060231835	1.067356323
12	0.600721595	0.608098621
13	0.360093698	0.352397372
14	0.931089965	0.92371294
15	0.600913805	0.585224284
16	0.329945098	0.339136313
17	0.98484421	1.000672496
18	0.75760374	0.77740328
19	0.127087532	0.146959007
20	0.861093034	0.880892574
21	0.742877411	0.77480406
22	0.145936521	0.119247721
23	1.037968136	1.007150612
24	1.15515455	1.117229542
25	0.447253474	0.409685871
26	0.377751821	0.415676829
27	0.500230288	0.540376758
28	0.261660995	0.217469289
29	1.267095814	1.228416796
30	1.666628252	1.678375518

CPU time of BAMMs for k = 2 is 0.07s and k = 3 is 0.04s

6.1 Graphical presentation of results

Using Microsoft Excel, the Absolute Random Error Results of BAMM for Problem 1 and 2 in Tables 1 and 2 are presented as;

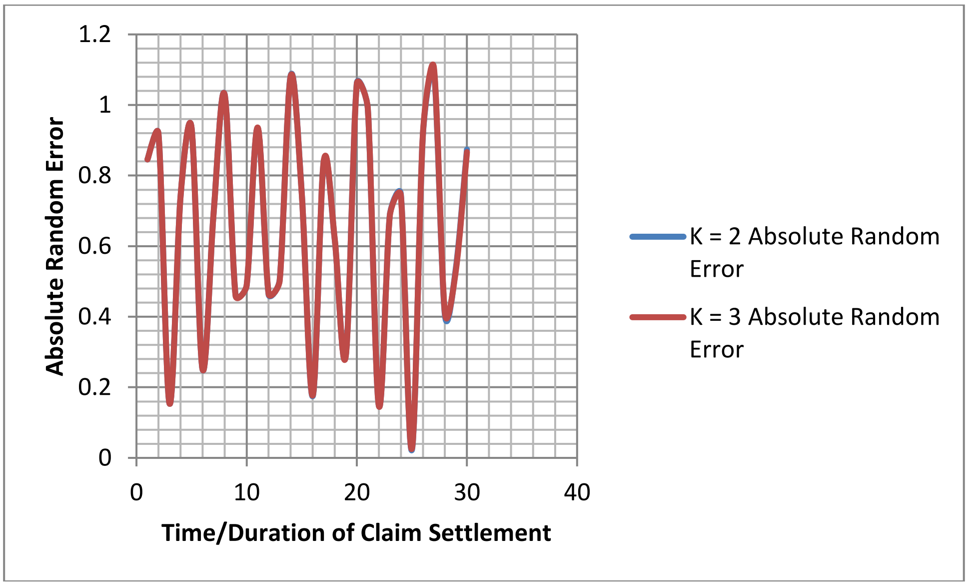

Figure 5. Advanced Stochastic Time-Delay DDEs Absolute Random Error Results for Problem 1

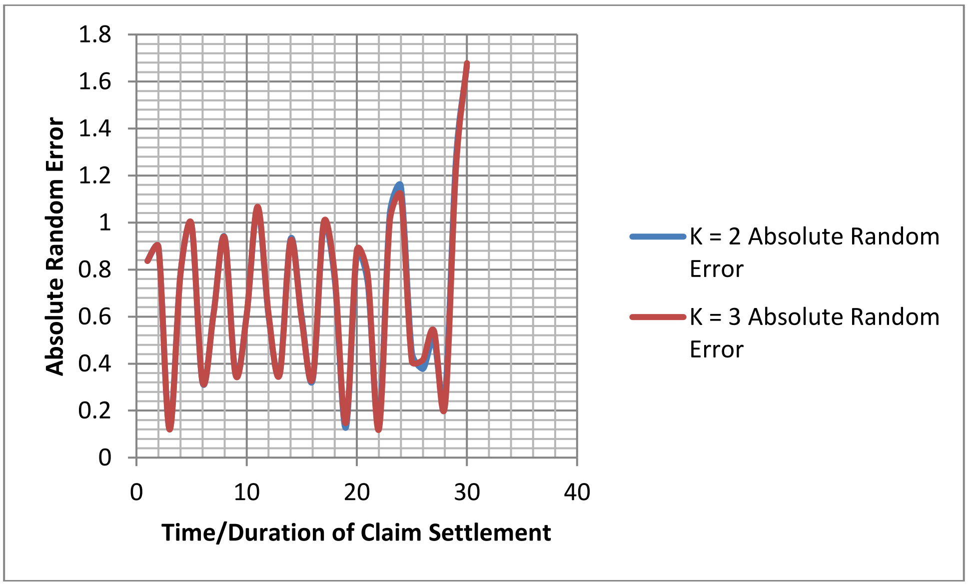

Figure 6. Advanced Stochastic Time-Delay DDEs Absolute Random Error Results for Problem 2

6.2 Comparison of results

In order to determine the accuracy, efficiency and advantage of our method BAMM, we compared the absolute maximum errors of our method with other existing methods in [6, 2,16]

Table 3: Comparison between the Maximum Absolute Random Errors (MARE) of our method BAMM for k = 2 and 3 with [6, 2, 16] for constant step size a = 0.01 for Problem 1

Numerical Method	COMPARED MAREs with [6,2,16]
BAMM MARE for k = 2	1.098762672
BAMM MARE for k = 3	1.106099222
CSSEMM MARE for k = 2	4.76E-02
CSSEMM MARE for k = 3	9.17E-02
CSSEMM MARE for k = 4	1.62E-01
EMM MARE for k = 2	1.84E-02
EMM MARE for k = 3	4.04E-03
EMM MARE for k = 4	9.73E-04
BSM MARE for k = 2	7.04E-01
BSM MARE for k = 3	7.04E-01
BSM MARE for k = 4	7.04E-01

Table 4: Comparison between the Maximum Absolute Random Errors (MARE) of our method BAMM for k = 2 and 3 with [6, 2, 16] for constant step size a = 0.01 for Problem 2

Numerical Method	COMPARED MAREs with [6,2,16]
HEBAMM MARE for k = 2	1.666628252
HEBAMM MARE for k = 3	1.678375518
CSSEMM MARE for k = 2	3.18E-02
CSSEMM MARE for k = 3	5.90E-02
CSSEMM MARE for k = 4	1.37E-01
EMM MARE for k = 2	1.09E-01
EMM MARE for k = 3	4.91E-02
EMM MARE for k = 4	2.44E-02
BSM MARE for k = 2	6.96E-01
BSM MARE for k = 3	6.96E-01
BSM MARE for k = 4	6.96E-01

6.3 Graphical presentation for compared results

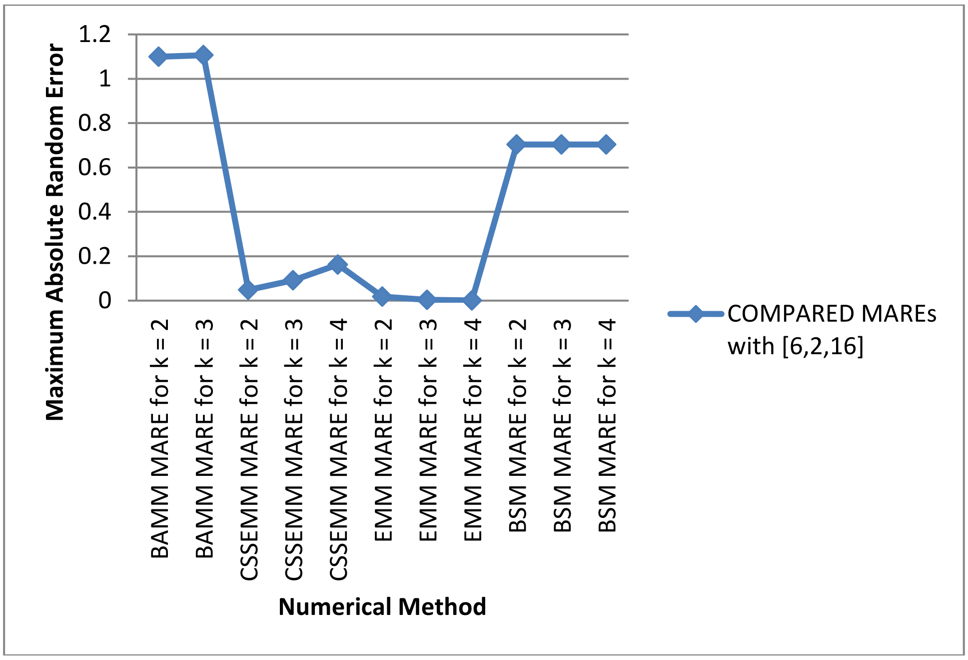

Figure 7. Compared MAREs of BAMM with [6, 2, 16] for Problem 1

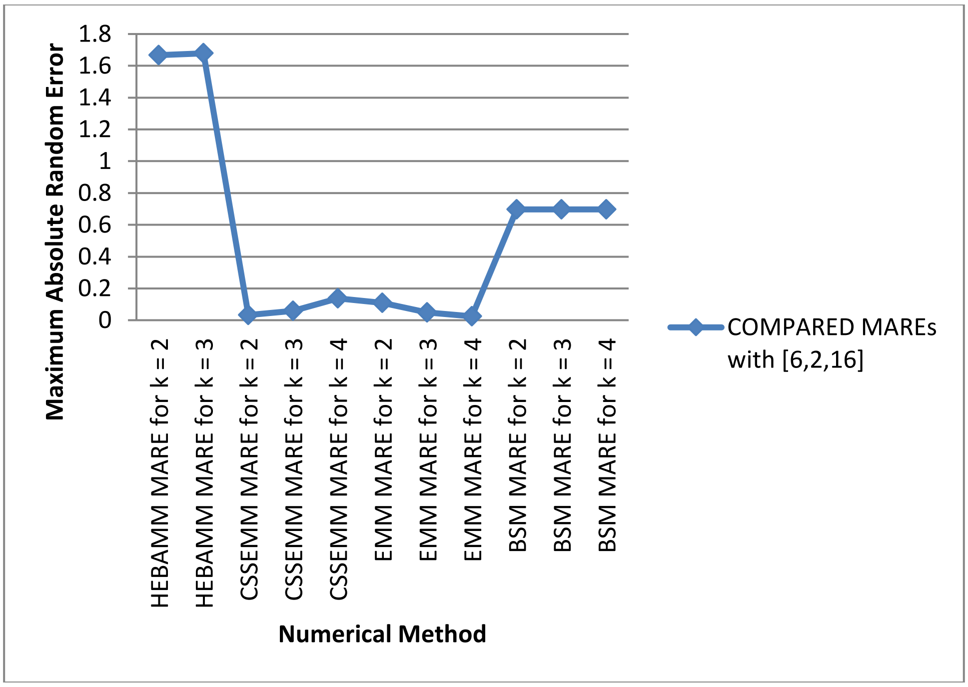

Figure 8. Compared MAREs of BAMMs with [6, 2, 16] for Problem 2

7 Conclusion

In this study, we have demonstrated that Block Adams Moulton Methods (BAMM) is suitable for the approximate solution of Advanced Stochastic Time-Delay Differential Equations (ASTDDEs). As shown in tables 1 to 4 and figs. 5 to 8, the numerical results revealed the effect of advanced stochastic time-delay on the policyholders claims settlement by the insurance industry which can lead to policyholders' dissatisfaction and claims dispute between the insurance companies and their customers. The policyholders and the insurance companies have big roles to play for prompt claims settlements by policyholders submitting their claims on time, storing of policyholders data in the cloud to avoid loss of insurance policy papers/documents, following the right pre-authorization claims process for cashless medical treatments, quick response by the organizations in the submission of policyholders claims to the insurance companies and regular supervision and recapitalization of the insurance industries by the government. Also, it was observed in tables 1 to 4 that the discrete schemes of higher step number $k=3$ of BAMM performed slightly better and faster than the lower step number $k=2$ when compared with other existing methods. Further research should be carried-out for step numbers $k=4,5,6...$ on the approximate solutions of ASTDDEs using BAMMs.

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