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Mathematical modeling of rape under the influence of human disturbance and noise

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Abstract

To leverage rape decline, a guide from epidemiological modeling was used to develop a new deterministic model for the dynamics of rape during covid-19 and the world at large. This was used to qualitatively assess the role of convictions and loss of immunity to rape on the transmission process of rape in a society. This study showed that loss of immunity to rape and its rate of previous convicted and jailed rape culprit individuals regress to being susceptible to rape cases induced a backward bifurcation when associated to rape reproduction number (R_{ef} <1) is less than one. This means that imprisonment of the convicted rape individuals will help to eradicate rape out from the entire world even at covid-19 pandemic. The study showcased that if the cause of backward bifurcation is removed, the rape-free equilibrium (R_e) of the model is globally asymptotically stable when the reproduction number is less than one. This study also showed that the rape endemic equilibrium (R_{ee}) is locally-asymptotically stable when the reproduction number is greater than one (R_{ef} >1). The analytical solutions suggest that Policymakers should provide policies that would check one time convicted and jailed individuals from being susceptible to rape after their released from prison. Self-recovered individuals should be assisted and engaged with employment as to avoid the stigmatization of rape.

Keywords: rape; Covid-19; pandemic; SCPV model; noise and human disturbance.

1 Introduction

Rape is the abuse of humanity and personality trait to the victims and society at large. It is a sexual assault that involves sexual intercourse or other forms of sexual penetration being carried out by physical force or coercion against the person's consent. Rape is an element of the crime of genocide when committed with the intention to destroy a targeted ethnic group partly or completely. It can be notable in various form as child rape and abuse, incest rape, sexual slavery, gang rape, prison rape, marital rape, acquaintance rape, war rape and statutory rape, [16,18]. Rape is predominantly committed by male factors against females. Globally, rape occurs in all continents of the world and the female partners are always helpless when involved according to [20, 21, 22] stated that about 35% of women worldwide have been raped in their life time.

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Several reasons have been identified for the causes of rape in a society. Thus, it can be caused by some of these factors drug abuse and alcohol addicts, lack of sex education, rape myths by men against women, the desire to control, parental negligence, Peer and family factors, Societal factors and poverty. Drug facilitated sexual assault is also known as predator-prey rape [12]. it is a sexual assault carried out after the victim has become incapacitated due to having consumed alcoholic beverages or other drugs, [13]. Alcohol has used to play an activating role during sexual assault, when the rapists must have taken some drugs like Cocaine, Marijuana and Tramadol which contributed to psychological havoc or trauma on the victims, [10,11, 14]. The major cost of rape is risk of being caught, punished, convictions and jailed. Nevertheless, the probability of being caught depends on the effectiveness of security and legal system of such countries, [15]. One may wish to read the detail article on rape and its possible mode of control in [1].

The various body of literature existing on the social, political and management aspects of rape and attempt to provide policies to curb rape. Scientifically, several attempts have been made to use mathematical models to study the dynamics of rape in a population. Amongst others were [9,17] used elementary model of population growth first order differential equation to study the growth rate of domestic violence and predictions using differential equations in a population; consideration was given in a community full of individuals prone to rape as case study. The work done by [7] studied rape as a disease epidemic process and used [5] classical SIR epidemiological mathematical models for modelling the spread of rape. He considered the population as made up of individuals susceptible to rape (S), rapists (I) and individuals who recover and are immune to rape (R) where the notations for the state variables were adjusted differently in [6]. [13] examined the impact of chaos theory in social systems afflicted with rape.

Currently, this study presents a new deterministic model that studies the population dynamics of rape in a population at covid-19 pandemic under the influence of human disturbance and noise. The model treats rape as a disease as was done by [19], thus, the model could fit well for an epidemiological setting even at covid-19 pandemic. In this present study, instead of having just three classes used by [7, 21], a modification of the SIR models to SCPVR models was made and I proposed dividing the population into five classes which includes the rape culprits (C), rape victims or prey (P) and individuals who are convicted and jailed for rape they committed (V). Also, the model considered human disturbance and noise factors. Hence, this paper is organized as follows: the model formulation is section 2 and section 3 provide discussion and conclusions.

2 Model Formulation

The total population at time t, denoted by N(t) is subdivided into five mutually exclusive compartments of susceptible individuals to rape (S(t)); individual for rape culprit (C(t)); individuals who are victims of rape or prey to rape (P(t)), rape individuals who are convicted and jailed (V(t)) and individuals who recover from rape (R(t)) respectively. So that the total population becomes

$$N(t) = S(t) + C(t) + P(t) + V(t) + R(t).$$

Rape interaction in the population is modelled using a standard incidence function. The population of individuals who are susceptible to rape S(t) is increased by the recruitment of a proportion ρ, $(0 \leq \rho \leq 1)$ of immigrants (who are susceptible) into the population at a rate γ, 'recovered' individuals who become susceptible to rape again at a rate π and convicted individuals who after their release become susceptible to rape again at a rate ψ. The susceptible population is decreased by rape at a rate λ that is the rape force of infection, where

$$\lambda = \frac{\beta C}{N-V} \ ,$$

and β is the rape transmission rate. The susceptible population is further decreased by natural death at a rate μ; natural removal occurs in all the compartments at this rate. Thus,

$$\frac{dS}{dt} = \rho\gamma - \lambda S - \mu S - \pi R - \psi V.$$

The population of rape culprit individuals C(t) is increased by the rape interaction that effects the subsequently infects susceptible individuals at the rate λ. Because the human disturbance or predation is negative on the culprit and positive on the victim or prey, so that it becomes $a_1 = Eq$ (q is the predating coefficient and E is total effort applied on predating on the population of the culprit). The noise term is assumed to be sinusoidal with amplitude A and frequency is ω of the noise, and phase shift φ. The population is decreased by trial, conviction and subsequently jailing which leads to removal from the society at a rate γ, recovery at a rate α and natural death μ. In this study, the noise term from culprit is called black noise. This is colored noise because it does not last long in the culprit and it shifts per time from when it was conceived and ends at the time action has been executed by the culprit; gives

$$\frac{dC}{dt} = \lambda S - (\gamma + \alpha + \mu)C - qEC + A\sin(\omega t + \varphi).$$

The population of rape victims or prey individuals P(t) is increased by the rape interaction, that effects subsequently infect susceptible individuals at the rate λ. Because the human disturbance or predation is positive on the victim or prey, so that it becomes $b_1 = Eq$ (q is the predating coefficient and E is total effort applied on predating on the population of the culprit as to stop the action). The noise term is assumed to be sinusoidal with amplitude A and frequency is ω of the noise The population is decreased by trial, conviction and subsequently trauma and sickness which leads to removal from the society at a rate δ, recovery at a rate α and natural death μ. In this study, the noise coming from the victims or prey is called green noise. This green noise is a limiting case of the black noise because it last long in the victims-prey; gives

$$\frac{dP}{dt} = \lambda S - (\delta + \alpha + \mu)P + A\sin\omega t.$$

The population of convicted and jailed individuals V(t) is increased by culprit individuals who are caught and convicted at the rate η (this population is in a form of isolation and without interaction with the outside world) and decreased by such individuals who are later become susceptible to rape after their release. Considering the period spent in jail at the rate **Ω**, convicted individuals who after serving their become resistant to rape at rate ξ and natural death μ. This gives

$$\frac{dV}{dt} = \eta C - (\xi + \Omega + \mu)V.$$

The population of rape individuals who were not caught and convicted but recover due to counselling and public enlightenment programmes on the need to shun rape considering that the society is an imperfect one where not all persons who commit rape face convictions. R(t) is increased by rape individuals who abandon a rape lifestyle at a rate α. This population is decreased by developing resistance to rape environment at a rate $\emptyset$, becoming susceptible to rape again after some time at a rate π and natural death μ, yields

$$\frac{dR}{dt} = \alpha C - (\emptyset + \pi + \mu)R.$$

Based on the above formulations and assumptions, the model for the dynamics and spread of rape at covid-19 in a population consist of the following system of nonlinear (deterministic) differential equations. Table 1 describes the associated state variables and parameters in the model.

$$\frac{dS}{dt} = \rho\gamma - \lambda S - \mu S - \pi R - \psi V, \qquad (1)$$

$$\frac{dC}{dt} = \lambda S - (\gamma + \alpha + \mu)C - qEC + A\sin(\omega t + \varphi), \qquad (2)$$

$$\frac{dP}{dt} = \lambda S - (\delta + \alpha + \mu)P + A\sin\omega t\ , \qquad (3)$$

$$\frac{dV}{dt} = \eta C - (\xi + \Omega + \mu)V, \qquad (4)$$

$$\frac{dR}{dt} = \alpha C - (\emptyset + \pi + \mu)R, \qquad (5)$$

with initial conditions

$S(0)= S_0$, $C(0) = C_0$, $P(0) = P_0$, $V(0) = V_0$, $R(0) = R_0$.

Some of the assumptions and features of this model are below.

1. Assume that we are dealing an imperfect society whereby not all culprit individuals are caught and convicted.
2. Assume that the culprit individuals who likely recovered due to counseling and enlightenment campaigns abandon their rape lifestyle and recover from rape tendencies.
3. Assume that the culprit individuals are not rape victims or prey
4. Incorporating a class of convicted individuals who are jailed and are in complete isolation from the outside.
5. Assume that the human disturbance is negative on the culprit and positive on the victim or prey; hence it was ignored on victim class.
6. Assume the noise term (white and colored noise) to be sinusoidal with amplitude A, frequency is ω and phase shift φ.

Table 1 shows the variables and parameters description.

Dependent Variables	Interpretation
S	Population of individuals susceptible to rape
C	Population of culprit individuals
P	Population of rape victims or prey individuals
V	Population convicted individuals who are jailed (isolated)
R	Population of individuals who recover from rape
Parameters	Interpretation
ρ	Recruitment rate
γ	Proportion of immigrants who are susceptible to rape
λ	Susceptible population is decreased by rape
μ	Natural mortality rate in all the compartments
π	Rate of the 'recovered' individuals become susceptible to rape
ψ	Convicted individuals become susceptible to rape after release
α	Rape recovery rate
β	Rape transmission rate
q	Predating coefficient by human beings
E	Total effort applied on predating the population of the culprit
A	Amplitude of the noise from culprit and victim individuals
ω	Frequency of the noise from culprit and victim individuals
φ	Phase shift of the noise from culprit
δ	Rate at which the victim individuals are removal from the society
Ω	Conviction rate
ξ	Rate at which convicted individuals become resistant to rape
η	Rate at which culprit individuals caught and convicted
$\emptyset$	Rate of developing resistance to rape environment

Table 1: Variables and parameters description

2.1 Basic properties

For model (1-5) to be meaningful, it is important to prove that all the state variables are non-negative for all time t. thus, the solutions of model (1-5) with positive initial data will remain positive for all $t \geq 0$.

2.1.1 Well-posed model and positivity of the solutions

Model (1) controls and monitors human population therefore all the state variables and parameters of the model are non-negative. Considering the domain

$$D = \left\{S, C, P, V, R \in R^5 : 0 \leq N \leq \frac{\rho\gamma}{\mu}\right\}. \quad (6)$$

It can be shown that the set of D is a positive invariant set and a global attractor of this system. That is any phase trajectory initiated anywhere in the non-negative region R_+^5 of the phase space eventually enters the region D and remains in D thereafter.

Let $S_0, C_0, P_0, V_0, R_0 \geq 0, S_0 + C_0 + P_0 + V_0 + R_0 = N_0$. Then, there exists solutions S(t), C(t), P(t), V(t), R(t) for the dynamical system (1) with initial data at t = 0, that defined for all $t \geq 0$. In fact, S(t), C(t), P(t), V(t), R(t) are non-negative and
S(t) + C(t) + P(t) + V(t) + R(t) =N(t) for all t. if C_0 =0, R_0 =0, V_0 =0, then $C(t) \equiv 0$ and $V(t) \equiv 0$.

Since N = S + C + P + V + R, it follows that the rate of change of the total population is given by

$$\frac{dN}{dt} = \rho\gamma - \mu N, \quad (7)$$

therefore, that $N(\dot{t}) \rightarrow \frac{\rho\gamma}{\mu}$ as $t \rightarrow \infty$. That is the total population is asymptotically constant. The well-posedness of the model follows a straight forward application of the classical theory in [11].

Lemma 2.1 *The region D is positively invariant for the model equations (1-5) and if* $C_0 > 0$*, then* $S(t) > 0$, $R(0) > 0$, $P(0) > 0$ *and* $C(0) > 0$ *for all* $t > 0$ *and V is bounded by* $\dot{V} = max\left\{V_0, \frac{\eta}{\mu+\xi+\Omega}\right\}$.

Proof. The right hand side of system (1-5) is continuously differentiable and hence it is locally Lipschitz and therefore there exists a unique solution S(t), C(t), P(t), V(t), R(t) to system (1-5) with the initial data S_0, C_0, P_0, V_0, R_0 that is defined on a maximal forward interval of existence, [11]. Consider the set $D \subset R_+^5$, since $C(0) \geq 0$ from system of equation (1-5), yields

$$C(t) \geq C_0 exp \int_0^{\infty} \left[\frac{\beta C}{N-V} - Eq + A\sin(\omega t + \varphi) - K_1\right] dt \quad (8)$$

$$P(t) = \left[P_0 + \int_0^t \eta C(\xi) e^{K_2(\xi - t_0)} d\xi\right] e^{K_2(t_0 - t)} (A \sin \omega t) \quad (9)$$

$$V(t) = \left[V_0 + \int_0^t \eta C(\xi) e^{K_3(\xi - t_0)} d\xi\right] e^{K_3(t_0 - t)} \quad (10)$$

$R(t) = \left[R_0 + \int_0^t \alpha C(\xi) e^{K_4(\xi - t_0)} d\xi\right] e^{K_4(t_0 - t)}$. (11)

So that $S(t) \geq 0$, $C(t) \geq 0$, $P(t) \geq 0$, $V(t) \geq 0$, $R(t) \geq 0, \forall\, t > 0$, where $K_1 = \gamma + \alpha + \mu$, $K_2 = \delta + \alpha + \mu$, $K_3 = \xi + \Omega + \mu$, $K_4 = \emptyset + \pi + \mu$.

To show that V is bounded by $\dot{V} = max\left\{V_0, \frac{\eta}{\mu+\xi+\Omega}\right\}$, is to show whether $V(t) \leq \omega, \forall\, t \geq 0$ and that $\omega \geq \frac{\eta}{K_3}$ if $V_0 \leq \omega$. Suppose that the above inequalities do not hold, then there exists a time, t_1, with $V'(t_1) > 0$ and $V'(t_1) > \omega$. Using the equation for V in (1), gives
$\frac{dV(t_1)}{dt} = \eta C(t_1) - K_3 V(t_1) \leq \eta(C(t_1) - 1) < 0$. (12)

Since $(C(t_1) - 1) < 0$ and $\eta > 0$. This is the contradiction which implies that $V(t) < \omega, \forall\, t \geq 0$. Suppose that $V(0) > \omega \geq \frac{\eta}{K_3}$. In order to show that $V(t) \leq V(0)$–$\forall\, t \geq 0$, we assume that the last inequality does not hold. Hence, there exists a time$t_2 > 0$ such that $V(t_2) \geq V(0)$ and

$V'(t_2) > 0$.

Since $V(t_2) > \frac{\eta}{K_3}$. Then

$\frac{dV(t_2)}{dt} = \eta C(t_2) - K_3 V(t_2) \leq \eta(C(t_2) - 1) < 0$, but $V'(t_2) > 0$. (13)

Hence, we reach a contradiction and V(t) is bounded from above by $\dot{V}$, where

$\dot{V} = max\left\{V_0, \frac{\eta}{K_3}\right\}$.

End of proof.

Lemma 2.1 guarantees the positivity of the different groups of individuals making up the total population, S, C, P, V and R for all time, $t \geq 0$.

3 Asymptotic stability of rape-free equilibrium (RFE)

The rape-free equilibrium of the model (1-5) is given by

$E_0 = (S^*, C^*, P^*, V^*, R^*) = \left(\frac{\rho\gamma}{\mu}, 0,0,0,0\right)$ (14)

Setting $A = 0$, depicting no noise and no rape action.
The linear stability of E_0 can be established using the next generation operator method on the model equ (1-5), [5,2,10]. Using the notations in [4], it follows that F and V, which stands for the new rape infection terms and remaining transition terms respectively are given by $F = \rho\beta$ and $V = \alpha + \mu + \gamma$.

It follows that the effective rape reproduction number of the model equation (1-5) denoted by $R_{rn} = FV^{-1} = \frac{\rho\beta}{\alpha+\mu+\gamma}$.
The result above follows from theorem 2 in [4].

Lemma 3.1 *The CFE (E_0) of the model equ (1-5) is locally-asymptotically stable (LAS) if* $R_{rn} < 1$ *and unstable if* $R_{rn} > 1$.

The threshold quantity, R_{rn} is the rape reproduction number. It represents the average number of secondary cases of rape generated by a typical rape individual in a population that is completely susceptible to rape (using a typical language in epidemiology, [14]. The implication of Lemma 3.1 is that when R_{rn} is less than one $(R_{rn} < 1)$, rape can be eliminated from the population if the initial sizes of the sub-population of the model are in basin of attraction of the rape-free equilibrium (E_0). Hence, a small influx of rape-infected individuals into the country will not generate large outbreaks of rape and it dies out in time completely.

3.1 Backward Bifurcation Analysis

To describe the bifurcation type in the model equations (1-5), it will go a long way to help in determining the factors that could hinder efforts in tackling rape in the population. Hence, the following results were assumed.

Theorem 3.1 That the model equation (1-5) exhibits a backward bifurcation at $R_{rn} = 1$ whenever a bifurcation coefficient denoted by a, is positive.

Proof.
Let $E_1 = (S^{**}, C^{**}, P^{**}, V^{**}, R^{**})$ represents any arbitrary endemic equilibrium of the model; that is an equilibrium in which at least one of the infected components is non-zero. The existence of backward bifurcation will be explored using the Manifold theory, [2,3]. To apply this theory, it is appropriate to carry out the following change of variables. Let: $S = x_1$, $C = x_2$, $P = x_3$, $V = x_4$, $R = x_5$. By using the vector notation, $X = \left(x_1, x_{2,} \dots . x_5\right)^T$, the model can be written in the form

$\frac{dX}{dt} = F(X)$, with $F = (f_1, f_2, \dots . . f_5)^T$, yields:

$$\dot{x}_1 \equiv f_1 = \gamma\rho - \lambda x_1 - \mu x_1 + \pi x_5 + \psi x_4 \quad (15)$$

$$\dot{x}_2 \equiv f_2 = \lambda x_1 - (\eta + \alpha + \mu)x_2 \quad (16)$$

$$\dot{x}_3 \equiv f_3 = \lambda x_1 - (\delta + \alpha + \mu)x_3 + \quad (17)$$

$$\dot{x}_4 \equiv f_4 = \eta x_2 - (\xi + \Omega + \mu)x_4 \quad (18)$$

$$\dot{x}_5 \equiv f_5 = \alpha x_2 - (\emptyset + \pi + \mu)x_5 \quad (19)$$

where $K_1 = \eta + \alpha + \mu + qE,\ K_2 = \delta + \alpha + \mu,\ K_3 = \xi + \Omega + \mu,\ K_4 = \emptyset + \pi + \mu,\ K_5 = qE$ and the force of infection is given by

$\lambda = \frac{\beta x_2}{N - x_4}$ and $N = x_1 + x_2 + x_3 + x_4 + x_5$.

Put $\beta = \beta^*$ as a bifurcation parameter. Solving for $\beta = \beta^*$ from $R_{rn} = 1$, gives

$$\beta = \beta^* = \frac{K_1}{\rho}. \tag{20}$$

The Jacobian of the transformed model equation (15–19) evaluated at the rape-free equilibrium (E_0) with $\beta = \beta^*$ is given by

$$J^* = J(E_0)_{|\beta-\beta^*} = \begin{pmatrix} -\mu & -\rho\beta^* & \pi & 0 & \xi \\ 0 & \rho\beta^* - K_1 & 0 & 0 & 0 \\ 0 & \alpha & -K_2 & 0 & 0 \\ 0 & 0 & \emptyset & -\mu & \psi \\ 0 & \eta & 0 & 0 & -K_3 \end{pmatrix} \tag{21}$$

The matrix J* has a right eigenvector as $w = (w_1, w_2, \ldots, w_5)^T$, where

$$\begin{aligned} w_1 &= \frac{-p\beta\, w_2 + \pi\, w_3 + \sum w_2}{\mu} \\ w_2 &= w_2 > 0, \\ w_3 &= \frac{\alpha}{K2} w_2 > 0, \\ w_4 &= \frac{\emptyset\alpha}{\mu K2} + \frac{\psi\eta}{\mu K3} w_2 > 0, \\ w_5 &= \frac{\eta}{k_3}\, w_2 > 0. \end{aligned} \tag{22}$$

Thus, J^* has a left eigenvector $v = (v_1, v_2, \ldots, v_5)$ satisfies that v.w = 1, with

$v_1 = v_3 = v_4 = v_5 = 0$, and $v_2 > 0$.

This follows from Theorem 4.1 in [2]. To compute the associated non-zero partial derivatives of F(x) evaluated at rape-free equilibrium, E_0 by defining the associated bifurcation coefficients, a and *b*, giving by:

$$a = \sum_{k,i,j=1}^{5} v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial xj}(0,0), \quad \text{and} \quad b = \sum_{k,i,=1}^{5} v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \beta^*}(0,0),$$

when computed gives

$$a = \frac{2\beta^* \mu v_2 w_2^2}{\gamma}\left\{\frac{1-\rho}{\mu}\left(\frac{\pi\alpha K_3 + \eta\xi K_2}{K_2 K_3}\right) - \left(\frac{(1-\rho)K_1}{\mu} + \frac{\eta\psi}{\mu K_3} + \frac{\emptyset\alpha}{\mu K_2} + \frac{\alpha}{K_2} + 1\right)\right\} \tag{23}$$

and

$$b = v_2 w_2 \rho > 0. \tag{24}$$

Hence, the bifurcation coefficient *b* is positive, it follows from Theorem 4.1 in [2] that the model equation (1-5) and the transformed model (15-19) will undergo a backward bifurcation if the backward bifurcation coefficient *a*, given by equation (23) is positive.

The occurrence of backward bifurcation has been observed in numerous disease transmission dynamics as in [2,3] showed a typical case of co-existence of a stable rape-free equilibrium and a stable rape endemic equilibrium when the associated rape reproduction number of the model is less than one. This implies that the model equation (1-5) is the typical tool of having a rape reproduction number (R_{rn}) less than one, while the jailed equation is to deteriorated rape in the population where it is common. Putting $\pi = \xi = 0$ in (23) and setting all parameters of the model (1-5) positive and $\beta^* > 0$, the bifurcation parameter *a* reduces to;

$$a = -\frac{2\beta^*\mu v_2 w_2^2}{\gamma}\left(\frac{(1-\rho)K_1}{\mu} + \frac{\eta\psi}{\mu K_3} + \frac{\emptyset\alpha}{\mu K_2} + \frac{\alpha}{K_2} + 1\right) < 0 \quad (25)$$

with $v_2 > 0$ and $w_2 > 0$.

From Theorem 4.1 in the model equation (1-5) does not undergo backward bifurcation, i.e. if $\pi = \xi = 0$. Then it shows that the rate at which individuals 'recover' from rape practices due to certain interventions like public enlightenment, counseling, jail e.t.c and previously convicted individuals become susceptible to rape for $\pi \neq 0$ and $\xi \neq 0$ are the causes of backward bifurcation in the rape transmission model. Therefore, the mathematical analysis in shows that previously convicted and recovered individuals becoming susceptible to rape will make it difficult to have rape eradicated from the population.

3.2 Global Asymptotic Stability: Special Case $\pi = \xi = 0$.

Considering the model equation (1-5) is with $\pi = \xi = 0$.

Theorem 3.2 The RFE of the model (1-5) with $\pi = \xi = 0$ is globally-asymptotically stable (GAS) in D whenever $R_{rn} < 1$.

Proof. Consider the model (1-5) with $\pi = \psi = 0$ and linear Lyapunow function $F = \rho C(t)$.
By Lyapunow derivative with respect to t.

$$\dot{F} = \rho\dot{C}$$

$$= \left[\frac{\rho\beta SC}{N-V} - (a - \mu - \eta)\right] C$$

$$= a + \frac{C}{\alpha+\mu+\eta}\left[\frac{\rho\beta}{\alpha+\mu+\eta}\,\frac{S}{N-V} - 1\right] \quad (26)$$

From (26), $S(t) \leq N(t) - V(t)$ in *D* for all $t > 0$, then

$$\dot{F} \leq \frac{1}{\alpha+m\mu+\eta}(R_{rn} - 1)C \quad (27)$$

Thus, $F \leq 0$ *if* $R_0 \leq 1$ with $\dot{F} = 0$, if and only if $C=0$. Therefore, *F* is *a* Laypnow function in *D* and it follows from LaSalle's Invariance Principle [8], that every solution to the equations in (1-5) with $\pi = \xi = 0$ with initial conditions in *D* converges to E_o as $t \to \infty$ i.e.

$(C(t), R(t), V(t) \to (0.0,0)$ as $t \to \infty$. Substituting $C = R = V = 0$ into equation (1-5) gives $S(t) \to \frac{\rho\gamma}{\mu}$ as $t \to \infty$. Hence $(S, C, P, V, R) \to \left(\frac{\rho\gamma}{\mu}, 0, \frac{(1-\rho)\gamma}{\mu}, 0, 0\right)$ as $t \to \infty$ for $R_{rn} \leq 1$, so that the RFE, E_0, is GAS in D if $R_{rn} \leq 1$ for the case $\pi = \xi = 0$.

The above result signifies that in the absence of recovered individuals and persons previously convicted and jailed for rape becoming susceptible to rape again, rape will be eliminated from the population of the society if the rape reproduction number or threshold Rrn <1.

3.3 Existence and stability of rape endemic equilibrium

To create the existence of the rape endemic equilibrium of the model equation (1 –5), let $E_1 = \{S^{**}, C^{**}, P^{**}, V^{**}, R^{**}\}$ represents any arbitrary rape endemic equilibrium of the model (1 –5). The equation (1-5) are solved in terms of the rape force of infection at a steady state to give

$$S^{**} = \frac{1}{\lambda^{**} - \mu}\left[\frac{\eta\xi K_2 C^{**} + \rho\gamma K_2 K_3}{K_2 K_3}\right] \quad (29)$$

$$C^{**} = \frac{\lambda^{**} K_2 K_3 \rho\gamma}{K_1 K_2 K_3 (\lambda^{**} + \mu) - \lambda^{**}(K_3\alpha\pi + K_2\eta\xi)}, \quad (30)$$

$$R^{**}\frac{\alpha C^{**}}{K_2};\ T^{**} = \frac{1}{\mu}\left[\frac{K_2 \emptyset\alpha C^{**} + \psi\eta K_2 C^{**} + K_2 K_3 (1-\rho)\gamma}{K_2 K_3}\right];\ V^{**} = \frac{\eta C^{**}}{K_3} \quad (31)$$

Recall, that the rape force of infection at steady state, λ^{**}, is expressed as

$$\lambda^{**} = \frac{\beta C^{**}}{S^{**} + C^{**} + R^{**} + T^{**}}$$

$$H\lambda^{**2-}\lambda^{**}(p\beta\mu K_2 K_3) = 0$$

where $H = \propto[(p-1)\pi + p(\mu + \emptyset)]K_3 + [\gamma((p-1)\varepsilon + p\psi) + (1-p)K_1)K_3]K_2$, and further reduces to

$$H\lambda^{**} - \mu K_1 K_2 (R_0 - 1) \quad (32)$$

$$\lambda^{**} = \frac{\mu K_1 K_2 K_3}{H}(R_0 - 1) \quad (33)$$

Hence, a unique rape endemic equilibrium exists (i.e. $\lambda^{**} > 0$) when $R_0 > 1$ as far as $H > 0$. Clearly, if $\pi = \varepsilon = 0$, then $H > 0$ and there exists a unique rape endemic equilibrium for this case. Thus, the following result is established.

Theorem 3.3 The model (1-5) has a unique endemic (positive) equilibrium when $\pi = \varepsilon = 0$ whenever $R_0 > 1$.

The possibility of the existence of a rape endemic equilibrium exists even $R_0 < 1$. This occurs when $H < 0$ which is possible when $\pi \neq 0$ and $\varepsilon \neq 0$ since $0 \leq 1$. This further proves the possibility of a backward bifurcation when $R_0 < 1$ whereby the rape free equilibrium will co-exist with a rape endemic equilibrium. Therefore, the general implication of this as stated before that individual who 'recover' from rape after an intervention programme and previously convicted individuals should strive not to become susceptible to rape again. The relevant government agencies may determine and formulate polices that could help with preventive strategies cum corrective measures.

3.4 Local Asymptotic Stability of the Rape Endemic Equilibrium

Since the total population is asymptotically constant, the result in [2] guarantee that the following system of equations from (34-38).

$$\frac{dS}{dt} = \rho\gamma - \lambda S - \mu S - \pi R - \psi V \quad (34)$$

$$\frac{dC}{dt} = \lambda S - (\gamma + \alpha + \mu)C - qEC + A\sin(\omega t + \varphi) \quad (35)$$

$$\frac{dP}{dt} = \lambda S - (\delta + \alpha + \mu)P + A\sin\omega t \quad (36)$$

$$\frac{dV}{dt} = \eta C - (\xi + \Omega + \mu)V \quad (37)$$

$$\frac{dR}{dt} = \alpha C - (\emptyset + \pi + \mu)R \quad (38)$$

where $\gamma = \frac{\beta C}{N-V,}$ with $N = \frac{\rho\gamma}{\mu}$, has the same asymptotic qualitative dynamics as those of system. Putting $K_2 = -qEC + A\sin(\omega t + \varphi)$, $K_4 = A\sin\omega t$

The Jacobian matrix for system (34-38) at the endemic equilibrium, ε_1 is

$$J(E_1) = \begin{pmatrix} -A-\mu & B & 0 & -D+\psi & \pi \\ A & 0 & B-K_1+K_6 & 0 & 0 \\ A & 0 & 0 & B-K_3+K_7 & 0 \\ 0 & \eta & 0 & -K_4 & 0 \\ 0 & \alpha & 0 & 0 & -K_5 \end{pmatrix}$$

where $A = \frac{\beta C^{**}}{N-V^{**}}, B = \frac{\beta S^{**}}{N-V^{**}}$ and $D = \frac{\beta S^{**}C^{**}}{(N-V^{**})2}$

Hence, the characteristic equation associated with the local stability of $J(\varepsilon_1)$ are:

$$A_4(\varphi)Q^4 - A_3(\varphi)Q^3 - A_2(\varphi)Q^2 + A_1(\varphi)Q + A_0 = 0 \quad (39)$$

where

$$\begin{aligned} A_4 &= 1, \\ A_3 &= B + \mu + K_2 + K_2 + K_3 \\ A_2 &= -D\gamma - B\mu + (A - B + \mu)K_3 + K_2(A-B+\mu+K_3) \\ &\quad +K_1(A + \mu + K_2 + K_3), \\ A_1 &= -D\gamma\mu + A\gamma\xi + A\alpha\pi + (-B\mu + (A+\mu)K_1)K_3 \\ &\quad +K_2(-D\gamma - B\mu + (A - B + \mu)K_3 + K_1(A + \mu + K_3), \\ A_0 &= +A\alpha\pi K_3 + K_2(\gamma(D\mu + A\xi) + (B\mu - (A+\mu)K_1)K_3) \end{aligned}$$

and Q_i represents the eigenvalues of the Jacobian with φ is the model's parameter vector.
The local stability of the rape endemic equilibrium is geared towards the roots of equation (39). Therefore, the rape endemic equilibrium point is locally asymptotically stable if the roots of equations (39) are negative real roots.

4 Numerical Simulations

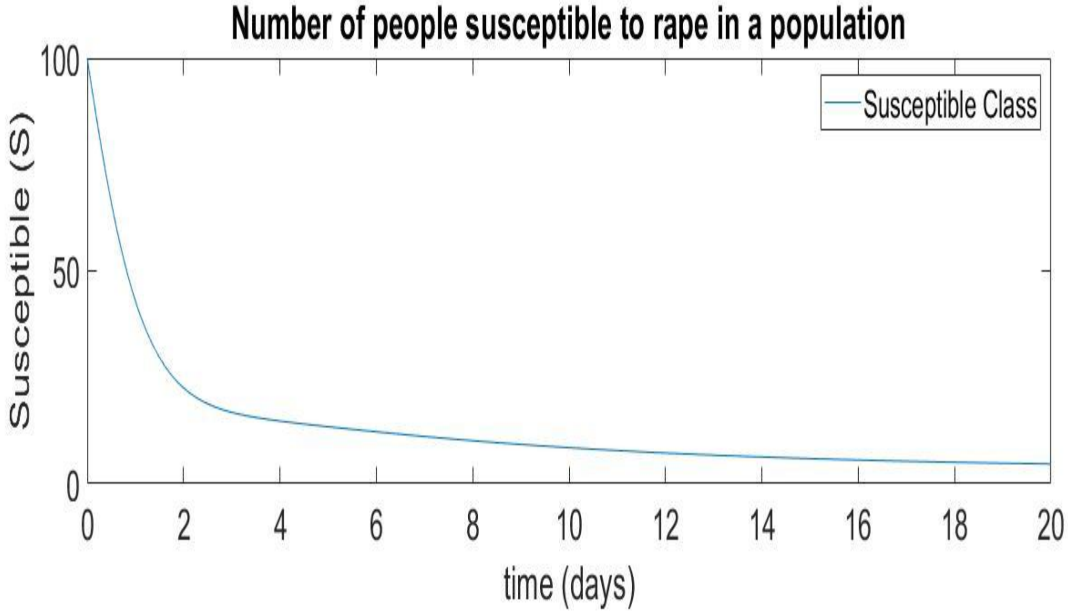

Figure 1: Graph of Susceptible Individuals to Rape cases in a Population. Parameter Values are: π=3.142, P=0.5, δ=0.90, ϱ=0.45, β=0.72, k_1=0.75, k_2=0.75, μ=0.005, φ=0.67, ϕ=0.81, γ=0.80, ξ=0.45, with initial condition (S(0), P(0), V(0), C(0), R(0)) = (100, 50, 50, 20, 10).

Figure 1 above showed that the susceptible population or individuals reduces slowly with respect time and move to either the culprit-victim individuals asymptotically.

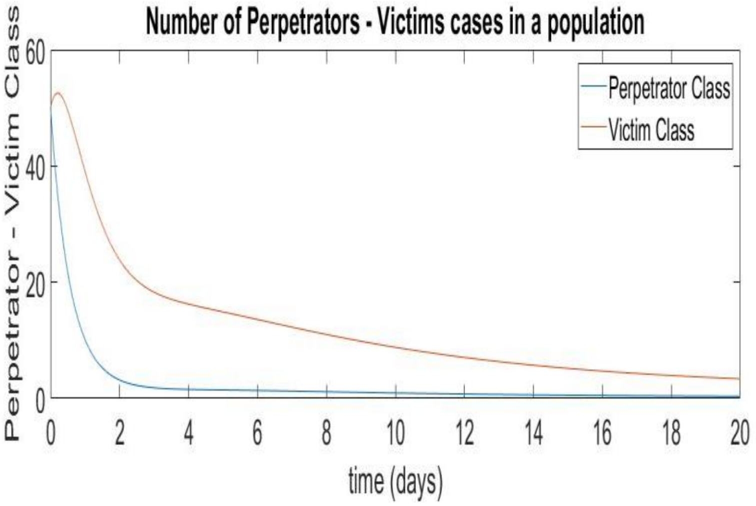

Figure 2: Graph of Rape Perpetrator-Victim cases in a Population. The parameter values are: π=3.142, P=0.5, δ=0.90, ϱ=0.45, β=0.72, k_1=0.75, k_2=0.75, μ= 0.005, φ=0.67, ϕ=0.81, γ=0.80, ξ=0.45, with initial condition (S(0), P(0), V(0), J(0), R(0)) = (100, 50, 50, 20, 10).

In figure 2, the graph depicts that rape victims do not go to extinction because in the natural the stigma lives with them mostly females, while the perpetrators or culprits sometimes vanishes as a result rape control measures and if the culprit are males.

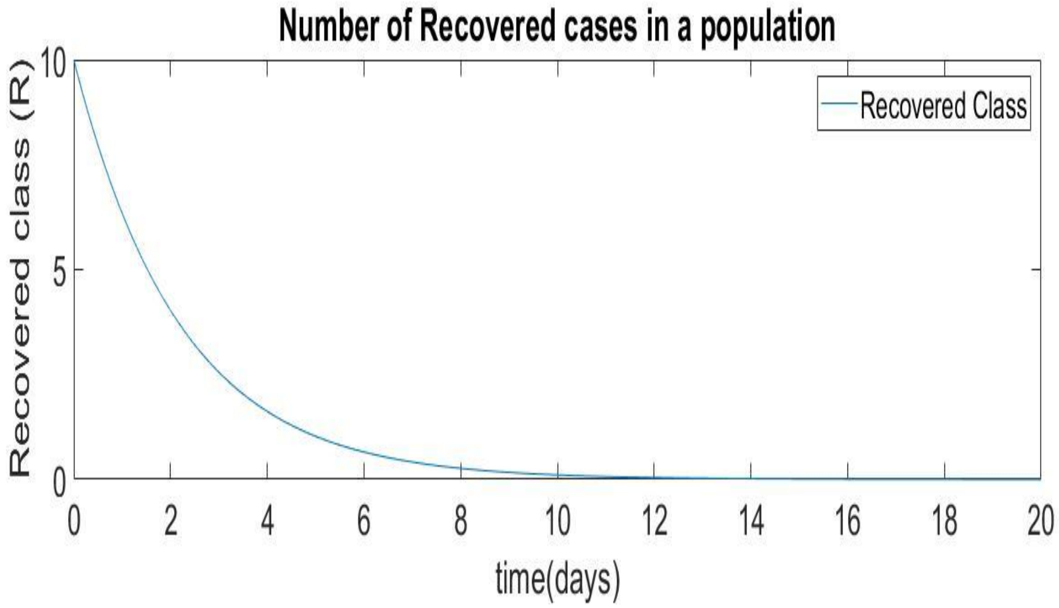

Figure 3: Graph of Recovered Individuals from Rape cases in a Population. The parameter values are: $\pi=3.142$, $P=0.5$, $\delta=0.90$, $\varrho=0$, $\beta=0.72$, $k_1=0.75$, $k_2=0.75$, $\mu= 0.005$, $\varphi=0$, $\phi=0.81$, $\gamma=0.80$, $\tau=0.45$, with initial condition $(S(0), P(0), V(0), C(0), R(0)) = (100, 50, 50, 20, 10)$.

Figure 3 above showed that the recovered individuals reduce to annihilation asymptotically with respect time through natural death or migration to another environment.

4.1 Discussions of Findings

The study designed and discussed the asymptotic behavior of a new deterministic model for the dynamics of rape in a population with noise and human disturbance. The model equation (1-5) has a locally-asymptotically stable rape free equilibrium whenever the associated rape reproduction number (R_{rn}) is less than one. This model undergoes the phenomenon of backward bifurcation, where the stable rape-free equilibrium co-exists with a stable rape endemic equilibrium. The analysis showed that this phenomenon is caused by the rates at which individuals who previously recovered from rape, following some intervention programme and individuals who previously were convicted and jailed for rape revert to being susceptible to rape.

It is also shown that the model has a globally-asymptotically stable rape-free equilibrium (DFE) when individuals who recover from rape and those previously convicted and jailed for rape never become susceptible to rape but have developed total resistance to rape. The Jailed model was identified as the general measure to curb and control rape, both at Covid-19 and other crisis respectively. To restrain rape globally, an SCPVR models was developed which implies Susceptible to rape S, Culprit C, Prey P, Convicted and Jailed because of rape committed V and recover from rape R, models. The study identifies two equilibrium points such as Pandemic free equilibrium state P_e and the rape free equilibrium state, R_e. The MATLAB ODE45 software was used for the simulations. However, the graphs show that the susceptible class increases with time in a ratio of 1:1 and the recovered class goes into extinction with the presence of human disturbance and jail class which is the control measure of rape cases in any given population. The results and simulations mean that the rape culprit goes to life imprisonment because of the action. The simulation of this study with noise showed a sinusoidal and an exponential increase; which implies that the action of the rape culprit was high on the victim.

We can see from the line graph that the recovered class tends to zero with respect to time. We can also affirm that the model is a good for the control of rape cases in a population within a stipulated time. The numerical simulation graph of this is depicted in rape cases with respect to time. The perpetrator-victim classes decline with respect to time also. This shows that the proposed model can be used to control the high rise in number of rape cases during the Covid-19 which is realistic to real life situation.

5 Conclusion

In conclusion, the findings from the model analysis, shows that the convicted and jail for life imprisonment is the only possible measure to mitigate and control the impact of rape cases in any rape society. This study could be extended to multiple rape scenarios that led to pregnancy.

Reference

[1] Baba I.A., Yusuf A. and Al-Shomrani M. (2021). A mathematical model for studying rape and its possible mode of control. Results in Physics, doi:doi.org/10.1016/j.rinp.2021.103917.
[2] Castillo-Chavez C. and Song B. (2004). Dynamical models of tuberculosis and their application. Mathematical Biosciences and Engineering, Vol. 2:361-404.
[3] Diekmann O., Heesterbeek J., and Metz J. (1990). On the definition and computation of the basic reproduction ratio R_0 in models for infectious disease in heterogeneous populations. Journal of Mathematical Biology, Vol. 28: 365-382.
[4] Hadeler K. and van dan Driessche P. (1997). Backward bifurcation in epidemic control. Mathematical Biosciences, Vol. 146:15-35.
[5] Hathroubi S. (2013). Epidemic rape and corruption: a bio-economic homology. ZBW-Leibniz Information Centre for Economics, available at http://hdl.handle.net/10419/73558.
[6] Kernack and McKendrick (1972). Classical SIR epidemiological mathematical models: simple logistics and population models. Oxford University Press, New York.
[7] Lasalle J. and Lefschetz S. (1976). The stability of dynamical systems. SIAM, Philadelphia
[8] Mimmo I. (2005). The mathematical modelling of epidemics. Journal of Mathematics Department, University of Trento.
[9] Neumann S. and Joseph E. (2018). Gang rape: examining peer support and alcohol fraternities sex crimes and paraphilia. Hickey, Eric W., 397-407.
[10] Nkuturum C. and Moses-Amini L. (2020). Effects of COVID-19 pandemic on students learning mathematics: Issues and prospects. Journal of Health, Applied Sciences and Management. Vol 4: 19 – 27.
[11] Nkuturum C. and Amini-Moses L. (2020). Health implication of COVID-19 on mathematics students. Journal of Health, Applied Sciences and Management, Vol 4: 126 – 138.

[12] Nkuturum C. (2016). Mathematical model of predator-prey relationship and human disturbance. International Journal of Educational Development, Vol. 6:13-26.
[13] Nkuturum C. (2017). Mathematical model of predator-prey relationship with human disturbance. https://www.repository.unn.edu.ng
[14] Okuonghae D. (2019). Theoretical analysis of a mathematical model for the dynamics of corruption: a guide from epidemiological modelling. Journal of the Nigerian Society for Mathematical Biology, Vol 2: 1-13.
[15] Perko I. (2000). Differential equations and dynamical systems. Springer. New York.
[16] Rastogi R.P. and Mathur P. (2013). Bifurcation phenomena and chaos in social systems afflicted with rape. Proc Indian Natn Sci Acad, Spl Issues, Part B, 79(4): 967-983.
[17] Triesman D. (2000). The causes of rape: a cross-national study. Journal of Public Economics, Vol. 76:399-457.
[18] van dan Driessche P. and Watmough J. (2002). Reproduction numbers and sub- threshold endemic equilibria for compartmental models of disease transmission. Mathematical Biosciences, Vol. 180:29-48.
[19] Waykar S.R. (2013). Mathematical modelling: a study of rape in a society. International Journal of Scientific and Engineering Research, Vol. 4(7): 303-318.
[20] Waykar SR. (2013). Mathematical modelling: a comparative mathematical study model based between corruption and development. IOSR Journal of Mathematics, Vol. 6(2):54-62.
[21] World Health Organization (WHO) (2020). Transmission of SARS-Cov-2: implications for infections, prevention and control, precautions and recommendations.
[22] World Health Organization (WHO) (2020). Laboratory testing for coronavirus disease 2019 (COVID-19) in suspected human cases: Interim Guidance.