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Application of stochastic model in estimation of stock return rates in capital market investments

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Abstract

The rigidity of financial assets lies on capital investments which geared towards bringing returns to the investors. Therefore, this paper studied the problem of system of Stochastic Differential Equations (SDE) of time-varying investments of Dangote cement PLC where multiplicative inverse effects, additive effects, additive inverse effects and multiplicative effects were used as key parameter in the model to obtain stock rate of returns. The problem was solved analytical by adopting Ito's theorem which gave four different investment solutions. In addition, the validity of analytical solutions was clearly confirmed graphically and discussed consequently.

Keywords: stock price; stochastic analysis; asset price; return rates and SDE

1 Introduction

In capital investments monies are been invested in a business such that the return rates will be appropriately utilized to cover day-to-day trading activities expenses. An investor may need additional capital assets; in order to improve in its trading events. So capital investments are established on the basis of: to acquire additional capital assets for expansion which enables the the business to increase in unit production, create new ideas on products or even add value to the business and to explore on technological advancements in order to increase efficiency and reduce costs and to replace worn-out assets. In absence of capital investments, trading business will definitely have a hard time getting off the ground.

Nevertheless, the effect of stock market operations and performance has enlarged significantly in the development of financial of markets as investors gain their normal returns to earn a living. However, stock trading is normally known in making high returns. Return on investment for capital market is a measure to properly evaluate gain of an investment. It is a major parameter often used by

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a trader or investors to regulate profitability of expenditure [1]. In stock market the dynamics of stock prices are reflected by uncertain movement of their value over time. That is why it better to properly understand the type of physical quantities to model in order to obtain realistic results of stock variables.

For instance [2] worked on stochastic model of the fluctuation of stock market price. Here conditions for determining the equilibrium price, sufficient conditions for dynamic stability and convergence to equilibrium of the growth rate of the value function of shares. In another dimension, [3] considered a stochastic model of price changes at the floor of stock market. In their research the equilibrium price and the market growth rate of shares were determined. See [4] for considerable extensions and constrains subsequently in this particular area of study. [5] Investigated the stochastic analyses of Markov chain in finite states. The transition matrix replicated the use of 3-states transition probability matrix which enables them to proffer precise condition of obtaining expected mean rate of return of each stock.

In similar manner, [6] examined stochastic analysis of stock market expected returns and Growth-rates. The precise conditions for obtaining the drifts, volatilities and Growth-rates of four different stocks were also considered. In the work of [7] the problem of stock price fluctuations using stochastic differential equations, principal component analysis and KS goodness of fit test were studied. The analytical solutions were detailed; the computational and graphical results were presented and discussed respectively; as if it that was not enough, [2] studied the stochastic analysis of stock market expected returns for investors. The detailed conditions for obtaining the drifts, volatilities and variances of four different stocks were considered. [8] Considered a combination of deterministic and stochastic systems with its random parameters in the model. The analytical solution to the proposed model is presented in detail which determined the insurance quantities or variables

However, [9] considered the unstable nature of stock market forces using proposed differential equation model. In the work of [10] studied stability analysis of stochastic model of price change at the floor of a stock market. In their research précised conditions are obtained which determines the equilibrium price and growth rate of stock shares.

The work in [11] considered stochastic analysis of the behavior of stock prices. Results reveal that the proposed model is efficient for the prediction of stock prices. In the same vein, [12] studied the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in their research the drift and volatility coefficients for the stochastic differential equations were determined.[13], built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data.[14] Studied the solution of differential equations and stochastic differential equations of time-varying investment returns; where precise conditions are obtained which govern stock return rate through multiplicative and multiplicative inverse trends series.

This paper, considered four systems of stochastic differential equations in time varying investments of Dangote cement PLC whose rate of returns is assumed to follow: multiplicative inverse effects, additive effects, additive inverse effects and multiplicative effects; all having quadratic functions over time. These investment equations were solved analytically independently by adopting Ito’s theorem and four solutions were obtained. To verify and to test for the validity of the analytical solutions, we used closing stock prices of Dangote cement PLC and estimate other relevant parameter values in the proposed solutions; this way the effects of rate of returns were displayed.

This paper is aimed at solving an accurate analytical solution of time independent equations by stating precise conditions of predicting stock returns in Dangote cement PLC which was not considered by previous efforts. This paper extends the works of [1] by solving the pricing effects of stock variables as quadratic functions in SDE domain.

This paper is arranged in following manners: Section 2.1 presents the mathematical formulation of the problem, method of solution is seen in Subsection 2.1.1, Results are seen in Section 3.1, while the discussion of results is presented in Section 3.1.1 and paper is concluded in Section 4.1.

2 Mathematical formulation of the problem

Here we Considered time varying investment of Dangote cement PLC through closing stock prices of 2018-2021 which said to have a complete probability space $\left(\Omega,,\mathrm{F}\wp\right)$ endowed with a Wiener process such that a finite time investment horizon $T>0$ .Investors are always geared towards maximizing investment returns. Therefore, we assumed the company's rate of returns to follow multiplicative inverse effects, additive effects, additive inverse effects and multiplicative effects; all having quadratic functions over time. Hence this rate of return is defined in the system of stochastic differential equations of (2- 5) below:

Thus the stochastic process describing the process is of the form:

$$dS(t)=\theta dt+\sigma dZ^{(1)}(\mathrm{t}), \qquad (1)$$

where θ is an expected rate of returns on stock, σ is the volatility of the stock, dt is the relative change in the price during the period of time and $Z^{(1)}$ is a Wiener process.

$$dS_{K_1}(t)=\left\{\left(\theta_1\theta_2\right)^{-1}\right\}^2 \mathrm{S}_{K_1}(t)dt+\sigma S_{K_1}(t)dZ^{(1)}(\mathrm{t}), \qquad (2)$$

$$dS_{K_2}(t)=\left(\theta_1+\theta_2\right)^2 \mathrm{S}_{K_2}(t)dt+\sigma S_{K_2}(t)dZ^{(2)}(\mathrm{t}), \qquad (3)$$

$$dS_{K_3}(t)=\left\{\left(\theta_1+\theta_2\right)^{-1}\right\}^2 \mathrm{S}_{K_3}(t)dt+\sigma S_{K_3}(t)dZ^{(3)}(\mathrm{t}), \qquad (4)$$

$$dS_{K_4}(\mathrm{t})=\left(\theta_1\theta_2\right)^2 S_{K_4}(t)dt+\beta S_{K_4}(t)dZ^{(4)}(\mathrm{t}), \qquad (5)$$

where $S_{K_1}(\mathrm{t}), S_{K_2}(\mathrm{t}), S_{K_3}(\mathrm{t})\ and\ S_{K_4}(\mathrm{t})$ are underlying stocks with the following initial conditions:

$$S_{K_1}(0)=S_{0(K_1)}, t>0 \qquad (6)$$

$$S_{K_2}(0)=S_{0(K_2)}, t>0 \qquad (7)$$

$$S_{K_3}(0)=S_{0(K_3)}, t>0 \qquad (8)$$

$$S_{K_4}(0)=S_{0(K_4})\ , t>0. \qquad (9)$$

Though, the price evolution of a risky assets are usually modeled as the trajectory of a risky assets that are usually of a diffusion process defined on some underlying probability space, with the geometric Brownian motion the paramount tool used as the established reference model [13].

Theorem 1.1: (Ito's formula) Let $(\Omega, \beta, \mu, \mathrm{F}(\beta)$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta)$ processing a quadratic variation(X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times2}(\Pi \times \square))$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau + f\frac{\partial u}{\partial x}dW(t).$$

2.1 Method of solution

The investment equation (2)-(5) consist of a system of stochastic differential equations and whose solutions are not trivial. We applying Theorem 1.1 solving for $\mathrm{S}_{K_1}(t),\ S_{K_2}(t),\ S_{K_3}(t),\ and\ S_{K_4}(t)$ To grab this problem we note that

$$\mathrm{S}_{K_1}(t),\ S_{K_2}(t),\ S_{K_3}(t)\ \text{ and } S_{K_4}(t) < \infty \text{ for all } S \in [0,1).$$

From (2) , Thus, generalizing and considering a function $f\left(S_{K_1}(t), t\right)$,hence it has to do with partial derivatives. Expansion of $f\left(S_{K_1}(t), \mathrm{dS}_{K_1}(\mathrm{t}), t+dt\right)$in a Taylor series about$\left(S_{K_1}(t), t\right)$ gives

$$df = \frac{\partial f}{\partial S_{K_1}(t)}dS_{K_1}(t) + \frac{\partial f}{\partial t}dt + \frac{1}{2}\frac{\partial^2 f}{\partial S_{K_1}{}^2(t)}dS_{K_1}^2(t) + \ldots \qquad (10)$$

Substituting (2) in (10) gives

$$df = \frac{\partial f}{\partial S_{K_1}(t)}\left\{\sigma S_{K_1}(t)dZ^{(1)}(\mathrm{t}) + \left\{(\theta_1\theta_2)^{-1}\right\}^2 S_{K_1}(t)dt\ \right\} + \frac{1}{2}\frac{\partial^2 f}{\partial S_{K_1}{}^2(t)}dS_{K_1}^2(t)\,\mathrm{dS}_{K_1}^2(\mathrm{t})$$

$$df = \sigma S_{K_1}(t)\frac{\partial f}{\partial S_{K_1}(t)}dZ^{(1)}(t) + \left(\left\{(\theta_1\theta_2)^{-1}\right\}^2 S_{K_1}(t)\frac{\partial f}{\partial S_{K_1}(t)} + \frac{1}{2}\sigma^2 S_{K_1}^2(t)\frac{\partial^2 f}{\partial S_{K_1}{}^2(t)}dS_{K_1}^2(t)\right)dt.$$

Now considering the SDE in (2).

Let $f\left(S_{K_1}(\mathrm{t})\right)=\ln \mathrm{S}_{K_1}(t)$, the partial derivatives becomes

$$\frac{\partial f}{\partial S_{K_1}(t)}=\frac{1}{S_{K_1}(t)},\ \frac{\partial^2 f}{\partial S_{K_1}^2(t)}=-\frac{1}{S_{K_1}^2(t)},\ \frac{\partial f}{\partial t}=0. \qquad (11)$$

Following Theorem 1.1(Ito's), substituting ( ) and simplifying gives

$$df=\left\{\left\{(\theta_1\theta_2)^{-1}\right\}^2-\frac{1}{2}\sigma^2\right\}dt+\sigma dZ^{(1)}. \qquad (12)$$

Since the RHS of (12) is independent of $f\left(S_{K_1}(t)\right)$,the stochastic is computed as follows:

$$f(S_{K_1}(t)=f_0+\int_0^t\left\{\left\{(\theta_1\theta_2)^{-1}\right\}^2-\frac{1}{2}\sigma^2\ \right\}dt+\int_0^t\sigma dZ^{(1)}(t)$$

$$=f_0+\left\{\left\{(\theta_1\theta_2)^{-1}\right\}^2-\frac{1}{2}\sigma^2\right\}t+\sigma dZ^{(1)}(t), \text{Since } f\left(S_{K_1}(t)\right)=\ln S_{K_1}(t) \text{ a found solution for } S_{K_1}(t) \text{ becomes}$$

$$\ln S_{K_1}(t)=\ln S_{o(K_1)}+\left\{\left\{(\theta_1\theta_2)^{-1}\right\}^2-\frac{1}{2}\sigma^2\right\}t+\sigma dZ^{(1)}(t), \ln S_{K_1}(t)-\ln S_{0(K_1)}=\left\{\left\{(\theta_1\theta_2)^{-1}\right\}^2-\frac{1}{2}\sigma^2\right\}t+\sigma dZ^{(1)}(t)$$

$$\ln\left(\frac{S_{K_1}(t)}{S_{0(K_1)}}\right)=\left\{\left\{(\theta_1\theta_2)^{-1}\right\}^2-\frac{1}{2}\sigma^2\right\}t+\sigma dZ^{(1)}(t)$$

$$S_{K_1}(t)=S_{0(K_1)}\exp^{\left\{\left(\left\{(\theta_1\theta_2)^{-1}\right\}^2-\frac{1}{2}\sigma^2\right)t+\sigma dZ^{(1)}(\mathrm{t})\right\}}. \qquad (13)$$

This is the complete solution of investment equation whose rate of returns and asset price valuation is multiplicative inverse effects with a quadratic function.

From (3), thus, generalizing and considering a function $f\left(S_{K_2}(t),t\right)$,hence it has to do with partial derivatives. Expansion of $f\left(S_{K_2}(t),\mathrm{d}S_{K_2}(\mathrm{t}),t+dt\right)$ in a Taylor series about $\left(S_{K_2}(t),t\right)$ gives

$$df=\frac{\partial f}{\partial S_{K_2}(t)}dS_{K_2}(t)+\frac{\partial f}{\partial t}dt+\frac{1}{2}\frac{\partial^2 f}{\partial {S_{K_2}}^2(t)}dS_{K_2}^2(t)+\ldots \qquad (14)$$

Substituting (3) in (14) gives

$$df = \frac{\partial f}{\partial S_{K_2}(t)}\left\{\sigma S_{_{K_2}}(t)dZ^{(2)}(\mathrm{t}) + \left(\theta_1+\theta_2\right)^2 S_{K_2}(t)dt \quad\right\} + \frac{1}{2}\frac{\partial^2 f}{\partial S_{K_2}{}^2(t)} dS^2_{_{K_2}}(t)\,\mathrm{dS}^2_{_{K_2}}(\mathrm{t})$$

$$df = \sigma S_{_{K_2}}(t)\frac{\partial f}{\partial S_{_{K_2}}(t)} dZ^{(2)}(t) + \left(\left(\theta_1+\theta_2\right)^2 S_{_{K_2}}(t)\frac{\partial f}{\partial S_{_{K_2}}(t)} + \frac{1}{2}\sigma^2 S^2_{_{K_2}}(t)\frac{\partial^2 f}{\partial S_{K_1}{}^2(t)} dS^2_{K_2}(t)\right)dt\,.$$

Now considering the SDE in (3).

Let $f\left(S_{K_2}(\mathrm{t})\right) = \ln \mathrm{S}_{K_2}(t)$, the partial derivatives becomes

$$\frac{\partial f}{\partial S_{_{K_2}}(t)} = \frac{1}{S_{K_2}(t)},\ \frac{\partial^2 f}{\partial S^2_{_{K_2}}(t)} = -\frac{1}{S^2_{_{K_2}}(t)},\ \frac{\partial f}{\partial t} = 0. \qquad (15)$$

Following Theorem 1.1 (Ito's), substituting (15) and simplifying gives

$$df = \left\{\left(\theta_1+\theta_2\right)^2 - \frac{1}{2}\sigma^2\right\}dt + \sigma dZ^{(2)}. \qquad (16)$$

Since the RHS of (16) is independent of $f\left(S_{K_2}(t)\right)$,the stochastic is computed as follows:

$$f(S_{K_2}(t) = f_0 + \int_0^t \left(\theta_1+\theta_2\right)^2 - \frac{1}{2}\sigma^2 \quad \}\,dt + \int_0^t \sigma dZ^{(2)}(t)$$

$$= f_0 + \left\{\left(\theta_1+\theta_2\right)^2 - \frac{1}{2}\sigma^2\right\}t + \sigma dZ^{(2)}(t), \text{Since } f\left(S_{K_2}(t)\right) = \ln S_{_{K_2}}(t) \text{ a found solution for } S_{K_2}(t) \text{ becomes}$$

$$\ln S_{K_2}(t) = \ln S_{o(K_2)} + \left\{\left(\theta_1+\theta_2\right)^2 - \frac{1}{2}\sigma^2\right\}t + \sigma dZ^{(2)}(t), \ln S_{_{K_2}}(t) - \ln S_{0(K_2)} = \left\{\left(\theta_1+\theta_2\right)^2 - \frac{1}{2}\sigma^2\right\}t + \sigma dZ^{(2)}(t)$$

$$\ln\left(\frac{S_{K_2}(t)}{S_{0(K_2)}}\right) = \left\{\left(\theta_1+\theta_2\right)^2 - \frac{1}{2}\sigma^2\right\}t + \sigma dZ^{(2)}(t)$$

$$S_{K_2}(t) = S_{0(K_2)} \exp^{\left\{\left(\left(\theta_1+\theta_2\right)^2 - \frac{1}{2}\sigma^2\right)t + \sigma dZ^{(2)}(\mathrm{t})\right\}}. \qquad (17)$$

This is the complete solution of investment equation whose rate of returns and asset price valuation is additive effects with a quadratic function.

From (4), thus, generalizing and considering a function $f\left(S_{K_3}(t),t\right)$,hence it has to do with partial derivatives. Expansion of $f\left(S_{K_3}(t),\mathrm{dS}_{K_3}(\mathrm{t}),t+dt\right)$in a Taylor series about$\left(S_{K_3}(t),t\right)$ gives

$$df=\frac{\partial f}{\partial S_{K_3}(t)}dS_{K_2}(t)+\frac{\partial f}{\partial t}dt+\frac{1}{2}\frac{\partial^2 f}{\partial {S_{K_3}}^2(t)}dS_{K_3}^2(t)+\ldots \qquad (18)$$

Substituting (4) in (18) gives,

$$df=\frac{\partial f}{\partial S_{K_3}(t)}\left\{\sigma S_{K_3}(t)dZ^{(3)}(\mathrm{t})+\left\{\left(\theta_1+\theta_2\right)^{-1}\right\}^2 S_{K_3}(t)dt\ \right\}+\frac{1}{2}\frac{\partial^2 f}{\partial {S_{K_3}}^2(t)}dS_{K_3}^2(t)\,\mathrm{dS}_{K_3}^2(\mathrm{t})$$

$$df=\sigma S_{K_3}(t)\frac{\partial f}{\partial S_{K_3}(t)}dZ^{(3)}(t)+\left(\left\{\left(\theta_1+\theta_2\right)^{-1}\right\}^2 S_{K_3}(t)\frac{\partial f}{\partial S_{K_3}(t)}+\frac{1}{2}\sigma^2 S_{K_3}^2(t)\frac{\partial^2 f}{\partial {S_{K_3}}^2(t)}dS_{K_3}^2(t)\right)dt\,.$$

Now considering the SDE in (4).

Let $f\left(S_{K_3}(\mathrm{t})\right)=\ln \mathrm{S}_{K_3}(t)$, the partial derivatives becomes

$$\frac{\partial f}{\partial S_{K_3}(t)}=\frac{1}{S_{K_3}(t)},\ \frac{\partial^2 f}{\partial S_{K_3}^2(t)}=-\frac{1}{S_{K_3}^2(t)},\ \frac{\partial f}{\partial t}=0\,. \qquad (19)$$

Following Theorem 1.1 (Ito's), substituting (19) and simplifying gives

$$df=\left\{\left\{\left(\theta_1+\theta_2\right)^{-1}\right\}^2-\frac{1}{2}\sigma^2\right\}dt+\sigma dZ^{(3)}\,. \qquad (20)$$

Since the RHS of (20) is independent of $f\left(S_{K_3}(t)\right)$,the stochastic is computed as follows:

$$f(S_{K_3}(t) = f_0 + \int_0^t \left((\theta_1+\theta_2)^{-1}\right)^2 - \frac{1}{2}\sigma^2 \quad \} dt + \int_0^t \sigma dZ^{(3)}(t)$$

$$= f_0 + \left\{\left((\theta_1+\theta_2)^{-1}\right)^2 - \frac{1}{2}\sigma^2\right\} t + \sigma dZ^{(2)}(t), \text{Since } f\left(S_{K_3}(t)\right) = \ln S_{K_3}(t) \text{ a found solution for } S_{K_3}(t) \text{ becom}$$

$$\ln S_{K_2}(t) = \ln S_{o(K_2)} + \left\{\left((\theta_1+\theta_2)^{-1}\right)^2 - \frac{1}{2}\sigma^2\right\} t + \sigma dZ^{(2)}(t), \ln S_{K_3}(t) - \ln S_{0(K_3)} = \left\{\left((\theta_1+\theta_2)^{-1}\right)^2 - \frac{1}{2}\sigma^2\right\} t + \sigma dZ^{(3)}$$

$$\ln\left(\frac{S_{K_3}(t)}{S_{0(K_3)}}\right) = \left\{\left((\theta_1+\theta_2)^{-1}\right)^2 - \frac{1}{2}\sigma^2\right\} t + \sigma dZ^{(3)}(t)$$

$$S_{K_3}(t) = S_{0(K_3)} \exp^{\left\{\left(\left((\theta_1+\theta_2)^{-1}\right)^2 - \frac{1}{2}\sigma^2\right) t + \sigma dZ^{(3)}(t)\right\}}. \tag{21}$$

This is the complete solution of investment equation whose rate of returns and asset price valuation is additive inverse effects with a quadratic function.

From (5), Thus, generalizing and considering a function $f\left(S_{K_4}(t), t\right)$,hence it has to do with partial derivatives. Expansion of $f\left(S_{K_4}(t), \mathrm{d}S_{K_4}(\mathrm{t}), t+dt\right)$ in a Taylor series about $\left(S_{K_4}(t), t\right)$ gives

$$df = \frac{\partial f}{\partial S_{K_4}(t)} dS_{K_4}(t) + \frac{\partial f}{\partial t} dt + \frac{1}{2}\frac{\partial^2 f}{\partial S_{K_4}{}^2(t)} dS^2_{K_4}(t) + \ldots \tag{22}$$

Substituting (5) in (22) gives

$$df = \frac{\partial f}{\partial S_{K_4}(t)}\left\{\sigma S_{K_4}(t) dZ^{(1)}(\mathrm{t}) + \left(\theta_1\theta_2\right)^2 S_{K_4}(t) dt \quad\right\} + \frac{1}{2}\frac{\partial^2 f}{\partial S_{K_4}{}^2(t)} dS^2_{K_4}(t)\, \mathrm{d}S^2_{K_4}(\mathrm{t})$$

$$df = \sigma S_{K_4}(t)\frac{\partial f}{\partial S_{K_4}(t)} dZ^{(4)}(t) + \left(\left(\theta_1\theta_2\right)^2 S_{K_4}(t)\frac{\partial f}{\partial S_{K_4}(t)} + \frac{1}{2}\sigma^2 S^2_{K_4}(t)\frac{\partial^2 f}{\partial S_{K_4}{}^2(t)} dS^2_{K_4}(t)\right) dt .$$

Now considering the SDE in (5).

Let $f\left(S_{K_4}(\mathrm{t})\right) = \ln \mathrm{S}_{K_4}(t)$, the partial derivatives becomes

$$\frac{\partial f}{\partial S_{K_4}(t)} = \frac{1}{S_{K_4}(t)}, \frac{\partial^2 f}{\partial S^2_{K_4}(t)} = -\frac{1}{S^2_{K_4}(t)}, \frac{\partial f}{\partial t} = 0 \tag{23}$$

Following Theorem 1.1 (Ito's), substituting (23) and simplifying gives

$$df = \left\{ \left(\theta_1\theta_2\right)^2 - \frac{1}{2}\sigma^2 \right\} dt + \sigma dZ^{(4)} \ . \qquad (24)$$

Since the RHS of (24) is independent of $f\left(S_{K_2}(t)\right)$,the stochastic is computed as follows:

$$f(S_{K_4}(t) = f_0 + \int_0^t \left(\theta_1\theta_2\right)^2 - \frac{1}{2}\sigma^2 \ \ \} dt + \int_0^t \sigma dZ^{(4)}(t)$$

$$= f_0 + \left\{ \left(\theta_1\theta_2\right)^2 - \frac{1}{2}\sigma^2 \right\} t + \sigma dZ^{(4)}(t), \text{Since } f\left(S_{K_4}(t)\right) = \ln S_{_{K_4}}(t) \text{ a found solution for } S_{K_4}(t) \text{ becomes}$$

$$\ln S_{K_4}(t) = \ln S_{o(K_4)} + \left\{ \left(\theta_1\theta_2\right)^2 - \frac{1}{2}\sigma^2 \right\} t + \sigma dZ^{(4)}(t), \ln S_{_{K_4}}(t) - \ln S_{0(K_4)} = \left\{ \left(\theta_1\theta_2\right)^2 - \frac{1}{2}\sigma^2 \right\} t + \sigma dZ^{(4)}(t)$$

$$\ln\left(\frac{S_{K_4}(t)}{S_{0(K_4)}} \right) = \left\{ \left(\theta_1\theta_2\right)^2 - \frac{1}{2}\sigma^2 \right\} t + \sigma dZ^{(4)}(t)$$

$$S_{K_4}(t) = S_{0(K_4)} \exp^{\left\{ \left(\left(\theta_1\theta_2\right)^2 - \frac{1}{2}\sigma^2 \right) t + \sigma dZ^{(4)}(\mathrm{t}) \right\}} . \qquad (25)$$

This is the complete solution of investment equation whose rate of returns and asset price valuation is multiplicative effects with a quadratic function.

3 Results

This Section presents the graphical results for the problems in (2)-(5) whose solutions are in (13)-(25). Hence the following parameter values were used in the simulation study:

$\text{Dangote}_{2018:}\theta_1\theta_2 = 21.7, S_{K_1} 0 = 128.91, \ \sigma = 2.5, \ dZ = 1.00, \ t = 1$

$\text{Dangote}_{2019:}S_{K_2} 0 = 166.11, \theta_1 + \theta_2 = 22.4,, \sigma = 2.5, \ dZ = 1.00, \ t = 1$

$\text{Dangote}_{2019:}S_{K_3} 0 = 270, \theta_1 + \theta_2 = 22.4,, \sigma = 2.5, \ dZ = 1.00, \ t = 1$

$\text{Dangote}_{2019:}S_{K_2} 0 = 190 \ , \theta_1\theta_2 = 22.4,, \sigma = 2.5, \ dZ = 1.00, \ t = 1$

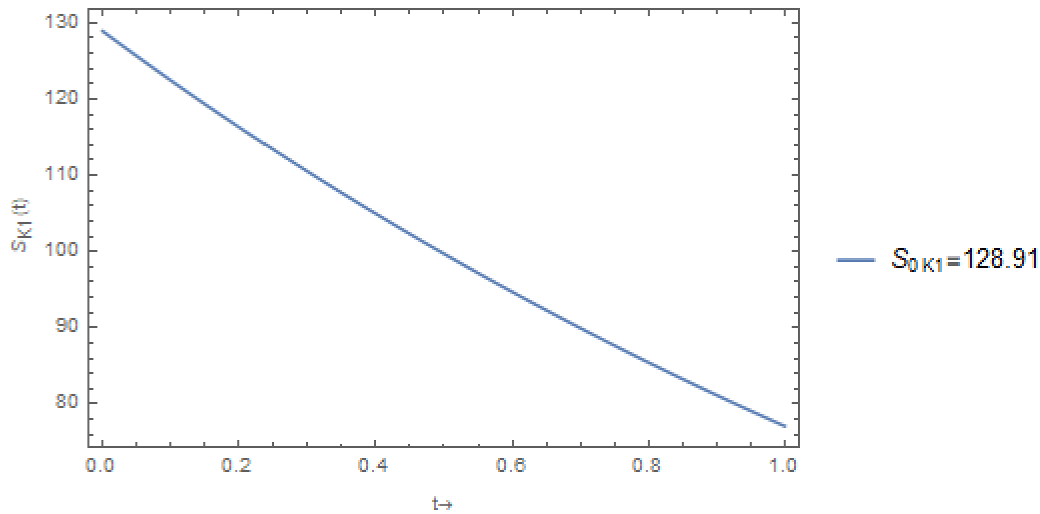

Figure 1: Plot of Dangote stock rate of returns which follows multiplicative inverse effects.

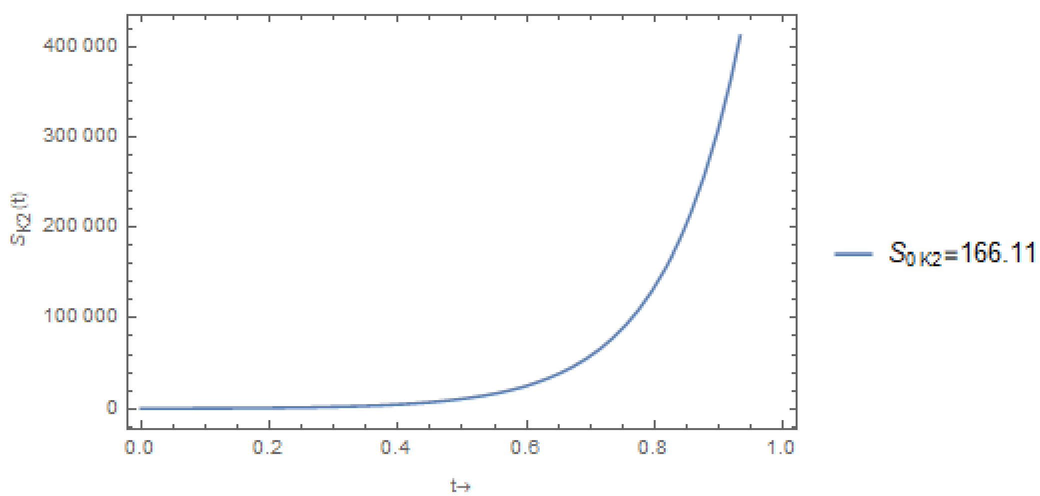

Figure 2: Plot of Dangote stock rate of returns which follows additive effects.

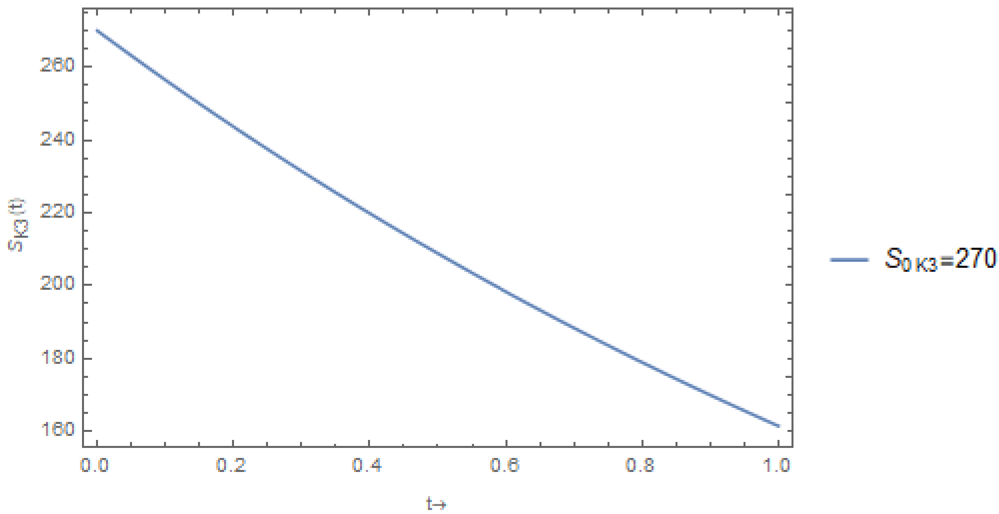

Figure 3: Plot of Dangote stock rate of returns which follows additive inverse effects.

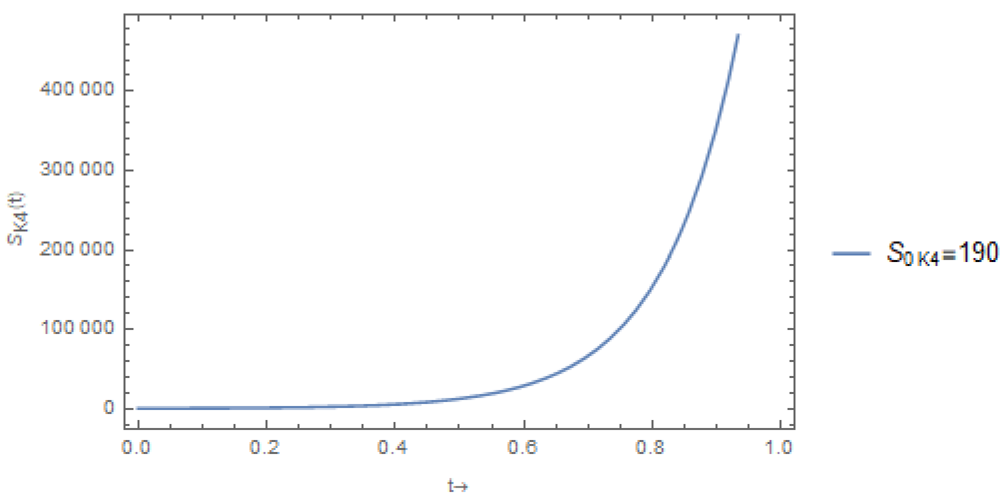

Figure 4: Plot of Dangote stock rate of returns which follows multiplicative effects

3.1 Discussion of results

As seen in Figures 1 and 2 respectively that the plots are linear; this means that there's a linear relationship between return rates and time of trading activities of price changes. Carefully examination of the plots shows that asset return increases linearly as time increases throughout the trading days, with the tendency of being 1. It also describes changes over short and long periods of time, which informs an investor for proper decision making. These results also agree with what the principle of multiplicative inverse and correlation as it affects trend line. Multiplicative inverse of

trend of a number is a number by which the multiplication result is 1. Therefore, in terms of prediction multiplicative inverse is more efficient and reliable than multiplicative effect. The range of regression is within$1 \leq r \leq -1$; by this formation means that total sure of high rate of returns in Dangote cement PLC which is indexed with millions of naira. This result is line with that of [1].
In Figures 2 and 4, it can be observed that the two plots have exponential growth of stock returns of time-varying investment. The trends stipulate that a little increase of time of investment; will also increase increase rate of return of capital investments. In the same vein, time decreases the value of asset return approaches to zero. This result agrees with the concept of multiplicative trend which indicates a non-linear trend (curved trend line). This informs investors that when return rates follow multiplicative trend it grows unboundedly. With this remark, profit making is inevitable throughout the trading days.

Nevertheless, stochastic systems are effective in the mathematical modeling of stock returns for long- and short-term investment plans worth millions of naira.

4 Conclusions

In absence of capital investments, trading business will definitely have a hard time getting off the ground. Therefore, this paper studied the problem of stochastic systems of time-varying investments of Dangote cement PLC where multiplicative inverse effects, additive effects, additive inverse effects and multiplicative effects were used as key parameter in the model to obtain stock rate of returns. The analytical solution of problem showed: (i) a little increase on time of investment; will also increase increases rate of return of capital investments,(ii) stock rates return increases linearly as time also increases,(iii) Multiplicative inverse and additive inverse effects is more effective and reliable than multiplicative effects and additive effect on the ground that it is a sure event (iv) Dangote cement PLC is making high rate of returns based on stochastic analysis of stock price data.
Finally, comparing two or more companies in terms of stock returns and applying stochastic systems will be an interesting area to explore.

Reference

[1] Amadi I.U. and Charles A. (2022). An analytical method for the assessment of asset price changes in capital market. International journal of Applied Science and Mathematical Theory, Vol. 8(2): 1-13.

[2] Osu B.A and Amadi I.U. (2022). A Stochastic Analysis of Stock Market Price Fluctuations for Capital Market. Journal of Applied Mathematics and Computation, Vol. 6(1): 85-95.

[3] Ugbebor O.O, Onah S.E., and Ojowo O. (2001). An empirical stochastic model of stock price changes. Journal of the Nigerian Mathematical Society, Vol. *20*: 95-101.

[4] Osu B.O. and Okoroafor A.C. (2007). On the measurement of random behavior of stock price changes. Journal of Mathematical Science Dattapukur, Vol. 18(2): 131-141.

[5] Amadi I.U, Ogbogbo C.P., and Osu B.O. (2022).Stochastic analysis of stock price changes as markov chain in finite states. Global Journal of Pure and Applied Sciences, Vol. 28: 91-98.

[6] Amadi I.U, Igbudu R., and Azor P.A. (2022).Stochastic analysis of the impact of growth-rates on stock market prices in Nigeria. Asian Journal of Economics, Business and Accounting, Vol. 21(24): 9-21.

[7] Wokoma D.S.A. (Bishop), Amadi, I.U., and Aboko I.S. (2022). A stochastic model for stock market price variation, Asian Research Journal of Mathematics, Vol. 18(4): 41-49.

[8] Osu B.O., Amadi I.U., and Azor P.A. (2022). analytical solution of time-varying investment returns with random parameters. Sebha University Journal of Pure and Applied Sciences, Vol. 21: 2.

[9] Davies I., Amadi I.U., and Ndu R.I. (2019). Stability analysis of stochastic model for stock market prices. International Journal of Mathematics and Computational Methods, Vol. 4: 79-86.

[10] Osu B.O., Okoroafor A.C., and Olunkwa C. (2009). Stability analysis of stochastic model of stock market price. African journal of Mathematics and Computer Science, 2(6): 98-103.

[11] Adeosun M.E., Edeki S.O., and Ugbebor O.O. (2015). Stochastic analysis of stock market price models: a case study of the Nigerian stock exchange(NSE). WSEAS transactions on Mathematics, Vol. 14: 353-363.

[12] Ofomata A.I.O., Inyama S.C., Umana R.A., and Omane A.O. (2017). A stochastic model of the dynamics of stock price for forecasting. Journal of Advances in Mathematics and Computer Science, Vol. 25(6): 1-24.

[13] Osu B.O. (2010). A stochastic model of the variation of the capital market price. International Journal of Trade Economics and Finance, Vol 1(3): 297-302.

[14] Amadi I.U. and Charles A. (2022). Stochastic analysis of time-varying investment returns in capital market domain. International Journal of Mathematics and Statistics Studies, Vol. 10(3): 28-38.

[15] Dmouj A. (2006). Stock price modeling: theory and practice. BIM Paper.

[16] Wets R.J.B. and Rios I. (2012). Modeling and estimating commodity prices: copper prices. JEL classification C53.

[17] Bachelier L. (1964). Theories de la speculative paris: Gauthier-Villar. Transplanted in cooler.