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CATANOVA analysis of academic performance of pupils in primary schools and districts - a tool for conflict resolution

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Abstract

This paper is an extract of the research findings that was carried out in Ofu local government area of Kogi state in 2021 which was used to resolve the conflict that broke out as a result of the establishment of Government Special Science College by the Kogi state Government. The bone of contention then was which of the three districts of the local government should have the new special science college. This necessitated the research to find out if performance depends on the district or not. An analysis of variance for categorical data (CATANOVA) was used in carrying out the test. It was discovered that performance depends on the district, and that pupils from Igalaogba district performed better than the other two districts. Hence the new Government Special Science College, should be sited in this particular district. All the findings were based on the performance of pupils in primary six using common entrance examination results for 2019 since there was no common entrance examination for 2020 due to covid-19 pandemic.

Keywords: districts; Chi- square; catanova; performance

1. Introduction

The growth of statistics has made itself felt in almost every phase of human endeavor. Statistics provides models that are needed to study situations involving uncertainties. One of such is that which confronted Ofu Local government area of Kogi state in 2021.

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The Kogi state government of Nigeria established a new special science college to be located in Ofu local government Area of the state. As usual every district began to give reasons why it should be located in their district. All efforts to result this problem to no avail. This necessitated a research to find out if performance is dependent on the district or not. This is because the district that performed better will be able to feed the college with the desired quality and quantity of students needed by the new school. There are three districts in Ofu Local Government. These include Ugwolawo, Igalaogba and Itobe districts.

This study is therefore aimed at finding out if performance across the districts is the same or not. Hence, this study investigates the comparative academic performances of primary six pupils across the districts of Ofu local government area of Kogi state.

1.1. Aim and objectives of the study

- i. To ascertain if performance is dependent on districts.
- ii. To determine which of the districts should have the school.
- iii. To resolve the conflict already created by this agitation.

1.2. Nature and scope of study

There are 53 primary schools in Ofu Local Government Area of Kogi State out of which samples of 20 primary schools were taken. The primary schools in kogi state like their counterparts in other states are controlled by the state primary Education Board which is now called state universal basic Education Board (SUBEB) which in turn is controlled by the state ministry of education. It is this state ministry of education that provides an approved syllabus to be followed by the primary schools and at the end of the six-year programme, the pupils sit for common entrance examination which qualify them to go to secondary schools. The examination is made up of three papers. These are: Paper 1(English language), paper 2 (Mathematics) and Paper 3 (General paper). At the end of the examination, successful candidates are classified into five groups – Distinction(A), Good (B), Credit (C), Fair (D) and Pass (E). Similarly, unsuccessful candidates are awarded (F). All findings are based on the analysis of this examination results from the sampled schools from the various districts.

1.3. Sampling techniques

Out of the 53 primary schools in Ofu Local government Area, I sampled 20 primary schools according to the classification below.

Sampling distribution of the primary schools in Ofu local government area

Districts	Number of schools
Ugwolawo	9
Igalaogba	6
Itobe	5

The researcher used stratified random sampling techniques to represent the entire Ofu Local Government Area. The local Government has three districts so that communities were used as the

stratifying factor. Ofu Local Government Area comprises of eighteen communities, which is grouped into three districts forming the strata, from which simple random sampling was used. Ugwolaro district consists of Ugwolaro, Ogegume, Ojofa, Obagwu, Umomi, Akpagidigbo, Alome, Ikagbo, and Ugbogbo, while Igalaogba district is made up of Aloma, Igah, Ejule, Ogbonicha, Ogbulu and Ikeje. Finally, Itobe district comprises of Ojodu, Adumu, Aloji, Itobe and Ochadamu. Also, I used proportional allocation of sample size according to the weight (number of the communities that each group consists in terms of number of primary schools in each group).

2. Literature review

Even though many studies have been carried out to ascertain the dependence of performance on one factor or the other, majority of them if not all were carried out using the results from examinations such as first school leaving certificate (FSLCE), Senior school certificate examination (SSCE), University matriculation Examinations (UME) among others as their only source of data for the analysis which this study frown at due to gross examination malpractice usually associated with the conduct and marking of the scripts by the authorities concern.

In order to ascertain if boys are more brilliant than girls, [5] investigated into the performance of boys and girls in a sample of 32 primary schools Nsukka Educational Zone. From the report of his findings, he asserted that boys are more brilliant than girls all within primary school age.

Another research carried out by [1] on how the shift system affects the academic performance of primary school pupils in Enugu using qualitative data based on 1978 only, he came up with the following results: first, that the morning session pupils performed better than the afternoons. Secondly, that this disparity in performance was as a result of the fact that shift one had relatively more qualified and trained teachers than shift two. [12] carried out research on school location and academic performance of pupils in English language in primary schools. He reported that there is a significant difference in the academic performance of pupils in urban and rural primary schools in English language.

3. Methodological framework

The methodological framework for this study is a two-way analysis of variance for categorical data (CATANOVA).

3.1. A two-way catanova with chi-square tests for interaction

Consider a two-way experimental situation with a k -dimensional vector of responses $\{(n_{ijk}) \mid K, i=1,2,\dots,I, j=1,2,\dots,J\}$ recorded in frequencies in the $(ij)^{\text{th}}$ plot. We assume a multinomial probability model, namely,

$$\Pr(\{n_{ijk}\} / \{\pi_{ijk}\}) = \frac{n!}{\prod_k n_{ijk}!} (\pi_{ijk})^{n_{ijk}} \quad \pi_{ijk} \geq 0, \sum_k \pi_{ijk} = 1$$

$$0 \leq \pi_{ijk} \leq 1$$

$$N_{ij} = \sum_k n_{ijk} \quad \text{is held fixed}$$

$n_{ijk}, n_{i'j'k}$ are independent $\forall i \neq i'$

The hypothesis to be tested are:

HR: $\prod_{ijk} = \prod_{jk}$, there is no row effect.

HC: $\prod_{ijk} = \prod_{ik}$, there is no column effect.

HRC: $\prod_{ijk} = \prod_{k}$, there is no row x column interaction.

The expectation of the sample estimate of $\prod_{ijk}$ is given by the linear model

$$E(\prod_{ijk}) = \mu_k + \tau_{ik} + \beta_{jk} + \lambda_{ijk},$$

where μ_k , β_{jk} and λ_{ijk} assume their usual meaning of constant, row effect, column effect and interaction respectively. The hypothesis, HR, HC, and HRC will be seen to be respectively equivalent to

$$H_i: \tau_{ik} = 0 \quad \forall i, H_b: \beta_{jk} = 0 \quad \forall j \text{ and } \lambda_{ijk} = 0 \quad \forall i, j.$$

The parameters: $\prod_{ijk}$ may be considered fixed or random with probability density $h(\prod_{ijk})$, over $\{0,1\}$ depending on whether I and J are random or not [13,14]. Specially,

$$g(\prod_{ijk}) \text{ if I and J are random, } 0 \leq \prod_{ijk} \leq 1,$$

$$h(\prod_{ijk}) = g(\prod_{jk}) \text{ if I is fixed and J random } 0 \leq \prod_{jk} \leq 1,$$

$$g(\prod_{ik}) \text{ if I is random and J fixed, } 0 \leq \prod_{ik} \leq 1.$$

We shall now consider the sum of squares (SS) for $\{n_{ijk}\}_k$. This may be obtained, [6,7,8,9,10,11] also [2,3], as the trace of the sample variance-covariance matrix. That is

$$SS(n_{ijk}) = \sum_k \text{var}(n_{ijk}).$$

Upon further simplification, we have that

$$TSS = n - \sum_k n_{2..k}/n, \text{ as the total sum of squares}$$

$$BRSS = n - \sum_{ik} n_{2i.k}/n_i, \text{ as the between row sum of squares}$$

$$BCSS = n - \sum_{jk} n_{2.jk}/n_j, \text{ as the between column sum of squares}$$

$$WUSS = n - \sum_{ijk} n_{2ijk}/n_{ij}, \text{ as the within unit sum of squares,}$$

where

$$N = \sum_i n_i = \sum_j n_j = \sum_{ij} n_{ij}$$

$$RSS = TSS - BRSS = \sum_k n_{2i.k}/n_i - \sum_k n_{2..k}/n$$

$$CSS = TSS - BCSS = n - \sum_{jk} n_{2.jk}/n_j - \sum_k n_{2..k}/n$$

$$NSS = BRSS + BCSS - TSS - WUSS$$

$$= n - \sum_{ijk} n_{2ijk}/n_{ij} - \sum_{ik} n_{2i.k}/n_i - \sum_{jk} n_{2.jk}/n_j + \sum_k n_{2..k}/n.$$

We may reject HR, HC and HRC respectively at specified α -level of error if

$$\chi^2_R \geq \chi^2_{(\alpha)} \text{ (I-1)(J-1)(K-1),}$$

where

$$\chi^2_R = (K-1)(n-1)RSS/TSS$$

$$\chi^2_C = (K-1)(n-1)css/TSS$$

$$\chi^2_{NT} = (K-1)(n-1)NSS/TSS$$

3.2. Materials and methods

Below is the data layout for a two – way analysis of variance for categorical data (CATANOVA). This was given by [4]

	Districts	Ugwolawo								Igalaogba								Itoke							
i	Responses	A	B	C	D	E	F	T	A	B	C	D	E	F	T	A	B	C	D	E	F	T			
	Males	n ₁₁	n ₁₁₂	n ₁₁₃	n ₁₁₄	n ₁₁₅	n ₁₁₆	n _{11.}	n ₁₁₂	n ₁₁₁	n ₁₁₁	n ₁₁₁	n ₁₁₁	n ₁₁₁	n ₁₁₁	n ₁₁₃₁	n ₁₁₃₂	n ₁₁₃₃	n ₁₁₃₄	n ₁₁₃₅	n ₁₁₃₆	n _{113.}			
	Females	n ₂₁₁	n ₂₁₂	n ₂₁₃	n ₂₁₄	n ₂₁₅	n ₂₁₆	n _{21.}	n ₂₂₁	n ₂₂₂	n ₂₂₂	n ₂₂₂	n ₂₂₂	n ₂₂₂	n ₂₂₂	n ₂₂₃	n ₂₂₃₂	n ₂₂₃₃	n ₂₂₃₄	n ₂₂₃₅	n ₂₂₃₆	n _{223.}			
	n.jk	n ₁₁₁	n ₁₁₂	n ₁₁₃	n ₁₁₄	n ₁₁₅	n ₁₁₆	n _{11.}	n ₁₂₁	n ₁₂₂	n ₁₂₂	n ₁₂₂	n ₁₂₂	n ₁₂₂	n ₁₂₂	n ₁₂₃	n ₁₂₃₂	n ₁₂₃₃	n ₁₂₃₄	n ₁₂₃₅	n ₁₂₃₆	n _{123.}			

Table 1: Data layout for two way catanova

	Ugwolawo							Igalaogba							Itoke						T							
sex	A	B	C	D	E	F	ni 1.	A	B	C	D	E	F	ni 2.	A	B	C	D	E	F	n i3 .	ni. k						ni. .
male	139	62	58	100	55	83	407	71	75	45	24	36	52	303	107	52	53	53	45	75	37	317	189	156	360	210	1047	
fem-ale	80	51	48	11	50	85	325	39	48	29	37	39	51	243	430	20	26	60	30	64	189	162	119	103	590	210	757	
Total	219	113	106	21	105	168	732	110	123	74	61	75	103	546	150	72	79	11	75	139	569	479	308	259	935	410	1804	

Table 2: Two – way catanova test on performance of male and female pupils of primary in Ofu local Government Area of Kogi State in common entrance examination for the year 2021
DISTRICTS

$$\begin{aligned}
 \text{Total sum of squares (TSS)} &= n - \sum_k n2..k/n \\
 &= 1804 - (479^2 + 308^2 + 259^2 + 93^2 + 255^2 + 410^2)/1804 \\
 &= 1804 - 633160/1804 \\
 &= 1804 - 350.9756 \\
 \text{TSS} &= 1453.0244.
 \end{aligned}$$

$$\begin{aligned}
 \text{Between sex sum of squares (BSSS)} &= n - \sum_{ik} n2i.k/n i \\
 &= 1804 - (317^2 + 189^2 + 156^2 + 39^2 + 136^2 + 210^2)/1047 + (162^2 + 119^2 + 103^2 \\
 &\quad + 54^2 + 119^2 + 200^2) / 757 \\
 &= 1804 - (214.5778 + 142.7886) \\
 &= 1804 - 357.3664 \\
 \text{BRSS} &= 1446.6336.
 \end{aligned}$$

$$\begin{aligned}
 \text{Between districts sum of squares (BDSS)} &= n - \sum_{jk} n2.jk/nj \\
 &= 1804 - (219^2 + 113^2 + 106^2 + 21^2 + 105^2 + 168^2)/732 + (110^2 + 123^2 + 74^2 \\
 &\quad + 11^2 + 75^2 + 139^2)/526 \\
 &= 1804 - (152.5355 + 96.4469 + 112.1521) \\
 &= 1804 - 361.1345 \\
 \text{BDSS} &= 1442.8655.
 \end{aligned}$$

$$\begin{aligned}
 \text{Row sum of squares (RSS)} &= \text{TSS} - \text{BDSS} \\
 &= 1453.0244 - 1446.6336 \\
 &= 6.3908 \\
 \text{RSS} &= 6.3908.
 \end{aligned}$$

$$\begin{aligned}
 \text{Column sum of squares (CSS)} &= \text{TSS} - \text{BRSS} \\
 &= 1453.0244 - 1442.8655 \\
 &= 10.1588
 \end{aligned}$$

Source of variance	Degree of freedom	Sum of squares
Row or factor levels or sexes	1	6.3908
Districts	2	10.1588
Sex x districts	10	1.3334
Residual	1790	1435.1414
Total	1803	1453.0244

Table 3: Catanova table

Ho: Performance is independent of the districts.
H1: Performance is dependent on the districts.

$$\begin{aligned}
 \text{The test statistic} &= \chi^2_{\text{cal}} = k - 1(n-1)\text{CSS} / \text{TSS} \\
 &= (6-1)1804 - 1) 10.1588 / 1453.0244 \\
 &= 63.0282.
 \end{aligned}$$

$$\chi^2_{\text{tab}} = \chi^2_{(j-1)(K-1)}^{(0.05)} = \chi^2_{(3-1)(6-1)}^{(0.05)} = 18.307.$$

Decision Rule: Reject H_0 if χ^2_{cal} is greater than χ^2_{tab}

Conclusion:

Since $\chi^2_{\text{cal}} = 63.0283 > \chi^2_{\text{tab}} = 18.307$, we reject H_0 and conclude that performance depends on the districts.

To determine which of the districts perform best, we compute the proportions of students that passed and failed in each districts as follows:

Proportion of pupils in Ugwolawo district that passed is

$$P_{\text{ug passed}} = n_{\text{ug pass}} / N_{\text{ug}} = 0.77.$$

Proportion of pupils in Ugwolawo district that failed is

$$P_{\text{ug failed}} = n_{\text{ug failed}} / N_{\text{ug}} = 168/732 = 0.23.$$

Proportion of pupils in Igalaogba district that passed is

$$P_{\text{ig passed}} = n_{\text{ig passed}} / N_{\text{ig}} = 443/546 = 0.81.$$

Proportion of pupils that failed in Igalaogba is

$$P_{\text{ig failed}} = n_{\text{ig failed}} / N_{\text{ig}} = 103/546 = 0.19.$$

Proportion of pupils in Itobe that passed is

$$P_{\text{it passed}} = n_{\text{it passed}} / N_{\text{it}} = 387/526 = 0.74.$$

Proportion of pupils in Itobe that failed is

$$P_{\text{it failed}} = n_{\text{it failed}} / N_{\text{it}} = 139/526 = 0.26.$$

The proportions of pupils that passed in each district in the decreasing order is as follows:

$$\text{Igalaogba} = 0.81$$

$$\text{Ugwolawo} = 0.77$$

$$\text{Itobe} = 0.74.$$

Similarly, the proportion of pupils that failed in each district in the increasing order is as follows:

$$\text{Igalaogba} = 0.19$$

$$\text{Ugwolawo} = 0.23$$

$$\text{Itobe} = 0.26.$$

4. Conclusion

From the foregoing, it shows that performance depends on the district. This means that performance of any chosen pupil depends on which district the pupil comes from. From the result obtained, pupils from Igalaogba district performed better than the other two districts with the Itobe district having the poorest. The researcher therefore recommended that the government special science college should be located at Igalaogba district.

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