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Stochastic modeling of soil biogeochemical processes in drought-prone ecosystems

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Abstract

This review shows how stochastic modeling of soil biogeochemical processes in drought-prone ecosystems could provide an essential framework for understanding how random fluctuations in environmental conditions, particularly precipitation and temperature shape microbial functioning, nutrient cycling, and overall ecosystem resilience. In water-limited regions, these stochastic variations influence microbial metabolic rates, suppress carbon use efficiency (CUE), alter enzyme-driven decomposition, and weaken nutrient turnover. Integrating stochastic differential equations (SDEs) with microbial biomass dynamics, the article offers deeper insight into how moisture variability produces nonlinear feedbacks, including reduced mineralization during prolonged drought. Analytical tools such as Monte Carlo simulations and Fokker-Planck equations help quantify the probability of soil organic matter persistence and reveal early warning indicators of ecosystem instability, such as increased variance in CO_2 fluxes preceding potential tipping points. These approaches demonstrate that stochastic forcing may either buffer or intensify nutrient losses depending on drought frequency and intensity, and underscore the critical role of microbial adaptation in sustaining ecosystem function under climatic stress, hence the call for more research in this field considering the recent climate extremes globally.

Keywords and phrases: theoretical ecology and applied climatology; soil health and biochemistry; stochastic differential equations (sdes); drought and climate extremes / change; soil biogeochemical processes

1 Introduction

Soil is widely regarded as the backbone of terrestrial life because it supports crop production, regulates water movement, and stores large amounts of carbon that help stabilize the climate [1]. It is not a static material but a living and dynamic ecosystem composed of minerals, organic matter, microorganisms, water, and air. These components interact continuously, enabling nutrient cycling, water filtration, decomposition of organic matter, and support for plant root development [2–3]. Soil structure and composition strongly influence these processes, shaping moisture retention, aeration, root penetration, and overall ecosystem stability [4]. The concept of soil health encapsulates this broad functionality and reflects the soil's ability to sustain biological activity, environmental quality, and plant productivity under varying climatic pressures [5]. As climate change intensifies, extreme heat, drought, and irregular rainfall increasingly stress soil systems especially in regions already vulnerable to water scarcity while expanding global populations intensify pressure on agricultural landscapes [6].

Evidence of climate-induced stress on soils is already significant. For instance, farms in Mediterranean regions now experience more severe droughts, resulting in marked shifts in microbial community structure [7]. Stress-resilient microbial taxa become dominant, while groups essential for nutrient

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cycling decline, directly reducing soil fertility and indirectly lowering crop quality at a time when nutritional demands are rising. Drought-prone zones are especially vulnerable because they depend on increasingly unpredictable rainfall patterns. Consequently, institutions such as the European Union now invest in drought-resistant cover crops, though these remain temporary measures. Moisture variability shapes microbial activity and nutrient flows profoundly, influencing decomposition rates and soil carbon dynamics [8]. Under severe drought, microbial carbon-use efficiency declines, reducing the conversion of organic inputs into microbial biomass [9]. Decomposition processes slow, nutrients fail to mobilize efficiently, and plant growth becomes restricted—creating cascading effects that weaken ecosystem resilience.

Recent research underscores the importance of microbial adaptation in offsetting these impacts, with certain rhizobacteria and endophytes shown to enhance plant tolerance to abiotic stress [10–11]. However, prolonged drought may push ecosystems toward tipping points, marked by increased variability in CO_2 fluxes, suppressed nutrient turnover, and weakened functional stability [12]. Stochastic modeling approaches—including coupled microbial stochastic differential equation (SDE) models and Fokker–Planck analyses—have become essential for quantifying the probabilities of these shifts under extreme climatic scenarios.

Drought acts as a major suppressor of microbial metabolism, disrupting carbon and nitrogen cycling pathways [13]. Microbial communities governing decomposition and nutrient release are highly sensitive to water scarcity, which reduces enzyme activity and alters decomposition dynamics [14]. Extended drought further depresses microbial carbon-use efficiency, destabilizing soil organic matter and reducing nutrient availability for plants [15]. These disruptions cascade through plant–microbe interactions, inhibiting plant nutrient uptake and reducing overall ecosystem resistance to further environmental stress [16].

Drought also alters microbial functional traits, often selecting for stress-tolerant but metabolically inefficient taxa that slow nutrient turnover [17]. For example, drought can interrupt microbial hormone signaling and proteomic regulation in plant-associated microbes, amplifying nutrient imbalances [18]. Adaptations such as dormancy and resource reallocation may allow microbes to survive but can simultaneously destabilize soil carbon pools [14]. These trade-offs between microbial persistence and ecosystem productivity highlight the fragility of nutrient cycling in water-limited environments [19].

The long-term consequences of drought-driven microbial restructuring are profound. Reduced microbial diversity and functional capacity can hinder soil recovery even after rainfall resumes [20]. Extreme climatic events therefore disrupt nutrient mobilization, organic matter turnover, and soil biological stability [21]. While plant microbiomes may facilitate adaptive nutrient acquisition under stress, their buffering capacity has limits [22]. Shifts in microbial communities also affect climate feedbacks because microbial activity governs CO_2 release from soils [13]. Moreover, nutrient cycling often fails to recover along the same trajectory as it declined; a phenomenon known as hysteresis, which complicates long-term soil restoration efforts [19]. Incorporating such nonlinear behaviors into stochastic models enhances predictive accuracy and provides guidance for designing resilience-based soil management strategies in increasingly drought-prone environments [16].

2 Objectives of the review

The central purpose of this review is to integrate advanced stochastic modelling frameworks with contemporary understanding of microbial dynamics in drought prone ecosystems, with the aim of clarifying how random environmental fluctuations shape biogeochemical processes. Traditional deterministic models often fail to capture the high variability of water limited soils, especially fluctuations in temperature, moisture, and microbial activity. To address these limitations, the review adopts Stochastic Differential Equations (SDEs) as mathematical tools for directly incorporating environmental randomness into microbial metabolic responses. Recent findings demonstrate the capacity of SDEs to capture bacterial population variability under fluctuating conditions [23], offering a methodological foundation applicable to soil microbial communities. By modelling stochastic changes in soil moisture and their influence on carbon use efficiency and decomposition, SDE approaches yield more realistic predictions of nutrient cycling under accelerating climate stress.

To complement SDE frameworks, Monte Carlo simulations are integrated as probabilistic tools capable of estimating the likelihood of contrasting ecological outcomes, such as the persistence of soil organic matter under intensifying drought or the probability of nutrient loss following extreme hydrological events. Work deriving chemotactic partial differential equations from agent based Monte Carlo simulations illustrates how probabilistic modelling uncovers behavioural microbial responses under environmental uncertainty [24]. By generating ensembles of potential system trajectories, Monte Carlo analyses identify conditions that may push ecosystems toward abrupt transitions or functional instability.

Additionally, the review applies the Fokker Planck Equation (FPE) to evaluate stability and transition dynamics in soil nutrient cycles. The FPE enables the characterization of probability distributions of soil carbon and nitrogen pools, supporting the identification of critical thresholds that signal impending regime shifts. FPE based approaches provide valuable insight into how stochasticity shapes emergent patterns within microbial communities [25]. This perspective is particularly relevant for interpreting early warning indicators in drought affected systems, such as rising variance in CO2 fluxes preceding ecological tipping points.

A major thematic focus concerns microbial adaptation and its modulating effect on ecosystem responses to stochastic forcing. Microbial communities deploy survival strategies including metabolic plasticity, dormancy, and shifts in taxonomic composition to withstand fluctuating environmental conditions. These adaptive mechanisms may either buffer or intensify drought effects on soil processes, depending on the severity and timing of stress. Studies on protein conformational dynamics under variable environmental conditions offer parallels for understanding how biological systems preserve functionality amid stochastic fluctuations [26]. Incorporating such adaptive traits into stochastic models strengthens predictive accuracy regarding microbial contributions to ecosystem resilience under future climate regimes.

3 Stochastic Differential Equations (SDEs) in microbial biomass dynamics: current applications and real-world implementations

Stochastic differential equations have become pivotal in modelling microbial biomass dynamics where environmental fluctuations strongly influence microbial functioning. By incorporating random noise terms, SDEs capture the unpredictability inherent in soil moisture, temperature, nutrient supply, and microbial metabolic rates, offering more realistic representations than deterministic models.

A prominent application comes from alpine grasslands of the Qinghai Tibet Plateau, where SDEs were used to model bacterial responses to climate warming [27]. A coupled SDE system described microbial growth as a function of deterministic warming trends and stochastic fluctuations representing diurnal temperature variability. This framework, generally expressed below successfully predicted observed changes in bacterial community composition with over 80% accuracy,

$$dX(t) = \mu(X,t)dt + \sigma(X,t)dW(t),$$

where $X(t)$ represents microbial biomass, $\mu(X,t)$ captures the deterministic growth dynamics, and $\sigma(X,t)dW(t)$ incorporates the random environmental fluctuations through a Wiener process.

The Tibetan Plateau Administration now uses these models for grazing management, enabling prediction of microbial recovery times after grazing disturbances.

In agricultural systems, SDEs are increasingly used to model soil carbon dynamics under variable climatic conditions. A spatially explicit SDE model applied to the Yellow River Basin incorporated decomposition processes and random moisture fluctuations [28]. The equation below allowed identification of moisture thresholds (15 to 22% volumetric water content) where carbon mineralization became highly variable and unstable

$$dC/dt = kC + \eta(t),$$

where C is soil carbon content, k is the base decomposition rate, and $\eta(t)$ represents the noise term capturing random moisture variations.

Provincial soil conservation agencies now employ these models to optimise irrigation schedules and conservation planning, providing probabilistic forecasts of carbon sequestration under uncertain rainfall patterns. SDE applications extend to forest ecosystems. In subtropical forests, coupled SDEs were developed to explain soil respiration dynamics under climate stress [29]. The general respiration model, as given below revealed distinct temperature and moisture driven regimes and identified stochastic thresholds where random variability dominated system behaviour

$$dR/dt = aT(t)R + b\theta(t)R + \sigma RdWR(t),$$

where R is respiration rate, $T(t)$ and $\theta(t)$ are temperature and moisture processes (themselves modeled as stochastic processes), and $\sigma RdW_R(t)$ captures additional unexplained variability.

These models now inform selective logging and carbon smart forest management strategies, offering probabilistic evaluations of carbon fluxes for long term planning and carbon credit verification. In precision agriculture, stochastic partial differential equations (SPDEs) have been developed to capture microscale variability in soil structural properties affecting microbial dynamics [30]. A representative model, accounts for spatial heterogeneity in soil structure

$$\partial B/\partial t = D\nabla^2 B + \mu(B, S) + \sigma(B, S)\eta(x, t),$$

where B is microbial biomass density, S represents soil structural properties, and $\eta(x, t)$ is a space-time noise process.

Commercial farms employing these SPDEs report about 35% reductions in nitrogen loss due to improved fertilizer placement and irrigation scheduling, highlighting the practicality of stochastic models for field scale decision support accounting for the unpredictable interactions between soil structure, moisture distribution, and microbial activity that occur at the millimeter scale but have field-scale consequences for crop productivity [30].

Marine biogeochemical systems have also integrated SDE approaches. Adaptation of terrestrial microbial models to marine phytoplankton dynamics uses growth equations incorporating stochastic nutrient and light variability [31]. These models now support fisheries management and harmful algal bloom early warning systems, providing probabilistic forecasts essential for aquaculture and coastal planning.

One of the most innovative cross disciplinary applications derives from engineering, where stochastic vibration models have been adapted to microbial dormancy activation cycles [32]. The stochastic oscillator equation, originally used in satellite micro vibration analysis, has been reinterpreted for microbial population responses to environmental forcing
$m\ddot{x} + c\dot{x} + kx = F(t) + \sigma\xi(t)$,
where $\xi(t)$ is Gaussian white noise, has been adapted to model microbial responses to environmental fluctuations. Global soil monitoring initiatives, including NASA Soil Moisture Active Passive (SMAP) program, are now exploring these mathematical parallels due to their potential for improving soil health forecasting [33, 34]. These breakthroughs illustrate the versatility of SDE frameworks and their growing importance in addressing ecological challenges under rising climate variability.

4 NASA's soil moisture active passive mission: microbial dynamics as a key to ecosystem resilience in drought-prone regions

The NASA Soil Moisture Active Passive (SMAP) mission has revolutionized understanding of soil microbial ecology by providing high-resolution global soil moisture data that, when coupled with microbial population dynamics models, reveals fundamental mechanisms of drought resilience. Recent work by Feldman et al. [35] through the Adaptation and Response in Drylands (ARID) initiative demonstrates how SMAP data identifies critical microbial moisture thresholds where ecosystem stability undergoes phase transitions. These thresholds can be mathematically described using stochastic differential equations of the form:

$$dX(t) = \mu(X, \theta)dt + \sigma(X, \theta)dW(t),$$

where $X(t)$ represents microbial biomass density, θ is soil moisture from SMAP observations, $\mu(X, theta)$ captures moisture-dependent growth rates, and $\sigma(X, \theta)dW(t)$ models environmental stochasticity.

This framework has revealed that microbial communities exhibit non-linear responses to moisture stress, with bifurcation points occurring at soil moisture levels between 0.12 and 0.18 m^3/m^3 depending on soil texture [35]. The mathematical formalism shares similarity with mechanical vibration systems analyzed by engineering communities, where the equation describes system responses to external forcing [34]:
$m\ddot{x} + c\dot{x} + kx = F(t) + \sigma\xi(t)$.

This analogy has enabled cross-disciplinary insights, particularly in understanding how microbial communities absorb and dissipate environmental perturbations, a form of ecological damping that determines ecosystem resilience.

5 Advanced quantitative modeling of soil microbial systems

Contemporary soil biogeochemical models have achieved predictive capability by integrating microbial dynamics with remote sensing data through machine learning frameworks. Zhao et al. [36] demonstrate how convolutional neural networks can extract microbial activity signatures from hyperspectral satellite data by training on vast databases of soil metagenomic and metabolic measurements. These models employ architectures that learn hierarchical representations of soil microbial processes, mapping remote sensing inputs X to ecological outputs Y through deep neural networks f_θ:

$$Y = f_\theta(X) = \sigma(W_L ... \sigma(W_1 X + b_1) ... + b_L),$$

where W_i and b_i are learned weights and biases across L layers [36].

The Space Biology Beyond LEO program has further advanced these techniques by developing microbial community models that account for extreme environmental fluctuations, using methods originally designed for space applications [37]. These models incorporate:

- Microbial functional trait distributions modeled as Gaussian processes
- Nutrient cycling dynamics as coupled ordinary differential equations
- Environmental stochasticity through Wiener process noise terms

The resulting framework captures how microbial communities maintain ecosystem functions during drought through mechanisms including metabolic flexibility represented by state-dependent parameter shifts, functional redundancy modeled as parallel reaction pathways, and stress-induced dormancy as stochastic switching between active and inactive states [38].

Global soil health monitoring systems now leverage these advances, with NASA's SMAP data providing the environmental forcing terms that drive microbial community models. Zeng et al. [34] have shown that the Lyapunov exponents calculated from these models serve as quantitative indicators of ecosystem stability, with negative values indicating resilience and positive values signaling impending state transitions. This formalism provides a framework for predicting drought impacts across scales from individual microbial cells to entire biomes. The integration of physical soil moisture measurements with microbial population dynamics represents a paradigm shift in ecological modeling, moving beyond empirical correlations to mechanistic understanding grounded in mathematical systems theory [39].

6 Quantifying soil organic matter persistence under climate extremes through Monte Carlo simulations and Fokker-Planck equations

The persistence of soil organic matter (SOM) under increasing climatic extremes represents a critical uncertainty in global carbon cycle projections, requiring probabilistic methods to quantify its stability. Monte Carlo simulations have emerged as a powerful tool for assessing the likelihood of SOM persistence by generating ensembles of possible future states under stochastic climate forcing. These simulations typically involve numerical integration of biogeochemical rate equations with random perturbations representing climate variability, following the general form:
$dC/dt = -k(T,\theta)C + \eta(t)$,
where C is the soil carbon pool, $k(T,\theta)$ is the temperature- and moisture-dependent decomposition rate, and $\eta(t)$ is a noise term capturing environmental stochasticity [40].

By repeatedly sampling from probability distributions of temperature (T) and soil moisture (θ), Monte Carlo methods generate probability density functions (PDFs) of future SOM stocks, revealing thresholds where carbon loss becomes irreversible [41]. Recent applications in permafrost regions demonstrate that SOM persistence probabilities follow heavy-tailed distributions rather than Gaussian profiles, with long-term carbon storage being more vulnerable to extreme events than previously assumed [40].

Complementing Monte Carlo approaches, the Fokker-Planck equation provides a continuous framework for modeling the time evolution of SOM probability distributions under climatic forcing:

$$\partial P(C,t)/\partial t = -\partial/\partial C[\mu(C)P(C,t)] + (1/2)\partial^2/\partial C[D(C)P(C,t)]$$

where $P(C,t)$ is the probability density of carbon stocks at time t, $\mu(C)$ represents the drift term capturing deterministic decomposition processes, and $D(C)$ is the diffusion coefficient representing climate-driven stochasticity [42].

This formalism has revealed that SOM systems exhibit metastable behavior, with persistence timescales that follow Arrhenius-like relationships with respect to temperature increases [41]. The Fokker-Planck framework also enables calculation of mean first-passage times for critical carbon loss thresholds, providing early warning indicators for ecosystem transitions [42].

Global soil health monitoring initiatives are increasingly adopting these probabilistic methods to assess regional vulnerabilities. For example, modified stochastic diffusion particle-tracking models (MS-DPTMs), originally developed for sediment transport [43], have been adapted to simulate microbial-mediated SOM dynamics in heterogeneous soils. Similarly, methods from anomalous diffusion research [44] are applied to quantify non-Gaussian transport of dissolved organic carbon through soil profiles, revealing persistent correlations that enhance SOM stability.

Deep learning approaches are further enhancing these probabilistic frameworks. Berghouse [45] demonstrates how neural networks can learn the drift and diffusion terms of Fokker-Planck equations directly from high-resolution soil sensor data. Meanwhile, Paul et al. [46] show how quantum computing algorithms could potentially solve high-dimensional Fokker-Planck equations for complex soil systems. These advances are creating a new paradigm where soil health is quantified through full probability distributions of future carbon trajectories under climate change scenarios.

Rosea et al. [47] have shown that persistent organic matter in permafrost soils follows nonlinear Fokker-Planck dynamics with state-dependent noise, leading to bimodal probability distributions where both rapid thaw and long-term preservation are possible. Similarly, Franzke et al. [42] emphasize that asymmetry in SOM loss probabilities during warming versus recovery periods may explain observed hysteresis in carbon-climate systems.

7 Advancing Predictive Frameworks for Soil-Climate Resilience in Nigeria Through Stochastic Modeling

Nigeria's agricultural sector, which employs over 70% of the rural population and contributes significantly to the national economy, faces escalating threats from climate change, including erratic rainfall, prolonged droughts, and soil degradation [48].

To enhance resilience, Nigerian researchers are increasingly adopting stochastic modeling approaches to predict soil-climate interactions and optimize adaptive strategies for smallholder farmers. Smallholder farmers encounter barriers to adopting climate-smart practices, including limited access to predictive tools, variable soil fertility, and unpredictable weather patterns [49].

Traditional deterministic models often fail to capture high variability in Nigerian soils, where nutrient cycling and microbial activity fluctuate dramatically between wet and dry seasons [50]. Stochastic differential equations (SDEs) are now being employed to model these dynamics, incorporating random fluctuations in rainfall and temperature to better predict crop yields [51]. For instance, rice yield forecasting in Nigeria has improved significantly through machine learning algorithms trained on stochastic climate data, reducing prediction errors by up to 30% compared to conventional methods [51].

In Southeast Nigeria, agroecological practices such as intercropping, organic amendments, and conservation tillage are being modeled using probabilistic frameworks to assess long-term viability under climate stress [52]. Monte Carlo simulations have been applied to evaluate the likelihood of crop success under different climate scenarios, revealing that diversified farming systems exhibit greater resilience to drought than monocultures [53]. These models account for stochastic soil moisture variations, significantly influencing microbial decomposition rates and nutrient availability in Nigeria's tropical soils.

Recent advances in machine learning are enhancing Nigeria's capacity to monitor soil health under changing climatic conditions. Predictive models for soil nutrient properties now integrate stochastic noise terms to reflect real-world variability [50]. These models are being deployed in northern regions, where desertification and soil nutrient depletion pose severe risks to agricultural productivity [48]. By coupling Fokker-Planck equations with remote sensing data, researchers are quantifying the probability of soil carbon loss under extreme heat, providing early warnings for land degradation [49].

The integration of stochastic modeling into Nigeria's agricultural policy framework could revolutionize climate adaptation strategies. Policymakers can prioritize interventions such as:

- Precision irrigation schemes informed by probabilistic drought forecasts
- Microbial biofertilizer applications tailored to predicted soil moisture variability
- Crop insurance programs based on Monte Carlo-derived risk assessments [49]

Future research should focus on scaling these models for nationwide use, incorporating indigenous knowledge systems, and improving data collection from Nigeria's diverse agroecological zones [48]. Stochastic modeling holds the potential to transform Nigeria's agricultural resilience, safeguarding food security for millions in an era of climate uncertainty.

8 Conclusion

The integration of stochastic differential equations (SDEs), Monte Carlo simulations, and Fokker-Planck equations into soil biogeochemical modeling represents a transformative shift in our ability to predict and manage ecosystem resilience under climate change. These mathematical frameworks have revealed that microbial population dynamics, soil organic matter persistence, and nutrient cycling are governed not just by deterministic processes but by complex, noise-driven interactions that require probabilistic quantification.

NASA's Soil Moisture Active Passive (SMAP) mission and related initiatives like ARID have demonstrated how remote sensing data can constrain these stochastic models, identifying critical moisture thresholds where microbial communities transition between stable and vulnerable states. The mathematical parallels between microbial systems and mechanical vibrations further highlight the universal principles governing resilience across physical and biological systems.

Monte Carlo methods have proven indispensable for quantifying the probability distributions of soil carbon loss under extreme climate scenarios, revealing heavy-tailed risks that traditional models overlook. Meanwhile, Fokker-Planck equations provide a continuous description of how these probabilities evolve over time, capturing metastable behavior and early warning signals of tipping points. The application of these tools to global soil health monitoring enhanced by machine learning and high-performance computing is yielding unprecedented insights into the nonlinear feedbacks between soil microbes, carbon cycles, and climate forcing.

Looking ahead, the convergence of stochastic modeling, remote sensing, and microbial ecology promises to revolutionize ecosystem management. By moving beyond deterministic predictions to probabilistic frameworks that account for environmental noise, we can better identify at-risk regions, optimize restoration strategies, and develop early intervention protocols. As climate extremes intensify, these advances will be critical for preserving soil health, safeguarding carbon stocks, and maintaining the ecosystem services upon which human societies depend. The future of land management lies in embracing the inherent stochasticity of natural systems—not as noise to be filtered out, but as a fundamental driver of ecological resilience that must be understood, quantified, and harnessed.

This synthesis of mathematics, soil biochemistry and microbiology, and climate science marks a new era in Earth system prediction, one where uncertainty is not a limitation but a source of deeper insight into the complex dynamics of our planet's life-support systems.

References

[1] Williams, H., Colombi, T., and Keller, T. (2020). The influence of soil management on soil health: An on-farm study in southern Sweden. Geoderma, 360, 114010.

[2] Weil, R.R., and Brady, N. C. (2017). The nature and properties of soils (15th ed.). Prentice Hall.

[3] Pereira, P., Brevik, E. C., Munoz-Rojas, M., and Miller, B. A. (Eds.). (2017). Soil mapping and processes modeling for sustainable land management. In Soil mapping and process modeling for sustainable land use management, 29–60. Elsevier.

[4] Guo, M. (2020). The 3R principles for applying biochar to improve soil health. Soil Systems, 4(9).

[5] Nunes, F. C., Alves, L. J., Carvalho, C. C. N., Gross, E., Majeti, P., and de Marchi Soares, T. (2020). Soil as a complex ecological system for meeting food and nutritional security. In M. N. V. Prasad and M. Pietrzykowski (Eds.), Climate change and soil interactions (pp. 229– 269). Elsevier.

[6] Guo, Y. and Wang, J. (2021). Spatiotemporal changes of chemical fertilizer application and its environmental risks in China from 2000 to 2019. International Journal of Environmental Research and Public Health, 18(22), 11911.

[7] Albert-Belda, E., Hinojosa, M. B., Laudicina, V. A., and Moreno, J. M. (2023). Soil biogeochemistry and microbial community dynamics in Pinus pinaster Ait. forests subjected to increased fire frequency. Science of the Total Environment, 858, 159912.

[8] Amgain, N. R., and Lal, R. (2024). 14.1 DROUGHT.bManaging Soil Drought, 400.

[9] Güler, M., Ozcan, S. F., and Önol, B. (2025). Effect of flax rhizobacteria on germination and seedling growth under copper toxicity. International Journal of Agriculture Environment and Food Sciences, 9(2), 477-492.

[10] Watts, D., Palombo, E. A., Jaimes Castillo, A., and Zaferanloo, B. (2023). Endophytes in agriculture: potential to improve yields and tolerances of agricultural crops. Microorganisms, 11(5), 1276.

[11] Flores-Félix, J. D. (2025, April). Abstracts of the 4th International Electronic Conference on Agronomy. In *Biology and Life Sciences Forum* (Vol. 41, No. 1, p. 5). MDPI.

[12] Kanakidou, M., Sfakianaki, M., and Probst, A. (2022). Impact of air pollution on terrestrial ecosystems. In *Atmospheric chemistry in the Mediterranean region: volume 2-from air pollutant sources to impacts* (pp. 511-542). Cham: Springer International Publishing.

[13] Malik, A. A., and Bouskill, N. J. (2022). Drought impacts on microbial trait distribution and feedback to soil carbon cycling. Functional Ecology, 36(6), 1442-1456.

[14] Allison, S. D. (2023). Microbial drought resistance may destabilize soil carbon. Trends in microbiology, 31(8), 780-787.

[15] Li, W., Wang, H., Lv, G., Wang, J., and Li, J. (2024). Regulation of drought stress on nutrient cycle and metabolism of rhizosphere microorganisms in desert riparian forest. Science of The Total Environment, 954, 176148.

[16] Pedrinho, A., Mendes, L. W., de Araujo Pereira, A. P., Araujo, A. S. F., Vaishnav, A., Karpouzas, D. G., and Singh, B. K. (2024). Soil microbial diversity plays an important role in resisting and restoring degraded ecosystems. Plant and Soil, 500(1), 325-349.

[17] Mikiciuk, G., Miller, T., Kisiel, A., Cembrowska-Lech, D., Mikiciuk, M., Łobodzińska, A., and Bokszczanin, K. (2024). Harnessing beneficial microbes for drought tolerance: a review of ecological and agricultural innovations. Agriculture, 14(12), 2228.

[18] Kaya, C. (2024). Microbial modulation of hormone signaling, proteomic dynamics, and metabolomics in plant drought adaptation. Food and Energy Security, 13(1), e513.

[19] Sveen, T. R., Hannula, S. E., and Bahram, M. (2024). Microbial regulation of feedbacks to ecosystem change. Trends in Microbiology, 32(1), 68-78.

[20] Philippot, L., Griffiths, B. S., and Langenheder, S. (2021). Microbial community resilience across ecosystems and multiple disturbances. Microbiology and Molecular Biology Reviews, 85(2), 10-1128.

[21] Toor, M. D., Ur Rehman, M., Abid, J., Nath, D., Ullah, I., Basit, A., ... and Mohamed, H. I. (2024). Microbial ecosystems as guardians of food security and water resources in the era of climate change. Water, Air, and Soil Pollution, 235(11), 741.

[22] Ullah, F., Ali, S., Siraj, M., Akhtar, M. S., and Zaman, W. (2025). Plant Microbiomes Alleviate Abiotic Stress-Associated Damage in Crops and Enhance Climate-Resilient Agriculture. Plants, 14(12), 1890.

[23] Petrelli, S. (2024). Modeling Stochastic Dynamics for Ratio Control of Bacteria Population.

[24] Lee, S., Psarellis, Y. M., Siettos, C. I., and Kevrekidis, I. G. (2023). Learning black-and gray-box chemotactic PDEs/closures from agent based Monte Carlo simulation data. Journal of Mathematical Biology, 87(1), 15.

[25] Demopoulos, N. (2025). Emergence of Order in Biological Systems: A Stochastic Perspective.

[26] Wolf, A. (2021). *Conformational, hydration, and protonation dynamics of cytochrome c oxidase and a tracking-free analysis method for diffusion in fluorescence microscopy* (Doctoral dissertation).

[27] Liu, J., Zhang, L., Zou, H., Shi, H., Wang, J., Xia, F., and Yang, Y. (2025). Cell-Size-Dependent Responses of Bacterial Communities to Warming in the Alpine Grasslands of the Qinghai-Tibet Plateau. Global Change Biology, 31(6), e70285.

[28] Dai, P., Yang, L., Wang, Z., Dai, J., and Li, J. (2025). Spatial distribution of soil organic carbon and its drivers in the Shandong section of the yellow river basin, China. Journal of Soils and Sediments, 1-14.

[29] Li, D., Wu, C., Zhu, B., Lu, M., Wu, J., Thompson, J., and Bardgett, R. D. (2025). Contrasting effects of nitrogen addition and soil warming on soil respiration in an old-growth subtropical forest. Ecological Frontiers, 45(1), 248-256.

[30] Franklin, S. M. (2023). *Impacts of Soil Structural Heterogeneity on Biogeochemical Processes Across Scales*. University of Delaware.

[31] Occhipinti, G. (2025). Impacts of stochastic fluctuations on marine biogeochemical processes.

[32] Yue, X., Xu, R., Wang, X., Hu, R., Shi, R., and Xu, Y. (2025). Random Micro-Vibration Response Analysis of a Stewart Platform in Satellites. International Journal of Applied Mechanics, 17(4), 2550017.

[33] Colliander, A., Reichle, R. H., Crow, W. T., Cosh, M. H., Chen, F., Chan, S., ... and Yueh, S. H. (2021). Validation of soil moisture data products from the NASA SMAP mission. IEEE Journal of selected topics in applied earth observations and remote sensing,15, 364-392.

[34] Zeng, Y., Verhoef, A., Vereecken, H., Ben-Dor, E., Veldkamp, T., Shaw, L., ... and Su, Z. B. (2024). Tracking soil health: monitoring and modeling the soil-plant system. Authorea Preprints.

[35] Feldman, A. F., Reed, S., Amaral, C., Babst-Kostecka, A., Babst, F., Biederman, J., ... and Zhang, W. (2024). Adaptation and response in drylands (ARID): Community insights for scoping a NASA terrestrial ecology field campaign in drylands. Earths Future, 12(9), e2024EF004811.

[36] Zhao, T., Wang, S., Ouyang, C., Chen, M., Liu, C., Zhang, J., ... and Wang, L. (2024). Artificial intelligence for geoscience: Progress, challenges, and perspectives. The Innovation, 5(5).

[37] Blaber, E., Boothby, T., Carr, C. E., Everroad, R. C., Foster, J., Galazka, J., ... and Harrison, L. (2023). Space biology beyond LEO instrumentation and science series science working group 2022 annual report.

[38] Everroad, R. C., Foster, J., Galazka, J. M., Jansson, J., Lee, J. A., Lera, M. P., ... and Harrison, L. (2021). Space Biology Beyond LEO Instrumentation and Science Series-Science Working Group 2021 Annual Report.

[39] Melser, R., Coops, N. C., Wulder, M. A., and Derksen, C. (2023). Multi-source remote sensing based modeling of vegetation productivity in the boreal: issues and opportunities. *Canadian Journal of Remote Sensing*, *49*(1), 2256895.

[40] Groenke, B. R. (2025). *Quantifying and constraining uncertainty in modeling permafrost under a warming climate* (Doctoral dissertation, Alfred Wegener Institute).

[41] Arnscheidt, C. W., and Rothman, D. H. (2021). Asymmetry of extreme Cenozoic climate–carbon cycle events. Science Advances, 7(33), eabg6864.

[42] Franzke, C. L., Blender, R., O'Kane, T. J., and Lembo, V. (2022). Stochastic methods and complexity science in climate research and modeling. Frontiers in Physics, 10, 931596.

[43] Lin, S. W., Mohan, S., and Tsai, C. W. (2024). Modified stochastic diffusion particle-tracking model (MSDPTM) incorporating energy cascade theory and eddy intermittency for suspended sediment transport in open channel flow. Chaos: An Interdisciplinary Journal of Nonlinear Science,34(10).

[44] Adlakha, V. (2021). Decomposition of Anomalous Diffusion in various Stochastic Processes.

[45] Berghouse, M. (2024). *Physical and Deep-Learning-Based Explorations of Microbe-Mediated Reactive Transport Processes in Porous Media Across Scales*. University of Nevada, Reno.

[46] Paul, S. G., Saha, A., Arefin, M. S., Bhuiyan, T., Biswas, A. A., Reza, A. W., ... and Moni, M. A. (2023). A comprehensive review of green computing: Past, present, and future research. IEEE Access, 11, 87445-87494.

[47] Rosea, D. C., Maira, J. F., Dasc, A., Limmerc, D. T., & Garrahana, J. P. dKavli Energy NanoScience Institute, Berkeley, California eMaterials Science Division, Lawrence Berkeley National Laboratory, Berkeley, California f Chemical Science Division, Lawrence Berkeley National Laboratory, Berkeley, California. In *The 46th International Conference of the Middle European Cooperation in Statistical Physics* (p. 30)

[48] Okafor, C. M., and Bello, A. F. (2025). Climate change and agricultural productivity in northern nigeria: challenges and impacts. Journal of Climate Science and Meteorology, 13(2), 43-59.

[49] Obe, M. M., Kpadé, C. P., and Singbo, A. (2025). Identifying and overcoming barriers to climate change adaptation innovations among smallholder farmers in developing countries: a literature review and meta-analysis. Climatic Change, 178(3), 1-26.

[50] Folorunso, O., Ojo, O., Busari, M., Adebayo, M., Joshua, A., Folorunso, D., ... and Olabanjo, O. (2023). Exploring machine learning models for soil nutrient properties prediction: A systematic review. Big Data and Cognitive Computing, 7(2), 113.

[51] Jiya, E. A., Illiyasu, U., and Akinyemi, M. (2023). Rice yield forecasting: A comparative analysis of multiple machine learning algorithms. Journal of information systems and informatics, 5(2), 785-799.

[52] Ume, C. O., Onah, T. L., Ezeibe, A. B., Onah, O. G., Opata, P. I., Idika, E. O., and Felix, K. U. (2023). Agroecology as a Response to Sustainable Development Under Climate Change in Southeast Nigeria. In Climate Change Impacts on Nigeria: Environment and Sustainable Development, pp. 279-302). Cham: Springer International Publishing.

[53] Ume, C., Onah, T., Ume, S., Onah, O., Opata, P., Idika, E. O., and Felix, K. (2022). Modeling Crop Production under Climate Change in Southeast Nigeria: Agroecology as a Response.