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Computational analysis of flow of an incompressible magnetohydrodynamic third grade fluid in an inclined cylindrical pipe with temperature-dependent viscosity

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Abstract

This present paper considers the MHD flow of third grade fluid in an inclined isothermal cylindrical pipe with temperature-dependent viscosity. The resulting governing momentum and energy equations of motion are highly nonlinear and are solved using perturbation technique. Vogel model viscosity is introduced to represent temperature-dependent viscosity. Results from the analysis show that increase in the third grade and the gravitational parameters increases the flow velocity while increase in the magnetic field reduces the flow velocity. It is observed that increase in third grade parameter increases the temperature at the wall of the cylindrical pipe. It is further observed that within the Vogel model viscosity, increase in the indices decreases the flow velocity. Results further show that increase in Vogel index increases the temperature at the walls of the cylinder.

Keywords and phrases: magnetohydrodynamic; temperature-dependent; third grade; inclined pipe

1 Introduction

Magnetohydrodynamic (MHD) flow of third grade fluid through pipes plays a significant role in different areas of engineering science and technology, such as the petroleum industry, biomechanics, and heat exchangers. Due to the non-linearity of the governing equations, exact solutions are difficult to obtain and are few in number under certain conditions. As a consequence of different physical structures of non-Newtonian fluids, there is not a single constitutive model which can predict all its salient features. In addition, considerable progress has been made in the study of heat and mass transfer in magnetohydrodynamic flow of non-Newtonian fluids due to its application in many devices, such as the MHD power generator and Hall accelerator.

Because of its importance in many fields, many researchers have examined it in different form amongst whom are [1]. The effects of magnetic field on MHD flow of third grade fluid with Ohmic heating were examined by [2]. The influence of variable viscosity and viscous dissipation on the flow of a third grade fluid in a pipe were carried out by [3]. [4] dealt with stability of a reactive third grade fluid in a cylinder.

[5] investigated entropy generation due to non-Newtonian fluid flow. They developed the entropy generation number due to heat transfer and fluid friction. They discovered that increase in the Brinkman number increases the fluid friction and heat transfer rate thereby increases the entropy number.

[6] studied reactive MHD flow of third grade fluid. They employed the weighted residual collocation method for their computation and analyzed the various theories thermo physical parameters and observed that the critical values of the Frank Kamenutskii parameter and observed that at a certain critical values of the parameter the solution seizes to be unique. [7] carried out a mathematical analysis of entropy generation in couple stress fluid flow through porous channel with fluid slippage.

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[8] carried out a computational study of reactive flow of an electrically conducting fluid with temperature dependent viscosity and axial magnetic field using the semi-implicit finite difference scheme.

[9],employed perturbation technique in analyzing the annular flow of an incompressible MHD third grade fluid in a rotating concentric cylinders with isothermal walls and Joule heating. [10], analyzed the flow of third grade fluid in an inclined cylindrical pipe with isothermal walls and Joule heating. [11], applied perturbation technique to analyze magnetohydrodynamic non-Newtonian fluid in an inclined cylindrical pipe. [12], applied regular perturbation technique to analyze Non-Newtonian Fluid Flow in an Incompressible Isothermal Cylindrical Piple and Temperature-Dependent Viscosity. [13], developed a mathematical model for the flow of water through a channel impregnated with a polymer gel that is treated as an elastic and deformable porous medium. [14] considered the case of volume flux and average velocity and applied Homotopy perturbation with slip condition for their analysis. In this research, the governing equations of momentum and energy balance was solved using the traditional regular perturbation method. Temperature and velocity distributions were analytically solved and the effects of magnetic field and other thermo-physical parameters were analyzed and results presented.

2 Mathematical formulation

Considering a steady flow of an incompressible MHD third grade fluid through a cylindrical pipe with isothermal wall. Axial pressure gradient was assumed to have induced the fluid motion and for magnetohydrodynamically developed flow, both the velocity and the temperature fields depend on r only and the governing equations for the momentum and energy balance can be represented as in [14]

$$\mu\frac{d^2u}{dr^2} + 6(\beta_2+\beta_3)\left(\frac{du}{dr}\right)^2\left(\frac{d^2u}{dr^2}\right) + \rho g\beta\sin\phi - \sigma B_0^2 u = -\frac{dp}{dz} \tag{1}$$

$$k\frac{d^2T}{dr^2} + \mu\left(\frac{du}{dr}\right)^2 + 2\beta\left(\frac{du}{dr}\right)^4 + \sigma B_0 u^2 = 0 \tag{2}$$

$$\frac{du}{dr}(0) = \frac{dT}{dr}(0) = 0, \quad u(a) = 0, \quad T(a) = T_0 \tag{3}$$

where u is the fluid velocity, T is the temperature of the cylinder, μ is the coefficient of dynamic viscosity of the fluid, T_a is ambient temperature, R is the radius of the pipe, r is the radial distance, ρ is the density of the fluid, B_0 is the magnetic field, P is the pressure, β_3 is the material coefficient relating to third grade parameter.

In order to solve equations (1-3), we introduce the following dimensionless parameters for non-dimensionalization:

$$\bar{u} = \frac{u}{u_0}, \quad \bar{r} = \frac{r}{\alpha}, \quad \theta = \frac{T}{T_0}, \quad \beta = \frac{(\beta_2+\beta_3)}{\alpha\mu}, \quad \sigma = \frac{\beta u^2}{\alpha^2\delta} \tag{4}$$

(5)

$$\frac{d^2u}{dr^2} + 6\beta\left(\frac{du}{dr}\right)^2\left(\frac{d^2u}{dr^2}\right) + \eta - ru = -1 \tag{6}$$

(7)

$$\frac{d^2\theta}{dr^2} + B_r\left(\frac{du}{dr}\right)^2 + 2\beta\left(\frac{du}{dr}\right)^4 + ru^2 = 0 \tag{8}$$

(9)

$$u(0)=0, \quad u(1)=0, \quad \theta(0)=0, \quad \theta(1)=0 \tag{10}$$

(11)

3 Method of Solution

In order to solving equations (5) and (6), with the condition equation (7), we introduce the approximate analytical solutions for the velocity and temperature profiles can be of the form

$$\begin{aligned} u(r) &= u_0(r)+\beta u_1(r)+O(\beta^2), \theta(r)=\theta_0(r)+\beta\theta_1(r)+O(\beta^3), \\ \eta &= \beta G, \tau=\beta M \end{aligned} \tag{12}$$

substituting equation (8) in equations (6) and (7), gives

$$\beta^0: \quad \frac{d^2u_0}{dr^2}=-1 \tag{13}$$

$$\beta: \quad \frac{d^2u_1}{dr^2}+6\left(\frac{du_0}{dr}\right)^2\frac{d^2u_0}{dr^2}+G-Mu_0=0 \tag{14}$$

$$\beta^0: \quad \frac{d^2\theta_0}{dr^2}+B_r\left(\frac{du_0}{dr}\right)^2=0 \tag{15}$$

$$\beta: \quad \frac{d^2\theta_1}{dr^2}+2B_r\left(\frac{du_0}{dr}\frac{du_1}{dr}\right)+2\left(\frac{du_0}{dr}\right)^4+Mu_0=0 \tag{16}$$

Solving equations (9-12) with the condition (7), yields

$$u(r)=\frac{1}{2}r-\frac{1}{2}r^2+\beta\left(\frac{3}{4}r^2-r^3+\frac{1}{4}r^4-\frac{1}{2}r^2G+M\left(\frac{1}{12}r^3-\frac{1}{24}r^4\right)-\frac{1}{24}Mr+\frac{1}{2}rG\right) \tag{17}$$

$$\begin{aligned} \theta(r)=-\Bigg(&\left(\frac{1}{6}r^3+\frac{3}{8}r^2-\frac{5}{24}\right)B_r+\beta\Bigg(-B_r\frac{1}{2}r^3-\frac{1}{2}r^4+\frac{1}{10}r^5-\frac{1}{3}r^3G+M\left(\frac{1}{24}r^4-\frac{1}{60}r^5\right) \\ &-\frac{1}{24}r^2M+\frac{1}{2}r^2G-\frac{1}{4}r^4+\frac{3}{10}r^5-\frac{1}{15}r^6+\frac{1}{6}r^4G-M\left(\frac{1}{40}r^5+\frac{1}{90}r^6\right)+\frac{1}{72}r^3M \\ &-\frac{1}{6}r^3G-\left(\frac{1}{32}r^2-\frac{7}{48}r^3+\frac{13}{144}r^4-\frac{7}{120}r^5+\frac{1}{30}r^6\right)-Q\left(\frac{1}{12}r^3-\frac{1}{24}r^4\right) \\ &+B_r\left(\frac{1}{12}r-\frac{1}{180}Mr+\frac{1}{6}rG\right)-\frac{1}{24}rG-\frac{97}{1440}r\Bigg)\Bigg) \end{aligned} \tag{18}$$

3.1 Temperature-dependent viscosity

3.1.1 Vogel model viscosity

In this research, the Vogel model viscosity is used to represent temperature-dependent viscosity and we apply the Massoudi and Christie [5] approach. The dimensionless equations for the temperature-dependent viscosity are

$$\frac{d\mu}{dr}\frac{du}{dr}+\mu\frac{d^2u}{dr^2}+6\beta\left(\frac{du}{dr}\right)^2\left(\frac{d^2u}{dr^2}\right)+\eta-\tau u=-1 \tag{19}$$

$$\frac{d^2\theta}{dr^2} + \mu B_r \left(\frac{du}{dr}\right)^2 + 2\beta B_r \left(\frac{du}{dr}\right)^4 + \tau u^2 = 0 \tag{20}$$

Considering Massoudi and Christie (1995),

$$\mu = \mu \exp\left(-\frac{Q}{A+\theta} - \theta_w\right) \tag{21}$$

Using Taylor series expansion of equation (17) becomes

$$\mu = \alpha \left(1 - \frac{Q\theta}{A^2}\right) \tag{22}$$

where

$$\alpha = \mu \left(\exp\left(\frac{Q}{A^2} - \theta_w\right)\right) \tag{23}$$

Q and A being parameters relating to Vogel model. The viscosity μ is assumed to be a function of temperature and equations (15) and (16) depends on viscosity. Taking the derivative of equation (19) with respect to r, gives

$$\frac{d\mu}{dr} = \alpha \frac{Q}{A^2}\frac{d\theta}{dr} \tag{24}$$

$$Q = \beta Q \tag{25}$$

Substituting equations (8), (18), (20) and (21) into equation (15) and (16), yields

$$\beta^0 : \alpha \frac{d^2 u_0}{dr^2} = -1 \tag{26}$$

$$\beta : \alpha \frac{d^2 u_i}{dr^2} + \alpha \frac{Q}{A^2}\frac{d\theta_0}{dr} - \alpha \frac{Q\theta_0}{A^2}\frac{d^2 u_0}{dr^2} + 6\left(\frac{du_0}{dr}\right)^2 \frac{d^2 u_0}{dr^2} + G - M u_0 = 0 \tag{27}$$

$$\beta^0 : \frac{d^2\theta_0}{dr^2} + \alpha B \left(\frac{du_0}{dr}\right)^2 = 0 \tag{28}$$

$$\beta : \frac{d^2\theta_1}{dr^2} + 2\alpha B_r \frac{du_0}{dr}\frac{du_1}{dr} - \alpha \frac{Q\theta_0}{A^2}\left(\frac{du_0}{dr}\right)^2 + 2B_r \left(\frac{du_0}{dr}\right)^4 + M u_0^2 = 0 \tag{29}$$

Solving equations (22-25) with the condition (7), yields

$$\begin{aligned} u(r) =& \frac{1}{2\alpha}r - \frac{1}{2\alpha}r^2 + \beta\Bigg(-\frac{Q}{A^2}\Bigg(-\left(\frac{1}{24\alpha}r^3 - \frac{1}{24\alpha}r^4 + \frac{1}{60\alpha}r^5\right)B_r + \frac{1}{48\alpha}r^2 B_r\Bigg) \\ &+ \frac{Q}{A^2}\Bigg(\left(\frac{1}{96\alpha^2}r^4 - \frac{1}{120\alpha^2}r^5 + \frac{1}{360\alpha^2}r^6\right)B_r + \frac{1}{144\alpha^2}r^3 B_r\Bigg) - \frac{3}{4\alpha^4}r^2 - \frac{1}{\alpha^4}r^3 + \frac{1}{2\alpha^4}r^4 \\ &- \frac{1}{2\alpha}r^2 G + M\left(\frac{1}{12\alpha^2}r^3 - \frac{1}{24\alpha^2}r^4\right) - r\Bigg(-\frac{Q}{A^2}\left(\frac{1}{240\alpha}B_r\right) + \frac{Q}{A^2}\left(\frac{17}{1440\alpha^2}B_r\right) \\ &+ \frac{1}{4\alpha^4} - \frac{1}{2\alpha}G - \frac{1}{24}M\Bigg)\Bigg) \end{aligned} \tag{30}$$

and

$$
\begin{aligned}
\theta(r) = & -\left(\frac{1}{8\alpha}r^2 - \frac{1}{6\alpha}r^3 + \frac{1}{12\alpha}r^4\right)B_r + \frac{1}{24\alpha}r^2 B_r \\
& + \beta\Bigg(-B_r\Bigg(-\frac{Q}{A^2}\left(-\left(\frac{1}{96\alpha}r^4 - \frac{1}{120\alpha}r^5 + \frac{1}{360\alpha}r^6\right)B_r + \frac{1}{288\alpha}r^2 B_r\right) \\
& + \frac{Q}{A^2}\left(\left(\frac{1}{480\alpha^2}r^5 - \frac{1}{720\alpha}r^6 + \frac{1}{2520\alpha^2}r^7\right)B_r + \frac{1}{288\alpha^2}r^7 B_r\right) \\
& - \frac{1}{4\alpha^2}r^3 + \frac{1}{4\alpha^4}r^4 + \frac{1}{10\alpha^2}r^3 - \frac{1}{6\alpha^2}r^2 G \\
& + M\left(\frac{1}{48\alpha^2}r^4 - \frac{1}{120\alpha^2}r^5\right) + \left(\frac{Q}{A^2}\left(\frac{1}{480\alpha^2}r^5 - \frac{1}{280\alpha^2}r^7\right)B_r\right) \\
& \left(-\frac{1}{8\alpha^2}r^3 + \frac{1}{4\alpha^2}r^2 G + \frac{1}{48\alpha^2}r^2 M\right)\Bigg)\Bigg) + \frac{Q}{A^2}\left(-\left(\frac{1}{80\alpha^2}r^5 - \frac{1}{90\alpha^2}r^6 + \frac{1}{252\alpha^2}r^7\right)B_r + \frac{1}{144\alpha^2}r^4 B_r\right) \\
& - \frac{Q}{A^2}\left(\left(\frac{1}{360\alpha^2}r^5 - \frac{1}{504\alpha^2}r^6 + \frac{1}{1680\alpha^2}r^7\right)B_r - \frac{1}{288\alpha^2}r^3 B_r\right) \\
& + \frac{1}{4\alpha^2}r^4 + \frac{3}{10\alpha^2}r^5 + \frac{2}{15\alpha^2}r^6 + \frac{1}{6\alpha}r^3 G \\
& - M\left(\frac{1}{40\alpha^2}r^3 - \frac{1}{90\alpha^2}r^6\right) + \left(\frac{Q}{A^2}\left(\frac{1}{720\alpha}r^7 B_r\right) + \frac{Q}{A^2}\left(\frac{17}{4320\alpha^2}r^7 B_r\right)\right. \\
& + \frac{1}{12\alpha^2}r^3 - \frac{1}{6\alpha}r^4 G - \frac{1}{72}r^3 M + \frac{Q}{A^2}\Bigg(-\left(\frac{1}{384\alpha^2}r^4 - \frac{1}{480\alpha^2}r^5 + \frac{1}{1440\alpha^2}r^6\right)B_r \\
& + \frac{1}{576\alpha^2}r^7 B_r\Bigg) + \frac{Q}{A^2}\left(-\left(\frac{1}{160\alpha^2}r^5 - \frac{1}{180\alpha^2}r^6 + \frac{1}{504\alpha^2}r^7\right)\right)B_r \\
& + \frac{1}{288\alpha^2}r^3 B_r - \frac{Q}{A^2}\Bigg(-\left(\frac{1}{240\alpha^2}r^5 - \frac{1}{252\alpha^2}r^6 + \frac{1}{672\alpha^2}r^7\right)B_r \\
& + \frac{1}{480\alpha^2}r^3 B_r - B_r\left(\frac{1}{16\alpha^2}r^3 - \frac{1}{6\alpha^2}r^4 + \frac{1}{4\alpha^2}r^4 - \frac{1}{5\alpha^2}r^5 + \frac{1}{15\alpha^2}r^6\right) \\
& - M\left(\frac{1}{48\alpha^2}r^4 - \frac{1}{40\alpha^2}r^5 + \frac{1}{120\alpha^2}r^6\right) \\
& + B_r\left(\frac{Q}{A^2}\left(\frac{37}{10080\alpha^2} - \frac{137}{60480\alpha^2}\right)B_r + G\left(\frac{1}{4\alpha^2} - \frac{1}{6\alpha^2}\right)\right) \\
& + \frac{1}{24\alpha^2}r^4 - \frac{19}{720\alpha^2}M - \frac{1}{72}M\Bigg) \\
& + \frac{Q}{A^2}\left(\frac{1}{2520\alpha^2} - \frac{1}{1920\alpha^2}\right)B_r - \frac{51}{240\alpha^2}B_r \\
& - \frac{1}{240\alpha^2}M\left(\frac{1}{24\alpha^2} - \frac{1}{720\alpha^2}\right)
\end{aligned}
\tag{31}
$$

4 Results and discussions

In figure 1, the third grade parameter is varied at a steady rate while the magnetic field and gravitational parameters are held constant. It is observed that increase in third grade parameter β increases the flow

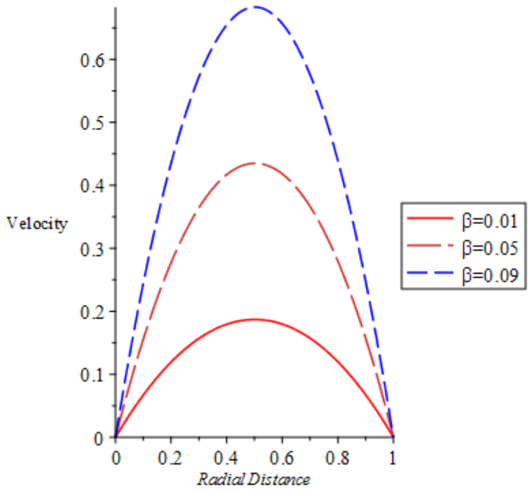

Figure 1: Velocity Profile For Variation of Non-Newtonian Parameter(β) When $G = 50$, $M = 10$

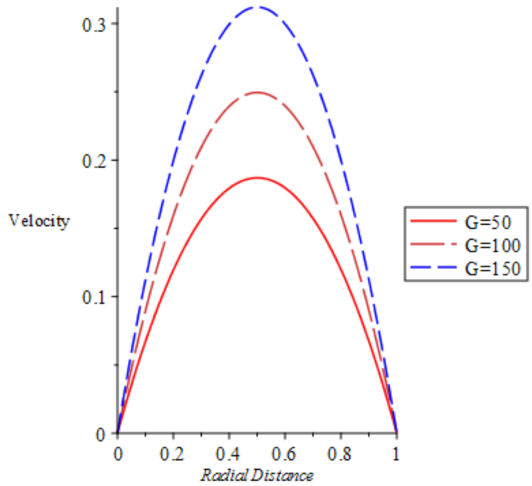

Figure 2: Velocity Profile For Variation of Gravitational Parameter(G) When $\beta = 0.01$, $M = 10$

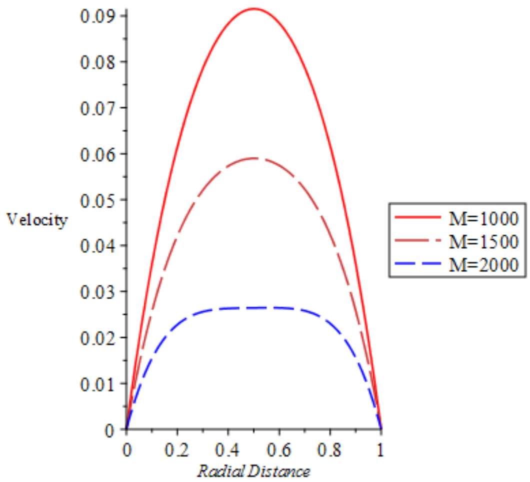

Figure 3: Velocity Profile For Variation of Magnetic Field Parameter(M) When $\beta = 0.01$, $G = 50$

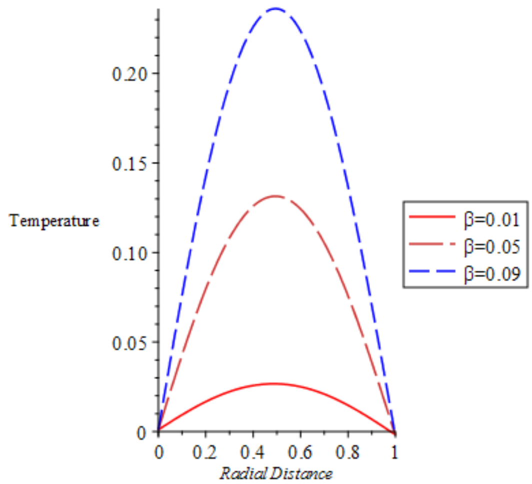

Figure 4: Temperature Profile For Variation of Non-Newtonian Parameter(β) When $B_r = 0.5$, $G = 200$, $M = 15$, $Q = 206$

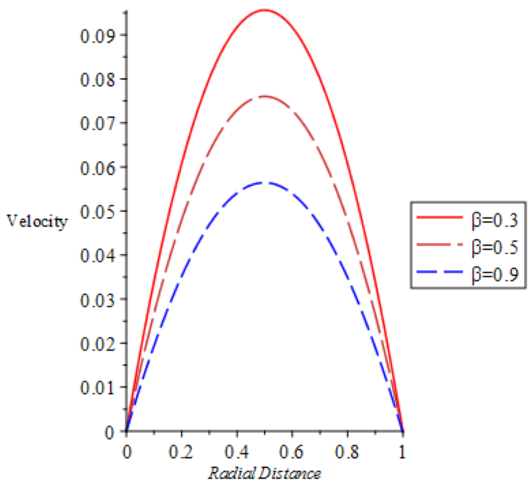

Figure 5: Velocity Profile For Variation of Non-Newtonian index (β) When $B_r = 0.5$, $\alpha = 1$, $G = 5$, $A = 5$, $Q = 206$

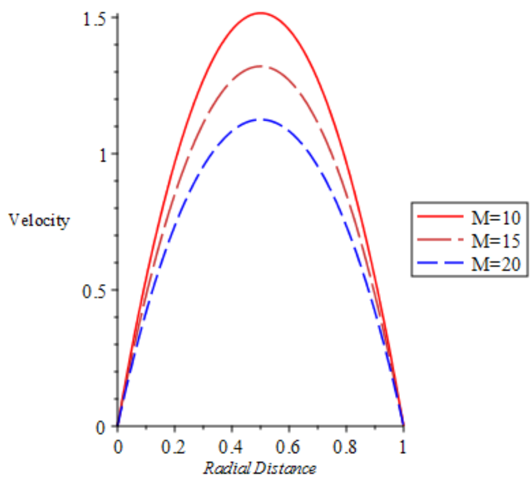

Figure 6: Velocity Profile For Variation of Vogel index (M) When $B_r = 0.5$, $\beta = 0.3$, $\alpha = 1$, $G = 5$, $A = 5$, $Q = 206$

velocity. It is observed in figure 2 that increase in the gravitational parameter increases the flow velocity when the third grade β and the magnetic field parameters M are kept constant at $\beta = 0.1$, $M = 1$ respectively.

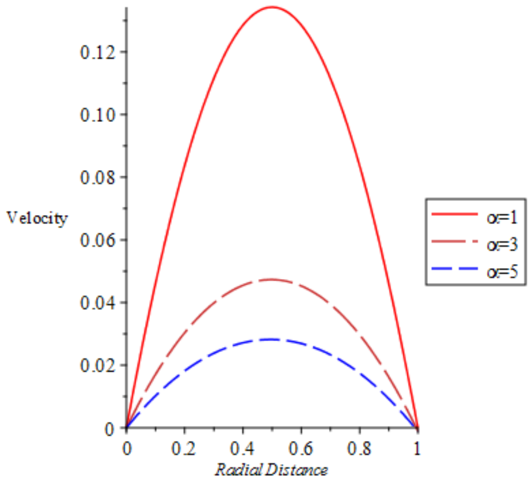

Figure 7: Velocity Profile For Variation of Vogel index (α)
When $B_r = 0.5$, $\beta = 0.3$, $\alpha = 1$, $G = 5$, $A = 5$, $Q = 206$

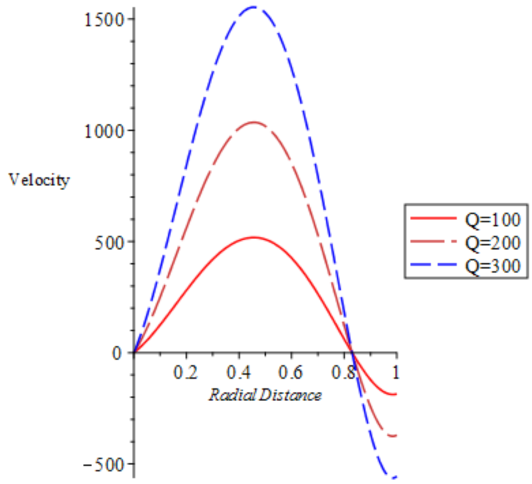

Figure 8: Velocity Profile For Variation of Vogel index (Q)
When $B_r = 0.5$, $\beta = 0.3$, $\alpha = 1.5$, $G = 0.5$, $A = 0.001$.

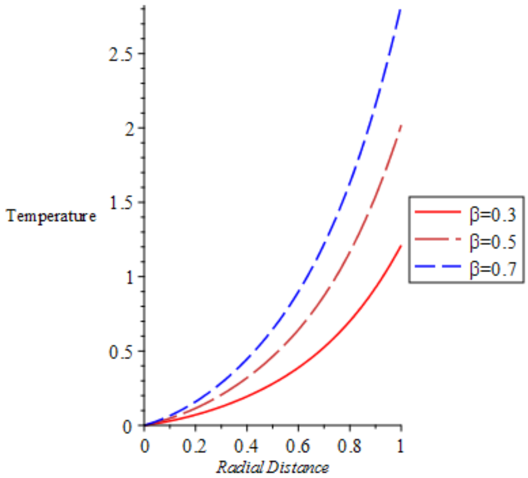

Figure 9: Temperature Profiles For Variation of Vogel index
(A) When $B_r = 0.5$, $\beta = 0.3$, $\alpha = 1.5$, $G = 0.5$, $Q = 10$.

Figure 3 is the velocity profiles for variation in magnetic field parameter M. Results show that increase in the magnetic field parameter M, reduces the flow velocity as a result of the MHD introducing additional drag force which converts kinetic energy into magnetic field. Figure 4 is the temperature profiles for various values of the third grade parameter β. It is observed that increase in the parameter β increases the temperature of the cylindrical walls. This is because the third grade parameter introduces nonlinearity in the temperature profiles resulting in a higher temperature gradient. Figure 5 shows the velocity profiles for different values of the Vogel model index γ reduces the flow velocity near the walls of the cylindrical pipe.

Figure 6 is the velocity profiles for various values of the Vogel model index γ when $\beta = 0.1$. Results show that a little increase in γ reduces the velocity of the fluid flow. Figure 7 shows the velocity profiles for different values of A. It is seen that increase in A drastically reduces the flow velocity. In figure 8, the Vogel model index Q is varied and other parameters are kept constant. Results indicate that increase in Q increases the velocity profiles with a point of inflexion at $\eta = 0.5$ along the radial distance before convergence.

In figure 9, the Vogel index β is varied to determine its effect on the temperature of the cylindrical pipe. It is noticed that a steady increase in β, increases the temperature of the cylinder at the walls.

5 Conclusion

The flow of an incompressible magnetohydrodynamic third grade fluid in inclined isothermal cylindrical pipe with temperature-dependent viscosity is considered. The resulting governing momentum and energy equations of motion are highly nonlinear and are solved using perturbation technique. Vogel model viscosity is introduced to represent temperature-dependent viscosity. Results from the analysis show that increase in the third grade and the gravitational parameters increases the flow velocity while increase in the magnetic field reduces the velocity of the fluid flow. It is observed that increase in third grade parameter decreases the temperature of the cylindrical pipe at the walls. It is further observed that within the Vogel model viscosity, increase in the indices η decreases the velocity of the fluid flow. Results further show that increase in Vogel index β increases the temperature at the walls of the cylinder.

Declarations

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2. Informed Consent Statement: Not applicable
3. Data Availability: Not applicable
4. Conflict of Interest Statement: No conflict of interest

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