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Sensitivity analysis of Smartphone Virus infection transmission dynamics

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Abstract

We proposed in this paper, a mathematical model for the sensitivity analysis of the Smartphone virus infection transmission dynamics using SEICQR. First, the model was well-posed as we proved that all its state variables are nonnegative for all time and then we determine a unique Smartphone virus infection free equilibrium state and indicate possibility of control of Smartphone virus infection. The basic Smartphone virus infection reproduction number, R_0, as a threshold parameter, in terms of the given model parameter was computed using the next generation method. Also, the sensitivity analysis of R_0 was determined to see parameters that positively contribute to the dynamics of Smartphone virus infection. The numerical simulation of the model was done using Runge-Kutta Fourth order (RK4) method in MATLAB to evaluate the effect of these parameters of the basic Smartphone reproduction number on the dynamics of the infection. The numerical results confirm the theoretical results of the sensitivity indices. Finally, these findings suggest that Smartphone virus infection will only be eradicated from the population with proper control strategies.

Keywords: smartphone virus; sensitivity analysis; basic reproduction number; infection transmission dynamics.

1 Introduction

The Smartphone is small enough to fit in a pocket and it enable users to have a device with a good processing power, internet access and functions of traditional mobile phones in one device. However, the wealth of private information which is stored on those devices made them an

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interesting target for malicious users and cyber criminals [1], who then invade these Smartphone with virus. Smartphone virus is a mischief program written to proliferate from one phone to another and it takes control of Smartphone devices by exploiting its vulnerability. Continuous improvement in Smartphone device hardware and Smartphone communication technologies has led to a highly interconnected world, but also a world grown highly vulnerable. Cyber criminals have proven significantly efficient in uncovering new vulnerabilities in popular applications (apps) [2]. As a result, more and more Smartphone malware families are introduced every year [3]. Just in 2017, more than 20 million malware samples have been detected [4], while other studies show that the chances for monetization is a key factor responsible for the rise of Smartphone malware [5, 6].

The cyber attacker who was able to install malware on some system can install or delete programs, modify files, download sensitive information and use them to impersonate the user's action and keystrokes, capture users screen, use camera and retrieve photos or videos, use the infected system as a source of Distributed denial of service (DDoS)) attack [1, 7]. It's a method where cyber criminals flood a network with so much malicious traffic that it cannot operate or communicate as it normally would. This causes the site's normal traffic, also known as legitimate packets, to come to a halt. Smartphone can become infected either through downloading infected files using the phone internet browser, transferring files between phones using Bluetooth interface, synchronizing with an infected computer, assessing an infected physical memory card, opening infected files attached to multimedia message service (MMS) or email [8, 9]. Zhang [10] discovered that Smartphone virus can also propagate through multi-layered network. He concluded that Smartphone viruses can use any network (3G, 4G, Wi-Fi or Bluetooth).

When an unsuspected user accepts the infected attachment file, using a phone susceptible to the virus, then the virus is installed and infects the target phone which then begins to perform as an attacked phone [9]. Smartphone virus can seize the victims mobile device by running malicious exploit and this infected device will in turn, scan and infect other Smartphones in the mobile network [11].

Some response mechanisms completely stop further virus propagation, but others simply slow the propagation rate of the virus [12]. Both types of response can be used. Ideally, the response mechanism would always quickly and completely stop Smartphone virus propagation. However, some viruses spread so quickly that a first response mechanism that slows the spread could buy time to enable activation of a second response mechanism that completely halts the propagation process. Actually, in phone user education response, the consent of the phone user to accept the message and infect the phone is required, therefore, a user not accepting an infected message has a direct effect on virus propagation. Also, other response mechanism such as immunization can prevent infection even if the user accepts the MMS message attachment [12]. A virus scans of all MMS attachments as they pass through an MMS gateway is completely effective against viruses with known virus signature. For that reason, after the new virus signature is added to the list of known viruses, the gateway virus scan is able to completely halt virus propagation [13].

The immunization response mechanism operates using software placed directly on each Smartphone [14]. After a Smartphone service provider detects virus that exploits a vulnerable Smartphone, the service provider begins developing a patch to fix that vulnerability [15]. Once the patch is developed, the immunization software resident in each Smartphone automatically installs any immunization patch available [10]. Bandwidth constraint prevents most Smartphone from receiving the patch simultaneously, as the patch is rolled out to the entire Smartphone population uniformly over a period of time [16]. The more servers that are dedicated to distributing these patches, the faster the deployment to susceptible Smartphones in the network [17]. When the patch arrives at a particular Smartphone, it becomes immunized from virus if not infected, or the patch stops further propagation attempts from Smartphone if it is already infected [18].

Because Smartphone viruses have potential to attack in many different ways, an optimal response strategy must incorporate mechanisms to counteract a wide variety of virus behaviors. The hope of stopping Smartphone virus before it degrades the utility and value of a Smartphone is quick and concerted action on the part of all universal mobile telecommunications service (UMTS) data networks that their mobile devices use; Wi-Fi networks have no such protection. And while some carriers already filter their MMS streams to remove messages bearing malicious attachments, all should do so [6].

Some of the manufacturers have joined the trusted computing group, which has been hammering on industry standards micro circuiting inside phones that will make it harder for malware to get a sensitive data in the devices memory or to hijack its payment mechanism. Symbian released a version of its operating system that does an improved job of protecting key files which requires software to obtain digital certificate from the company. This Symbian system refuses to install programs not accompanied by a certificate. Unless disabled by a user the system effectively excludes all mobile malware discovered to date [6].

2 Related Literature

In Smartphone network environment, virus attack is a serious threat and is still multiplying as time continues to pass and it is very necessary to control these virus attacks in Smartphone. A number of mathematical models have been formulated to simulate, analyze and understand Smartphone virus propagation and its control strategies. In related works, Guillen et al [19] used the theory of optimal control that is associated to systems of ordinary differential equations to find a new function to control malware epidemic through time, while Liu and Zhong [20] employed the optimal control theory to study malware immunization strategies, aiming to keep the total economic loss of security investment and infection loss as low as possible.

Ruitenbeek and Stevens [13] proposed response mechanisms for mobile phone viruses and used their model to obtain insight on the effectiveness of the virus mitigation technique. To study the fundamental spreading patterns that characterize a mobile virus outbreak, Wang et al [11] modeled the mobile benign worm based on two stages of repairing mechanisms. Chinebu et al [21] proposed an SVEIQRS model that control virus attack through vaccination, quarantine and treatment. Their model has a globally asymptotically stable virus free equilibrium when $R_0 < 1$ and a unique endemic equilibrium when $R_0 > 1$. Yang et al [22] proposed two different propagation models of mobile phone viruses under the complex network.

Our aim in this paper is to highlight the seriousness of Smartphone virus propagation, particularly exposed$\big(E(t)\big)$Smartphone that is infected without symptom which can transmit the infection to susceptible$\big(S(t)\big)$ Smartphone. Likewise, there can be transmission from infected Smartphone$\big(I(t)\big)$ to susceptible. In addition, we included complication with virus infected Smartphone $\big(C(t)\big)$. Furthermore, we have the quarantined Smartphone$\big(Q(t)\big)$ and finally the recovered Smartphone$\big(R(t)\big)$.

A continuous mathematical model we proposed in this paper and it describes the dynamics of a population infected by Smartphone virus. Moreover, we gave the model in section 3 and some basic properties of the model in section 4. Lastly, we presented the discussion and conclusion in section 5.

3 Mathematical Model Formulation

We consider a mathematical model SEICQR that describe the Smartphone virus infection transmission dynamics. The entire Smartphone population denoted by $N(t)$ was divided into six compartments.

3.1 Description of the Smartphone Model Compartments

The schematic diagram of the proposed Smartphone virus transmission dynamics is shown in Figure 1:

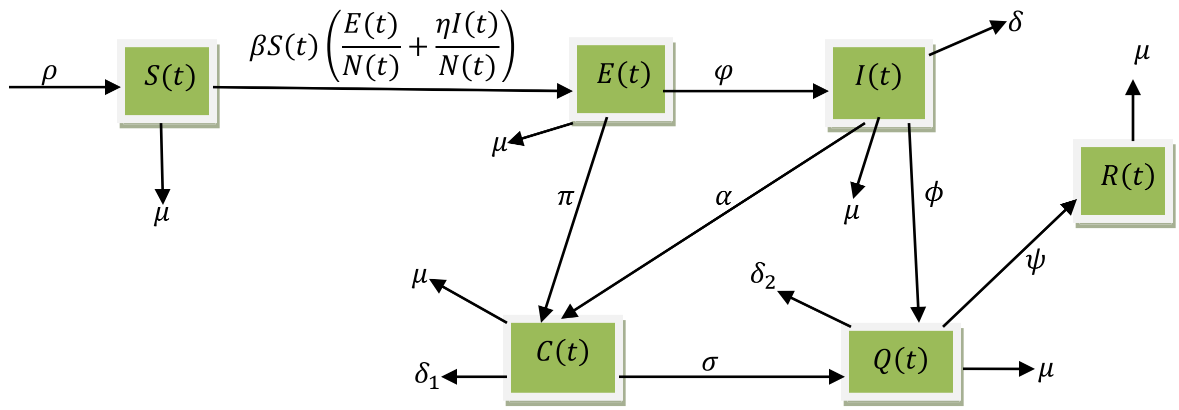

Figure 1: Schematic Diagram of the Model for Smartphone virus infection transmission dynamics.

Let us denote the force of infection by

$$\lambda = \beta\left(\frac{\eta E(t) + I(t)}{N}\right), \tag{1}$$

where, β is the Smartphone virus infection transmission rate and η is the modification factor for the exposed Smartphones.

Compartment $(S(t))$ represents the number of Smartphone infected with virus. This compartment is increased by ρ, which denotes the incidence of susceptible Smartphone. The compartment is decreased by the amount μ (natural Death, i.e., Smartphones that can never be repaired again), $\frac{\beta\eta S(t)E(t)}{N}$ (The number of Smartphone that are infected with the virus by contact with the exposed Smartphone) and $\frac{\beta S(t)I(t)}{N}$ (The number of Smartphone that were infected with the virus by communication with infected Smartphone). Thus, we have

$$\frac{dS(t)}{dt} = \rho - \mu S(t) - \beta\left(\frac{\eta E(t) + I(t)}{N}\right)S(t) \tag{2i}$$

Compartment $(E(t))$ represents the number of exposed Smartphones, that is, those that are infected without symptoms but can transmit the infection. The compartment $(E(t))$ is increased by the amount of $\frac{\beta\eta S(t)E(t)}{N}$ and $\frac{\beta S(t)I(t)}{N}$. It is decreased by natural death, denoted with μ, the number of Smartphones that become infectious, φ, and also by number of Smartphones that have upgraded to the development of rapid and dangerous virus, thereby, having complications due to antivirus deficiency and is denoted by π. This gives

$$\frac{dE(t)}{dt} = \beta\left(\frac{\eta E(t) + I(t)}{N}\right)S(t) - (\mu + \varphi + \pi)E(t) \qquad (2ii)$$

Compartment $(I(t))$ represents the number of infected Smartphones. The compartment $(I(t))$ is increased by the number of exposed Smartphones that becomes infectious and is denoted by φ. This compartment is decreased by α, that is, Smartphones that have severe complications such as Apps crashing over and over again. It is also decreased by, μ, natural death rate, δ, death rate due to virus infection and ϕ, progression rate from infected to quarantine. Therefore, we obtain

$$\frac{dI(t)}{dt} = \varphi E(t) - (\mu + \alpha + \phi + \delta)I(t) \qquad (2iii)$$

Compartment $(C(t))$ represents the number of infected Smartphones with complications. This compartment is increased by π and α, that is, progression rate from exposed and infected respectively to infected with complications. It is decreased by natural death, μ, death rate due to virus infection with complication δ_1 and progression rate from infected with complications to quarantine, σ. Then, we get

$$\frac{dC(t)}{dt} = \pi E(t) + \alpha I(t) - (\mu + \delta_1 + \sigma)C(t) \qquad (2iv)$$

Compartment $(Q(t))$ represents the number of Quarantined Smartphones with follow-up treatment. This compartment is increased by ϕ and σ, that is, progression rate from infected and infected with complications respectively to quarantined. It is decreased by the movement from quarantined to recovered Smartphones, ψ, natural death rate, μ and quarantined Smartphones that cannot be repaired again, δ_2. This, provides us with

$$\frac{dQ(t)}{dt} = \phi I(t) + \sigma C(t) - (\mu + \delta_2 + \psi)Q(t) \qquad (2v)$$

Compartment $(R(t))$ represents the number of recovered Smartphones. This is increased by recovery rate, ψ and is decreased by natural death rate, μ. Thus, we have

$$\frac{dR(t)}{dt} = \psi Q(t) - \mu R(t) \qquad (2vi)$$

Based on the above formulations from equations $(2i) to\ (2vi)$ and the schematic diagram, we thus present the Smartphone virus infection transmission dynamics model by the following systems of differential equation:

$$\left.\begin{aligned}
\frac{dS(t)}{dt} &= \rho - \mu S(t) - \lambda S(t) \\
\frac{dE(t)}{dt} &= \lambda S(t) - (\mu + \varphi + \pi)E(t) \\
\frac{dI(t)}{dt} &= \varphi E(t) - (\mu + \alpha + \phi + \delta)I(t) \\
\frac{dC(t)}{dt} &= \pi E(t) + \alpha I(t) - (\mu + \delta_1 + \sigma)C(t) \\
\frac{dQ(t)}{dt} &= \phi I(t) + \sigma C(t) - (\mu + \delta_2 + \psi)Q(t) \\
\frac{dR(t)}{dt} &= \psi Q(t) - \mu R(t)
\end{aligned}\right\} \quad (3)$$

where the initial state are $S(0) \geq 0,\ E(0) \geq 0,\ I(0) \geq 0,\ C(0) \geq 0,\ Q(0) \geq 0,\ R(0) \geq 0$.

4 Basic Properties of the Model

4.1 Positivity of Solutions

For Smartphone virus infection transmission dynamics model system (3) to be epidemiologically and mathematically meaningful, it is important to prove that all its state variables are positive for all time. Therefore, solution of model system (3) with non negative initial data remains non-negative for all time $t \geq 0$. Thus, the total Smartphone population can be defined as

$$N(t) = S(t) + E(t) + I(t) + C(t) + Q(t) + R(t) \quad (4)$$

Theorem 1: If $S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, C(0) \geq 0, Q(0) \geq 0\ and\ R(0) \geq 0$, the solutions $S(t), E(t), I(t), C(t), Q(t)\ and\ R(t)$ of system (3) are non negative for all $t \geq 0$.

Proof: It follows from equation (2i) that

$$\frac{dS(t)}{dt} = \rho - \mu S(t) - \beta\left(\frac{\eta E(t) + I(t)}{N(t)}\right)S(t)$$

$$\geq -\mu S(t) - \lambda S(t)$$

$$\frac{dS(t)}{dt} + (\mu + \lambda)S(t) \geq 0$$

Let $\mu + \lambda = F(t)$, then we have

$$\frac{dS(t)}{dt} + F(t)S(t) \geq 0$$

Multiplying both sides of this inequality by $\exp\left(\int_0^t F(s)ds\right)$ we get

$$\exp\left(\int_0^t F(s)ds\right).\frac{dS(t)}{dt} + F(t)\exp\left(\int_0^t F(s)ds\right)S(t) \geq 0$$

Then,

$$\frac{d}{dt}\left(S(t)\exp\left(\int_0^t F(s)ds\right)\right) \geq 0$$

Integrating this inequality from 0 to t we obtain:

$$\int_0^t \frac{d}{dt}\left(S(s)\exp\left(\int_0^t (\mu + \lambda)ds\right)\right)ds \geq 0$$

Therefore,

$$S(t) \geq S(0)\exp\left(\int_0^t (\mu + \lambda)ds\right)$$

This implies that $S(t) > 0\ \forall\, t > 0$. Similarly, it can be proved that $E(t) \geq 0, I(t) \geq 0, C(t) \geq 0, Q(t) \geq 0\ and\ R(t) \geq 0$ and this is done using the remaining sub equations of (3) and we have

$$E(t) \geq E(0)\exp\left(\int_0^t (\mu + \varphi + \pi)de\right)$$

$$I(t) \geq I(0)\exp\left(\int_0^t (\mu + \alpha + \phi + \delta)di\right)$$

$$C(t) \geq C(0)\exp\left(\int_0^t (\mu + \delta_1 + \sigma)dc\right)$$

$$Q(t) \geq Q(0)\exp\left(\int_0^t (\mu + \delta_2 + \psi)dq\right)$$

$$R(t) \geq R(0)\exp\left(\int_0^t (\mu)dr\right)$$

This completes the proof. It is important to note that system (3) will be analyzed in a feasible region

$$\Phi = \left\{(S,E,I,C,Q,R) \in \mathbb{R}_+^6 : S+E+I+C+Q+R \leq \frac{\rho}{\mu}\right\} \qquad (5)$$

which can easily be verified to be positively invariant with respect to model system (3). Therefore, model system (3) is epidemiologically and mathematically well posed in Φ [23].

4.2 The Basic Smartphone Virus Infection Reproduction Number, R_0

To compute the basic Smartphone virus infection reproduction number, we first calculate the Smartphone virus infection free equilibrium state of the model by setting the left hand side of system (3) to zero and obtain the Smartphone virus infection free equilibrium state as follows

$$\mathrm{H}_0^* = \big(S_0(t), E_0(t), I_0(t), C_0(t), Q_0(t), R_0(t)\big) = \left(\frac{\rho}{\mu}, 0, 0, 0, 0, 0\right)$$

The rate of appearance of new virus infection in the population is given by the new infection terms in the exposed compartment [24][25][26]. Then new infections in compartments$E(t)\ I(t)\ and\ C(t)$ is given by

$$F := \begin{pmatrix} \frac{\beta\eta\rho}{\mu} & \frac{\beta\rho}{\mu} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

While the remaining transmission terms in compartments$E(t)\ I(t)\ and\ C(t)$ is given by

$$V := \begin{pmatrix} A & 0 & 0 \\ -\varphi & B & 0 \\ -\pi & -\alpha & G \end{pmatrix}$$

$$where\ A = \mu + \varphi + \pi, B = \mu + \alpha + \phi + \delta,\ \ G = \mu + \delta_1 + \sigma$$

The inverse of the transmission terms in compartments$E(t)\ I(t)\ and\ C(t)$ and FV^{-1} are respectively given by

$$V^{-1} \rightarrow \begin{pmatrix} \frac{1}{A} & 0 & 0 \\ \frac{\varphi}{A \cdot B} & \frac{1}{B} & 0 \\ \frac{\pi \cdot B + \varphi \cdot \alpha}{A \cdot B \cdot G} & \frac{\alpha}{B \cdot G} & \frac{1}{G} \end{pmatrix} \quad \text{and} \quad F \cdot V^{-1} \rightarrow \begin{pmatrix} \frac{\beta\eta\rho}{A\cdot\mu} + \frac{\varphi\cdot\beta\rho}{A\cdot B\cdot\mu} & \frac{\beta\rho}{B\cdot\mu} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Therefore the basic Smartphone virus infection reproduction number R_0 is given by:

$$R_0 = \frac{\beta\eta\rho}{\mu A} + \frac{\varphi\beta\rho}{\mu AB} = \frac{\beta\eta\rho}{\mu(\mu+\varphi+\pi)} + \frac{\rho\varphi\beta}{\mu(\mu+\varphi+\pi)(\mu+\alpha+\phi+\delta)}$$

$$R_0 = \frac{\beta\rho}{\mu(\mu+\varphi+\pi)}\left(\eta + \frac{\varphi}{(\mu+\alpha+\phi+\delta)}\right) \tag{6}$$

4.3 Endemic Equilibrium Point

Here we discuss the point at which the virus infection cannot be totally eradicated from the population. We therefore let H_1^{**} be the Smartphone virus infection endemic equilibrium of the model. Thus, we have

$$H_1^{**} = \left(S_1(t), E_1(t), I_1(t), C_1(t), Q_1(t), R_1(t)\right)$$

$$\left.\begin{aligned}
S_1(t) &= \frac{\rho}{\mu+\lambda^{**}} \\
E_1(t) &= \frac{\lambda^{**}\rho}{A(\mu+\lambda^{**})} \\
I_1(t) &= \frac{\lambda^{**}\varphi\rho}{AB(\mu+\lambda^{**})} \\
C_1(t) &= \frac{\lambda^{**}\rho}{AG(\mu+\lambda^{**})}\left(\pi + \frac{\alpha\varphi}{B}\right) \\
Q_1(t) &= \frac{\lambda^{**}\rho}{AK(\mu+\lambda^{**})}\left(\frac{\phi\varphi}{B} + \frac{\sigma\pi}{G} + \frac{\sigma\alpha\varphi}{BG}\right) \\
R_1(t) &= \frac{\lambda^{**}\psi\rho}{\mu AK(\mu+\lambda^{**})}\left(\frac{\phi\varphi}{B} + \frac{\sigma\pi}{G} + \frac{\sigma\alpha\varphi}{BG}\right)
\end{aligned}\right\} \tag{7}$$

But the force of infection $\lambda^{**} = \beta\left(\frac{\eta E(t)+I(t)}{N}\right)$, substituting (7) into the force of infection gives

$$\lambda^{**}\left(a_2(\lambda^{**})^2 + a_1\lambda^{**} + a_0\right) = 0 \tag{8}$$

$$\left.\begin{gathered}
a_2 = \mu AB^2GK + \mu ABGK + \mu\pi AB^2GK + \mu\alpha\varphi ABK + \mu\varphi ABG + \mu\sigma\pi AB^2 + \mu\sigma\alpha\varphi AB \\
+\phi\varphi\psi ABGK + \sigma\pi\psi AB^2GK + \sigma\varphi\psi\alpha AB \\
a_1 = \mu^2AB^2GK + \mu^2\varphi ABGK + \mu^2\pi AB^2K + \mu^2\varphi\alpha AB + \mu^2\varphi ABG + \mu^2\sigma\pi AB^2 + \mu^2\varphi ABGK \\
+\mu\phi\varphi\psi ABGK + \mu\sigma\pi\psi AB^2GK + \mu\sigma\varphi\psi\alpha AB^2 \\
a_0 = \mu^2A^2B^2GK(1-R_0)
\end{gathered}\right\}$$

The root $\lambda^{**} = 0$ of (8) correspond to Smartphone virus infection free equilibrium H_0^*. It is important to know that the coefficient a_0 is always positive if and only if $R_0 < 1$ and negative if and only if $R_0 > 1$.

4.4 Boundedness of Solution

Theorem 2: Assume that the initial condition of Smartphone virus transmission dynamics of model system (3) $S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, C(0) \geq 0, Q(0) \geq 0 \; and \; R(0) \geq 0$ with the set $\Phi = \left\{(S, E, I, C, Q, R) \in \mathbb{R}_+^6 : S + E + I + C + Q + R \leq \frac{\rho}{\mu}\right\}$ satisfying $N(0) \leq \frac{\rho}{\mu}$. Then, whenever the solution exists on an interval Φ, it satisfies the following bundedness

$$N(t) \leq \frac{\rho}{\mu}$$

Proof: From equation (4) we obtain the total Smartphone population as

$$N(t) \leq \rho - \mu N - \delta I - \delta_1 C - \delta_2 Q \tag{9}$$

Applying Barkhoff and Rota [29] theorem to equation (9) and having in mind that in the absence of virus infection, $\delta = \delta_1 = \delta_2 = 0$, then we have

$$0 \leq N(t) \leq \frac{\rho}{\mu} \; as \; t \to \infty \tag{10}$$

Since $I(t) \geq 0$ we have from (5) that

$$N(t) \leq \rho - \mu N$$

Using comparison theorem by Lakshmikanthan et al [30], Zhang [31], it can be shown that

$$N(t) \leq N(0)e^{-\mu t} + \frac{\rho}{\mu}(1 - e^{-\mu t}) \tag{11}$$

Whenever $(0) \leq \frac{\rho}{\mu}$, we have $N(t) \leq \frac{\rho}{\mu}$, and Consequently, $I(t) \leq \frac{\rho}{\mu}$.

4.5 Existence of Solution

Theorem 3: The system (3) that satisfies a given initial condition $S(0), E(0), I(0), C(0), Q(0), R(0)$ has a unique solution.

Proof: Let

$$Y = \begin{bmatrix} S(t) \\ E(t) \\ I(t) \\ C(t) \\ Q(t) \\ R(t) \end{bmatrix} \text{ and } \mathfrak{D}(Y) = \begin{bmatrix} \frac{dS(t)}{dt} \\ \frac{dE(t)}{dt} \\ \frac{dI(t)}{dt} \\ \frac{dC(t)}{dt} \\ \frac{dQ(t)}{dt} \\ \frac{dR(t)}{dt} \end{bmatrix}$$

So that system (11) can be written in the following form:

$$\mathfrak{D}(Y) = VY + F(Y) \tag{12}$$

Where

$$V := \begin{pmatrix} -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & -A & 0 & 0 & 0 & 0 \\ 0 & \varphi & -B & 0 & 0 & 0 \\ 0 & \pi & \alpha & -G & 0 & 0 \\ 0 & 0 & \phi & \sigma & -K & 0 \\ 0 & 0 & 0 & 0 & \psi & -\mu \end{pmatrix} \quad \text{and}$$

$$F(Y) = \begin{bmatrix} -\beta\left(\frac{\eta E(t) + I(t)}{N}\right)S(t) \\ +\beta\left(\frac{\eta E(t) + I(t)}{N}\right)S(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{where } A = \mu + \varphi + \pi, \qquad B = \mu + \alpha + \phi + \delta, \;\; G = \mu + \delta_1 + \sigma \text{ and } \; K = \mu + \delta_2 + \psi$$

$$F(Y_1) - F(Y_2) = \begin{bmatrix} -\beta\left(\frac{\eta E(t) + I(t)}{N}\right)S(t) \\ +\beta\left(\frac{\eta E(t) + I(t)}{N}\right)S(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -\beta\left(\frac{\eta E(t) + I(t)}{N}\right)S(t) \\ +\beta\left(\frac{\eta E(t) + I(t)}{N}\right)S(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

$$\begin{aligned} |F(Y_1) - F(Y_2)| \\ &= \left|-\beta\left(\frac{\eta E_1(t) + I_1(t)}{N}\right)S_1(t) + \beta\left(\frac{\eta E_2(t) + I_2(t)}{N}\right)S_2(t)\right| \\ &+ \left|\beta\left(\frac{\eta E_1(t) + I_1(t)}{N}\right)S_1(t) - \beta\left(\frac{\eta E_2(t) + I_2(t)}{N}\right)S_2(t)\right| \end{aligned}$$

$$\leq 2\beta\left(|\eta S_1(t)|\left|\frac{E_1(t) - E_2(t)}{N}\right| + |S_1(t) - S_2(t)|\left|\frac{\eta E_2(t) + I_1(t)}{N}\right| + |S_2(t)|\left|\frac{I_1(t) - I_2(t)}{N}\right|\right)$$

$$\leq 2\beta\left(\left|\frac{\eta E_1(t)S_1(t) + I_1(t)S_1(t)}{N}\right| - \left|\frac{\eta E_2(t)S_2(t) + I_2(t)S_2(t)}{N}\right|\right)$$

$$\leq \Gamma(|Y_1 + Y_2|)$$

By letting equation (10) be equal to $\Pi_0 \in \mathbb{R}_+^6$ $and \lim_{t\to\infty} Sup\, N(t) \leq \Pi_0$, which concludes that $S(t), E(t), I(t), C(t), Q(t), R(t) \leq \Pi_0$ $as\ t \to \infty$, where

$$\Gamma = 2g\Pi_0$$

$$|\mathfrak{D}(Y_1) - \mathfrak{D}(Y_2)| \leq \|\mathrm{V}\||Y_1 - Y_2| + \Gamma|Y_1 - Y_2| \leq \Pi|Y_1 - Y_2|$$

$$where \qquad \Pi = Max(\Gamma, \|V\|) < \infty$$

Therefore,it follows that the function $\mathfrak{D}$ is uniformly Lipschitz continuous and with the restriction on $S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, C(0) \geq 0, Q(0) \geq 0$ and $R(0) \geq 0$, we see that a solution of system (12) exists [27].

4.6 Sensitivity Analysis of the Model R_0

Let the normalized forward sensitivity index of the basic reproduction number of the model R_0, which is differentiable with respect to a given parameter ω, be defined by

$$\Gamma_{\omega}^{R_0} = \frac{\partial R_0}{\partial \omega} \cdot \frac{\omega}{R_0}$$

Thus, we compute an analytical expression for the sensitivity of R_0, with the value of $R_0 = 0.627$using equation (6), to each parameter that it includes. The value of the parameters and the sensitivity indices obtained are represented in table 1. Remember that the Sensitivity index may depend on many parameters of the system, but also can be constant, independent of any parameter [28].

Table 1: The effect of the parameters on R_{02}.

Parameters	Value	Effect on R_{02}
ρ	100	$+1$
μ	0.057	-14.301887
β	0.0065	$+1$
η	0.0002	$+0.046864$
φ	0.0045	$+0.906593$
π	0.04	-0.375563
α	0.21	-0.010300
ϕ	0.83	-0.040708
δ	0.01	-0.000490

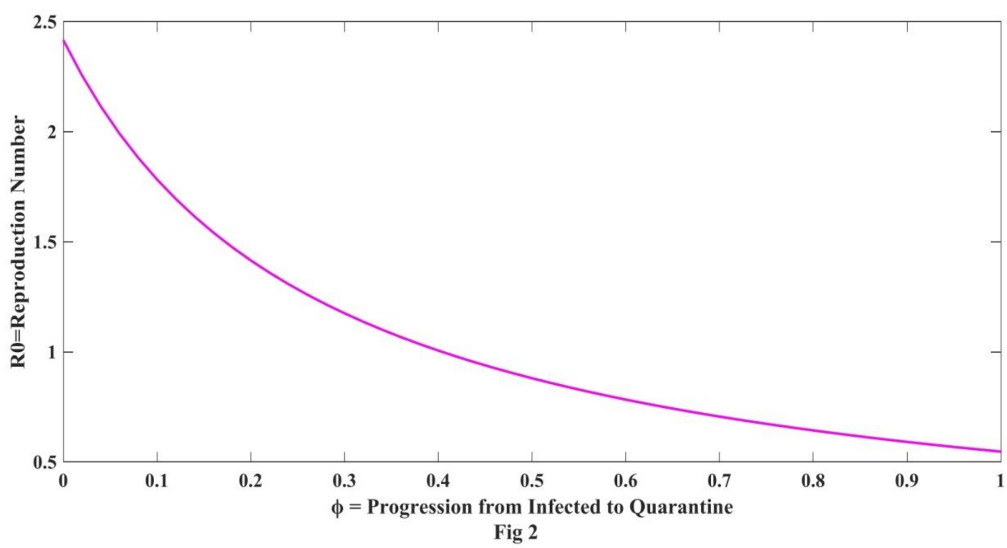

Figure 2: Impact of ϕ (progression from infected to quarantine) on the Smartphone virus infection transmission dynamics in the population

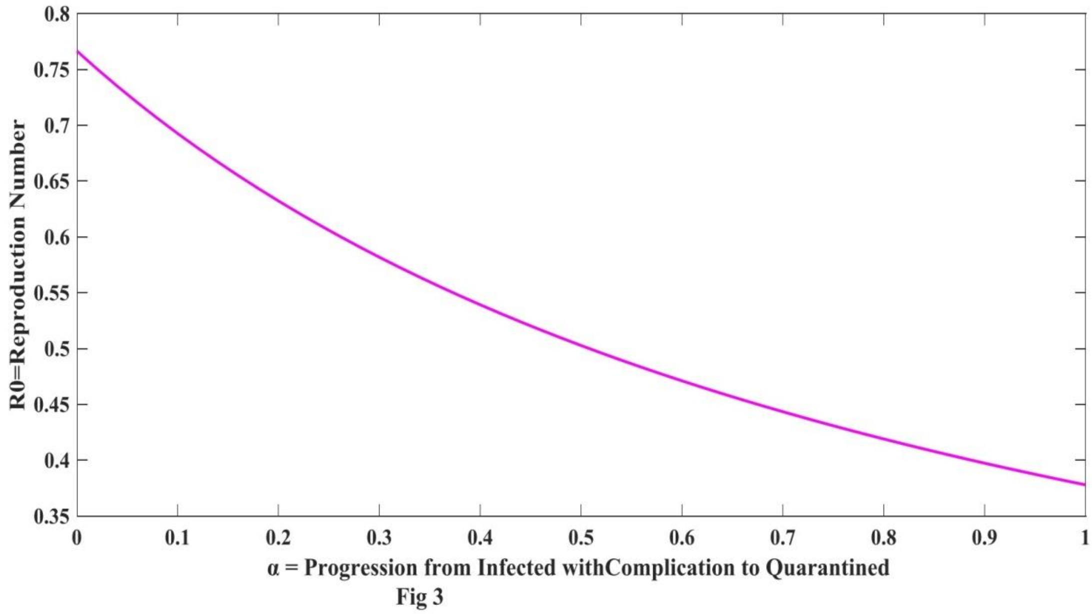

Figure 3: Impact of α (Smartphones with severe complications such as Apps crashing over and over again) on the Smartphone virus infection transmission dynamics in the population

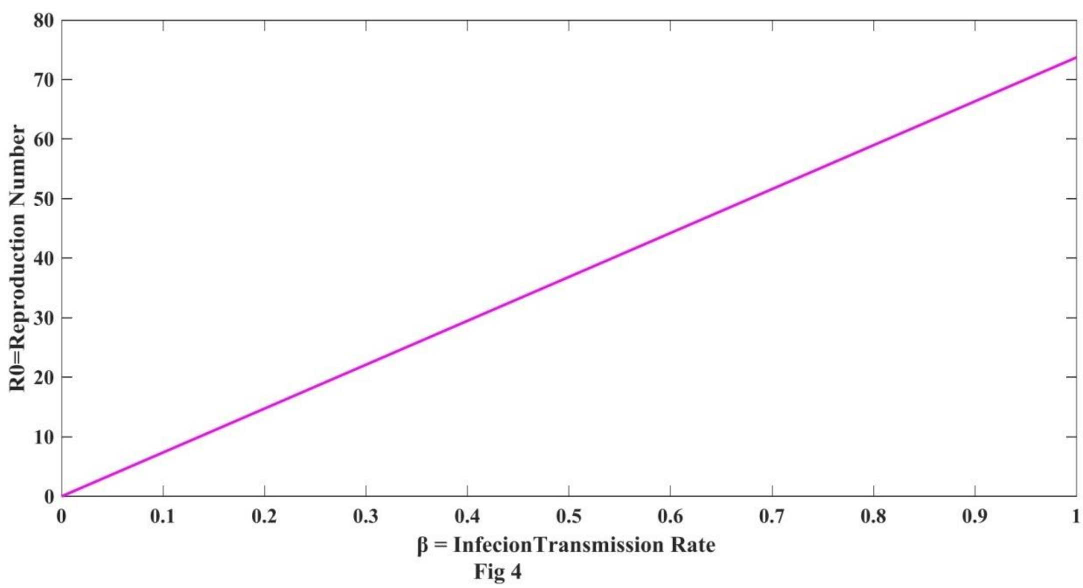

Figure 4: Impact of β (Smartphone virus infection transmission rate) on the Smartphone virus infection transmission dynamics in the population.

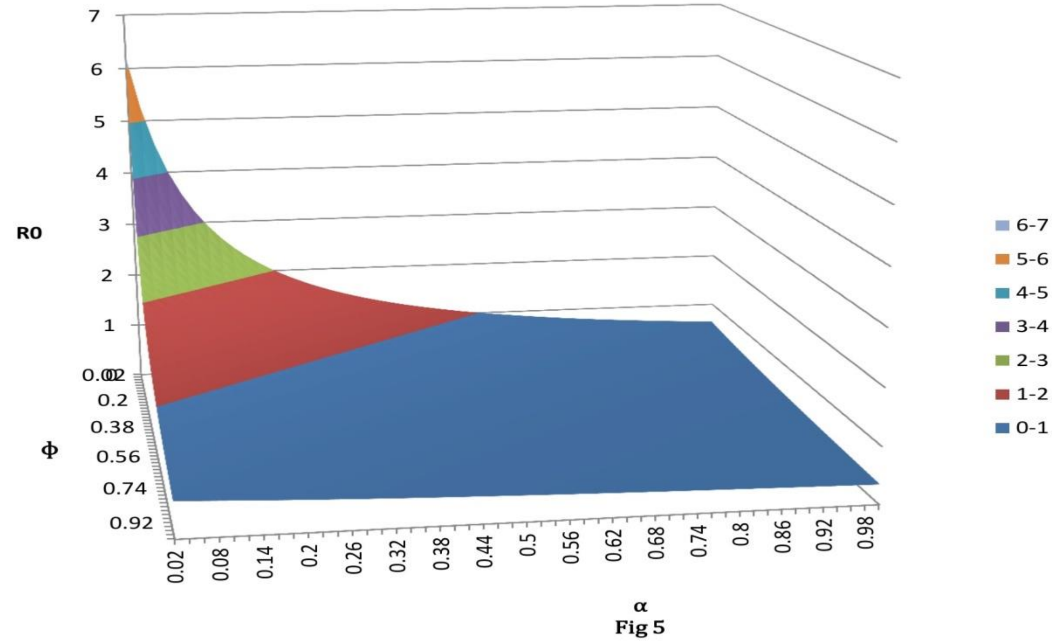

Figure 5: Impact of ϕ (progression from infected to quarantine) and α (Smartphones with severe complications such as Apps crashing over and over again) on the dynamics of Smartphone virus infection in the population.

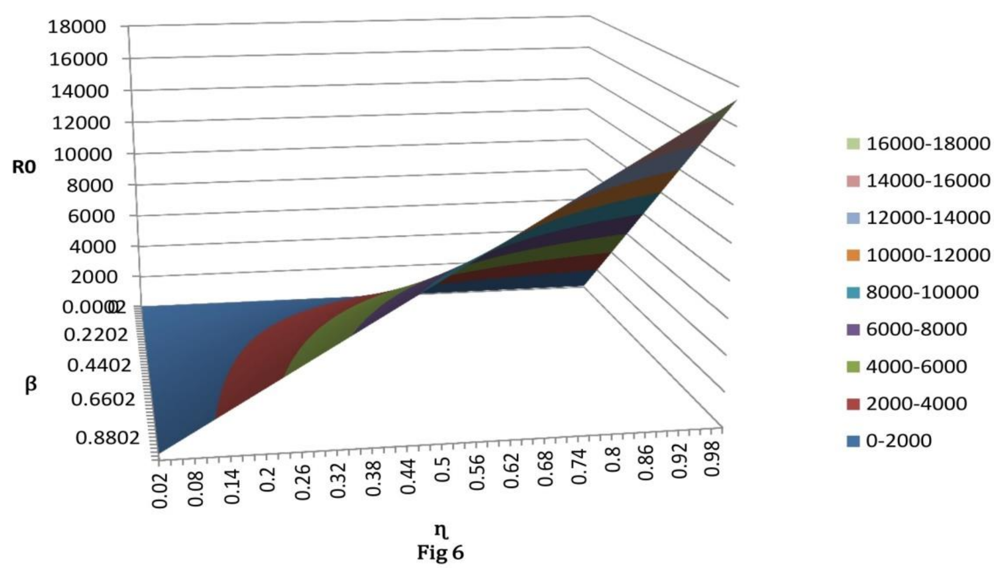

Figure 6: Impact of β (Smartphone virus infection transmission rate) and η (modification factor for the exposed Smartphone) on the dynamics of Smartphone virus infection in the population.

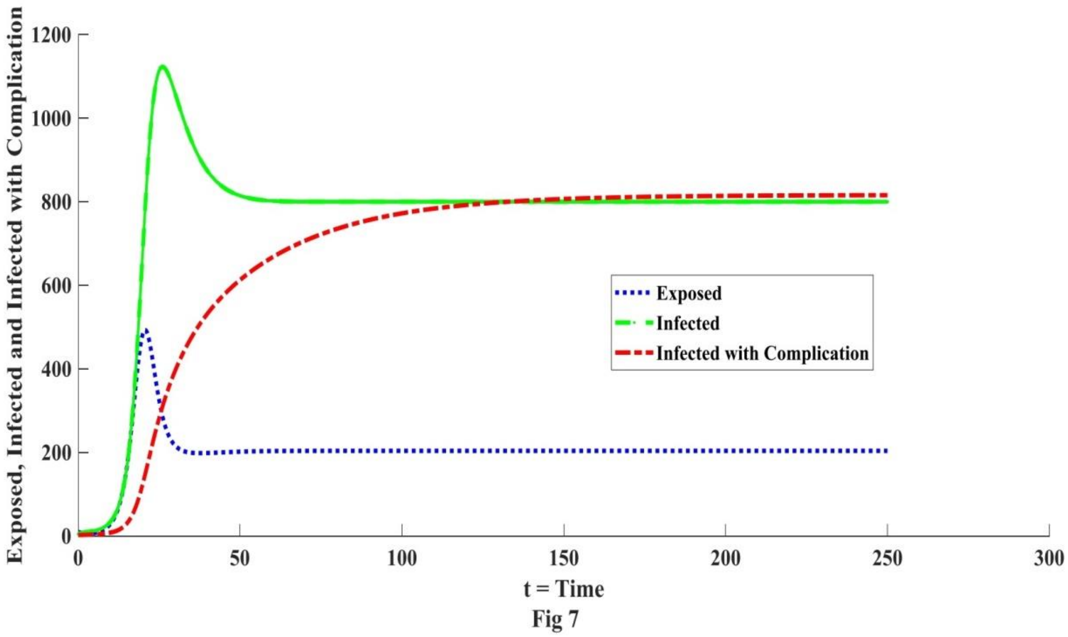

Figure 7: Simulation of the dynamic behavior of model system (3) (Exposed, Infected and Infected with complication) generated in Matlab.

5 Discussion and Conclusion

5.1 Discussion

From Table 1 above, we observe that $\Gamma_{\beta}^{R_0} = \Gamma_{\rho}^{R_0} = +1$. This implies that increasing (decreasing) β or ρ or both by a certain percentage always increases (decreases) R_0 by the same percentage. Thus, sensitivity indices with positive sign show that the value of R_0 increases as these values increase, while the remaining indices that are negative, indicates that the value R_0 decreases as they increase. The estimation of a sensitive parameter should be carefully done, since a small perturbation in such parameter leads to relevant quantitative changes. On the other hand, the estimation of a parameter with a small value for the sensitivity index does not require as much attention to estimate, because a small perturbation in that parameter leads to small changes [29]. This simply means that Smartphone will be infected, repaired (disinfected with antivirus) and eventually recover and retain the infection or obtain immunity. These parameters with negative index reduce the rate at which the infection multiplies in Smartphone population and the virus may eventually be eradicated.

In this work, the analytic results obtained are discussed as follows. The basic Smartphone virus infection reproduction number, R_0,which is the threshold parameter that predicts whether an infection can spread were used as a guide to Smartphone manufacturers and antivirus software developers on the amount of effort needed to control or eradicate virus infection in Smartphone. The results of the numerical simulation of the sensitivity analysis of the basic Smartphone virus infection reproduction number were discussed in this section. The parameter values and sensitivity indices are shown in table 1. The main aim is to assess the feasibility of the dynamics of Smartphone virus infection for different value of the parameters of the basic reproduction number.

Figures 2, 3 and 5 confirm our theoretical analysis that negative sensitivity indices contribute in decreasing R_0 as their value increases. Similarly, figures 4 and 6 represents positive sensitivity indices and it shows that as the value of these parameters increases, R_0 is also increased. Figure 7 shows that the prevalence of Smartphone virus infection increases to a climax, decreases and remains almost stable through the horizon at an alarming rate. Actually the climax indicates the maximum point of the epidemic and the almost stable horizon indicates endemicity of Smartphone virus infection. Therefore, the study confirms that without adequate control measure, Smartphone virus will continuously invade and destroy Smartphones and the virus cannot be eradicated in the population.

5.2 Conclusion

In this work, we computed the sensitivity analysis of the basic reproduction number to study Smartphone virus infection transmission dynamics. The model parameters are given in section (3) and the model equations was derived with the aid of the schematic diagram in figure 1. The Smartphone virus free equilibrium state of the model was determined. The basic Smartphone virus infection reproduction number R_0, for the model was computed using next generation method. The model showed that the stable Smartphone virus infection free equilibrium coexists with the Smartphone virus infection endemic equilibrium in the population. The model suggests that without adequate control measure, Smartphone virus will continuously invade and destroy Smarphones and the virus cannot be eradicated in the population.

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