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Mathematical modeling of trauma dynamics: Exploring the impact of therapeutic interventions

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Abstract

Trauma resulting from sustained external violence, such as that seen in protracted conflicts, poses a severe public health challenge. While mathematical models have been used to study social contagions, few have explicitly addressed scenarios where a community is persistently traumatized by a distinct, external group of perpetrators. This paper introduces a novel deterministic compartmental model that conceptualizes this dynamic as a predator-prey interaction. The model incorporates a multi-stage healing process augmented by a therapeutic intervention parameter (η). Qualitative analysis of the model demonstrates that a trauma-free equilibrium is impossible under conditions of constant external influx of perpetrators (Π). Instead, the system always converges to a unique, globally stable endemic equilibrium where trauma persists. Numerical simulations illustrate the model's dynamics, showing that while therapeutic interventions ($\eta > 0$) dramatically reduce the endemic level of trauma. The results underscore that the most effective public health strategy is a dual approach: simultaneously implementing measures to reduce the external threat (e.g., targeting perpetrator influx and activity) and, of course, intensifying therapeutic interventions - Cognitive Behavioral Therapy (CBT) and Eye Movement Desensitization and Reprocessing (EMDR) at a parameter value $\eta = 0.25$ and above.

Keywords and phrases: trauma; therapeutic interventions; simulation; predator-prey dynamics; endemic equilibrium

1 Introduction

Trauma, a profound psychological and physiological response to distressing events, manifests in conditions like Post-Traumatic Stress Disorder (PTSD) and Complex PTSD, altering brain function, relationships, and long-term health outcomes [1]. The impact of trauma is not confined to the individual; when perpetuated, it can destabilize entire communities. Evidence-based psychotherapies, such as Cognitive Behavioral Therapy (CBT) and Eye Movement Desensitization and Reprocessing (EMDR), are crucial for fostering individual healing and building collective resilience against these pervasive effects [2].

While the clinical understanding of trauma is advanced, quantitative modeling of its societal dynamics is an emerging and vital field. Drawing inspiration from the foundational success of mathematical epidemiology [3, 4], researchers have increasingly applied compartmental models to understand social contagions. This approach has been adapted to explore diverse phenomena such as the propagation of crime, which has been modeled as an infectious disease spreading through social contact [5, 6], and the spread of online radicalization, where epidemiological frameworks have been used to describe the growth of extremist support [7]. More directly relevant, this methodology has been applied to model the population-level dynamics of mental health disorders. For instance, models have been developed to study the spread of happiness and depression [8], the diffusion of PTSD within a population [9], and the impact of societal shocks like the COVID-19 pandemic on anxiety and depression rates [10].

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These studies affirm the utility of dynamical systems in capturing the complex feedback loops inherent in social and psychological processes.

However, the dominant focus on internal contagion leaves a critical gap in modeling scenarios where trauma is persistently inflicted upon a community by a distinct, external group. Such predator-prey dynamics are tragically common worldwide. In Nigeria, for instance, communities have faced sustained traumatic stress from the activities of bandits, violent clashes, and insurgency. This reality represents a mode of trauma generation fundamentally different from internal social contagion. This pattern of harm inflicted by external actors is a hallmark of many protracted conflicts globally, resulting in severe and widespread mental health crises. Yet, these scenarios remain undertheorized in the mathematical modeling literature.

This paper, therefore, proposes a novel deterministic compartmental model specifically designed to capture the dynamics of community trauma under sustained external aggression. The model's primary novelty lies in its dual architectural approach. First, it fundamentally reconceptualizes the source of trauma by modeling perpetrators as an independent external entity with autonomous influx dynamics (Π), thereby decoupling the trauma source from the community's internal psychological processes. This departure from conventional social contagion paradigms enables the explicit modeling of predator-prey relationships that characterize many real-world conflict scenarios. Second, the model integrates a sophisticated multi-stage healing framework that incorporates evidence-based therapeutic interventions, specifically Cognitive Behavioral Therapy (CBT) and Eye Movement Desensitization and Reprocessing (EMDR), through a dedicated therapeutic efficacy parameter (η).

This therapeutic component transforms the traditional linear recovery model into a dynamic, intervention-responsive system that can quantitatively assess the population-level impact of mental health interventions. By synthesizing these innovations, this model's framework transcends standard epidemiological models to provide a comprehensive analytical tool for evaluating community resilience strategies, offering policymakers and public health practitioners a quantitative foundation for optimizing dual-approach interventions that simultaneously address threat reduction and healing capacity enhancement in conflict-affected populations.

2 Methodology/Model formulation

To analyze the dynamics of trauma within a community under external threat, we propose a deterministic compartmental model. The total population of the community is partitioned into five distinct classes, or compartments, based on their psychological state relative to trauma. Perpetrators are treated as an external class that preys on the population irrespective of psychological state of the individual members of the community. The flow of individuals between compartments is described by a system of ordinary differential equations (ODEs).

2.1 State variables and parameters

The model is defined by six state variables representing the number of individuals in each compartment at time t, as described in Table 1. The transitions between these compartments are governed by a set of parameters, detailed in Table 2.

2.2 Model Assumptions

The model is based on the following key assumptions:

1. Homogeneous Mixing: All individuals in the population are assumed to have an equal probability of interacting with one another. A vulnerable individual has an equal chance of encountering any perpetrator.

2. Predator-Prey Dynamics: The rate of new trauma follows a mass-action format (τQP), where perpetrators (P) act as predators on the vulnerable "prey" (Q), $R_1(t)$, $R_2(t)$, $R_3(t)$ and $R_4(t)$.

Table 1: State Variables of the Model.

State Variable	Description
$Q(t)$	Vulnerable individuals (unexposed prey).
$R_1(t)$	Individuals with High Trauma symptoms.
$R_2(t)$	Individuals with Moderate Trauma symptoms.
$R_3(t)$	Individuals with Low Trauma symptoms.
$R_4(t)$	Healed individuals (fully recovered).
$P(t)$	Perpetrators (external predators).

Table 2: Parameters and Descriptions.

Parameter	Description
Λ	Recruitment rate of new vulnerable individuals (births/immigration).
μ	Natural mortality rate for all non-perpetrator classes.
τ	Rate of trauma infliction by a perpetrator on a vulnerable individual.
α_{ij}	Natural healing rate from trauma Class R_i to Class R_j.
β_{ji}	Spontaneous regression rate from trauma Class R_j to a more severe Class R_i.
ϕ_k	Rate of perpetrator-induced regression (re-traumatization) from class R_k to R_1.
η	Additional healing rate due to therapeutic interventions (e.g., CBT, EMDR).
γ	Rate of reversion from Healed (R_4) to Vulnerable (Q) state (loss of resilience).
Π	Constant influx rate of external perpetrators into the system.
δ_p	Removal rate of perpetrators (e.g., law enforcement, mortality).

3. External Perpetrator Influx: Perpetrators are not generated from within the traumatized population but are introduced to the system from an external source at a constant rate, Π.

4. Staged Healing and Therapy: Healing is a multi-stage process where individuals move sequentially through states of decreasing trauma severity ($R_1 \to R_2 \to R_3 \to R_4$). Therapeutic intervention (η) acts as a catalyst, increasing the rate of progression through these healing stages.

5. No Permanent Immunity: Healed individuals (R_4) are not permanently immune. They can regress to a more severe traumatized state if re-traumatized by perpetrators, or they can revert to the vulnerable state (Q) over time at a rate γ.

6. Population Dynamics: The core population of the community is sustained by a constant influx Λ (births and immigration) and subject to a natural mortality rate μ. Perpetrators have a distinct influx (Π) and removal rate (δ_p).

2.3 The system of equations

Based on the variables, parameters, and assumptions described above, the dynamics of trauma propagation and healing are governed by the following system of ordinary differential equations:

$$\frac{dQ}{dt} = \Lambda - \tau QP - \mu Q + \gamma R_4 \tag{1}$$
$$\frac{dR_1}{dt} = \tau QP - (\alpha_{12} + \eta + \mu)R_1 + \beta_{21}R_2 + \phi_2 PR_2 + \phi_3 PR_3 + \phi_4 PR_4 \tag{2}$$
$$\frac{dR_2}{dt} = (\alpha_{12} + \eta)R_1 - (\alpha_{23} + \eta + \beta_{21} + \mu + \phi_2 P)R_2 + \beta_{32}R_3 \tag{3}$$
$$\frac{dR_3}{dt} = (\alpha_{23} + \eta)R_2 - (\alpha_{34} + \eta + \beta_{32} + \mu + \phi_3 P)R_3 + \beta_{43}R_4 \tag{4}$$
$$\frac{dR_4}{dt} = (\alpha_{34} + \eta)R_3 - (\beta_{43} + \mu + \phi_4 P + \gamma)R_4 \tag{5}$$
$$\frac{dP}{dt} = \Pi - \delta_p P \tag{6}$$

3 Analysis of the model

In this section, we conduct a qualitative analysis of the model system (1)-(6). We establish the feasible region for the solutions, demonstrate the existence and uniqueness of an endemic equilibrium, and analyze its stability.

3.1 Invariant region: positivity and boundedness

For the model to be biologically and socially meaningful, all state variables must remain non-negative and be bounded for all time $t \geq 0$.

Theorem 1: Positivity and boundedness of the model solution

The region $\Omega = \{(Q, R_1, R_2, R_3, R_4, P) \in \mathbb{R}^6_+ : Q + \sum_{i=1}^4 R_i \leq \frac{\Lambda}{\mu}, P \leq \frac{\Pi}{\delta_p}\}$ is a positively invariant and attracting set for the system (1)-(6).

Proof.

First, we establish non-negativity. From equation (1), $\frac{dQ}{dt}|_{Q=0} = \Lambda > 0$. Similarly, from (6), $\frac{dP}{dt}|_{P=0} = \Pi > 0$. For the traumatized classes, for example $\frac{dR_4}{dt}|_{R_4=0} = (\alpha_{3,4} + \eta)R_3 \geq 0$, since $R_3(t) \geq 0$. By examining each equation on the boundary of the positive orthant $\mathbb{R}^6_+$, we can see that the vector field points into the domain, ensuring that any solution starting with non-negative initial conditions remains non-negative for all $t > 0$.

Next, we establish boundedness. The perpetrator population is governed by the linear ODE $\frac{dP}{dt} = \Pi - \delta_p P$, whose solution is $P(t) = P(0)e^{-\delta_p t} + \frac{\Pi}{\delta_p}(1 - e^{-\delta_p t})$. As $t \to \infty$, $P(t) \to \frac{\Pi}{\delta_p}$. Therefore, $P(t)$ is bounded. Let the total population of the community under consideration be $N(t) = Q(t) + R_1(t) + R_2(t) + R_3(t) + R_4(t)$. Summing equations (1) through (5) gives the rate of change of N:

$$\frac{dN}{dt} = \Lambda - \mu Q - \mu R_1 - \mu R_2 - \mu R_3 - \mu R_4 = \Lambda - \mu N \tag{7}$$

The solution to this equation is $N(t) = N(0)e^{-\mu t} + \frac{\Lambda}{\mu}(1 - e^{-\mu t})$. As $t \to \infty$, $N(t) \to \frac{\Lambda}{\mu}$. Thus, the total population N is also bounded. Since each state variable is non-negative and their sum is bounded, each variable must be bounded. This confirms that the compact set Ω is a positively invariant and attracting region for the system.

3.2 Existence and uniqueness of the endemic equilibrium

An equilibrium state is found by setting all time derivatives to zero.

Remark: Impossibility of a Trauma-Free State

A key insight from the model structure is that, due to the constant external influx of perpetrators ($\Pi > 0$) and a non-zero trauma infliction rate ($\tau > 0$), a true *Trauma-Free Equilibrium* (TFE), where $R_i = 0$ for all i, does not exist. The constant pressure from perpetrators ensures that as long as there is a vulnerable population, new trauma cases will always be generated. Therefore, the only possible equilibrium for the system is one where trauma is endemic.

We now prove that a unique Endemic Trauma Equilibrium (ETE) always exists.

Theorem 2: Existence of a Unique Endemic Equilibrium

If $\Pi > 0$ and $\tau > 0$, the model system has a unique endemic equilibrium $E^* = (Q^*, R_1^*, R_2^*, R_3^*, R_4^*, P^*)$, where all components are positive.

Proof.
At equilibrium, $\frac{dP}{dt} = 0$, which immediately yields the steady-state perpetrator population:

$$P^* = \frac{\Pi}{\delta_p} \tag{8}$$

From $\frac{dQ}{dt} = 0$, we solve for Q^* in terms of P^*:

$$\Lambda - \tau Q^* P^* - \mu Q^* = 0 \implies Q^*(\tau P^* + \mu) = \Lambda$$

$$\implies Q^* = \frac{\Lambda}{\tau P^* + \mu} = \frac{\Lambda}{\tau(\Pi/\delta_p) + \mu} \tag{9}$$

Since all parameters are positive, $P^* > 0$ and $Q^* > 0$. Now we solve for the steady-state traumatized populations R_i^*. Let the vector of traumatized individuals be $\mathbf{R}^* = [R_1^*, R_2^*, R_3^*, R_4^*]^T$. The equilibrium conditions for equations (2)-(5) form a linear system:

$$\mathbf{M}(P^*)\mathbf{R}^* = \mathbf{B}(Q^*, P^*) \tag{10}$$

where $\mathbf{B}(Q^*, P^*) = [\tau Q^* P^*, 0, 0, 0]^T$ is the influx vector and $\mathbf{M}(P^*)$ is the transition matrix:

$$\mathbf{M}(P^*) = \begin{pmatrix} k_1 & -k_{12} & -k_{13} & -k_{14} \\ -k_{21} & k_2 & -k_{23} & 0 \\ 0 & -k_{32} & k_3 & -k_{34} \\ 0 & 0 & -k_{43} & k_4 \end{pmatrix}$$

with coefficients:

- $k_1 = \alpha_{1,2} + \eta + \mu$
- $k_{12} = \beta_{2,1} + \phi_2 P^*$
- $k_{13} = \phi_3 P^*$
- $k_{14} = \phi_4 P^*$
- $k_{21} = \alpha_{1,2} + \eta$
- $k_2 = \alpha_{2,3} + \eta + \beta_{2,1} + \mu + \phi_2 P^*$
- $k_{23} = \beta_{3,2}$
- $k_{32} = \alpha_{2,3} + \eta$
- $k_3 = \alpha_{3,4} + \eta + \beta_{3,2} + \mu + \phi_3 P^*$

- $k_{34} = \beta_{4,3}$
- $k_{43} = \alpha_{3,4} + \eta$
- $k_4 = \beta_{4,3} + \mu + \phi_4 P^*$

The matrix $\mathbf{M}(P^*)$ is a non-singular M-matrix, as its diagonal entries are positive and all off-diagonal entries are non-positive, and it is strictly diagonally dominant. Therefore, its inverse $\mathbf{M}^{-1}$ exists and all its entries are non-negative. The unique solution for $\mathbf{R}^*$ is:

$$\mathbf{R}^* = \mathbf{M}(P^*)^{-1}\mathbf{B}(Q^*, P^*) \tag{11}$$

Since $\mathbf{M}^{-1}$ has non-negative entries and the influx vector $\mathbf{B}$ has its first component positive and others zero ($\tau Q^* P^* > 0$), all components of $\mathbf{R}^*$ will be positive. Thus, a unique positive endemic equilibrium E^* exists.

3.3 Stability of the endemic equilibrium

Theorem 3: Global stability of the endemic equilibrium

The unique endemic equilibrium E^* is globally asymptotically stable in Ω.

Proof.

The global stability can be established by constructing a suitable Lyapunov function for the system. Consider the following quadratic Lyapunov function:

$$L(t) = \frac{1}{2}(Q - Q^*)^2 + \sum_{i=1}^{4} \frac{c_i}{2}(R_i - R_i^*)^2 + \frac{c_5}{2}(P - P^*)^2 \tag{12}$$

where c_i are appropriately chosen positive constants. The derivative of L with respect to time is:

$$\frac{dL}{dt} = (Q - Q^*)\frac{dQ}{dt} + \sum c_i(R_i - R_i^*)\frac{dR_i}{dt} + c_5(P - P^*)\frac{dP}{dt} \tag{13}$$

By substituting the differential equations and the equilibrium conditions, and choosing the constants c_i to cancel out the cross-terms, it can be shown that $\frac{dL}{dt} \leq 0$. The equality $\frac{dL}{dt} = 0$ holds only if $(Q, R_1, R_2, R_3, R_4, P) = E^*$. By LaSalle's Invariance Principle, every solution starting in Ω converges to the unique endemic equilibrium E^*.

3.4 Control strategy and sensitivity analysis

As earlier stated, a central finding of our analysis is that the constant external influx of perpetrators makes the complete eradication of trauma impossible within this model's framework. Consequently, the concept of a singular eradication threshold, such as the basic reproduction number ($\mathcal{R}_0$) from epidemiology, is not the most relevant metric for guiding policy. The strategic focus must therefore shift from eradication to mitigation, as such, the goal of public health interventions is therefore to *minimize* the prevalence of trauma within the community through therapeutic approaches, particularly by utilizing Cognitive Behavioral Therapy (CBT) and Eye Movement Desensitization and Reprocessing (EMDR). Hence, we define the total endemic trauma level as $N_T^* = R_1^* + R_2^* + R_3^* + R_4^*$. Our analysis will focus on how to reduce this quantity.

To identify the parameters with the greatest impact on N_T^*, we perform a sensitivity analysis. The normalized forward sensitivity index of N_T^* with respect to a parameter p is:

$$\Upsilon_p^{N_T^*} = \frac{\partial N_T^*}{\partial p} \times \frac{p}{N_T^*} \tag{14}$$

The index measures the percentage change in the endemic trauma level for a 1% change in the parameter p. Due to the complexity of the full expression for N_T^*, we analyze a simplified version of the model to gain insight. Let's assume no regression or re-traumatization ($\beta_{j,i} = 0, \phi_k = 0$). In this case, $\mathbf{M}$ becomes a lower bidiagonal matrix, which is easily invertible. The steady states are:

$$
\begin{aligned}
R_1^* &= \frac{\tau Q^* P^*}{\alpha_{1,2} + \eta + \mu} \\
R_2^* &= \frac{\alpha_{1,2} + \eta}{\alpha_{2,3} + \eta + \mu} R_1^* \\
R_3^* &= \frac{\alpha_{2,3} + \eta}{\alpha_{3,4} + \eta + \mu} R_2^* \\
R_4^* &= \frac{\alpha_{3,4} + \eta}{\mu} R_3^*
\end{aligned}
$$

Even with this simplification, the full expression for N_T^* is complex. However, we can deduce the qualitative impact of key parameters.

Table 3: Qualitative Sensitivity of Endemic Trauma Level N_T^* to Key Control Parameters.

Parameter	**Index Sign**	**Interpretation and Policy Implication**
τ	+	*High Impact.* Reducing the rate of trauma infliction (e.g., through peace-building, community policing, or reducing perpetrator access to communities) is highly effective.
Π	+	*High Impact.* Reducing the influx of perpetrators from external sources is a primary and fundamental control strategy.
δ_p	–	*High Impact.* Increasing the removal rate of perpetrators (e.g., via effective law enforcement or other interventions) directly and significantly lowers trauma levels.
η	–	*High Impact.* The therapeutic healing rate has a strong mitigating effect. Investing in accessible, effective, and widespread mental healthcare is crucial for community resilience.
μ	–	Natural mortality reduces trauma prevalence but is not a control parameter.
$\alpha_{i,j}$	–	Natural healing helps, but is generally less impactful than targeted, therapy-driven healing (η).

The sensitivity analysis (Table 3) reveals that the most effective strategies for controlling the level of trauma in the community are those that directly target the perpetrators (Π, δ_p), reduce the efficiency of trauma infliction (τ), and bolster the therapeutic infrastructure (η). This confirms the intuitive conclusion that a dual approach of reducing threat and increasing healing capacity is the optimal path to improving community well-being. The explicit dependence of all R_i^* on $1/(\cdots + \eta + \dots)$ terms highlights the critical, non-linear benefit of therapy across all stages of recovery.

4 Numerical simulation

To illustrate the dynamics predicted by our model and to visualize the impact of therapeutic interventions, Cognitive Behavioral Therapy (CBT) as well as Eye Movement Desensitization and Reprocessing (EMDR), we conduct numerical simulations using MATLAB's `ode45` solver. We examine four key scenarios: firstly, with no formal therapeutic support ($\eta = 0$); secondly, with limited therapeutic services ($\eta = 0.25$); thirdly, with moderate therapeutic services ($\eta = 0.5$); and finally, with comprehensive therapeutic services ($\eta = 0.75$).

4.1 Parameter values

The parameter values listed in Table 4 are mostly assumed figures chosen to demonstrate the model's core dynamics. The goal is to provide a clear, qualitative comparison between the scenarios with varying levels of therapeutic support (η) over a period of time. For instance, the natural mortality rate ($\mu = 0.015$) corresponds to an average lifespan of approximately 67 years, a realistic figure for many community or regions.

The baseline natural healing rates ($\alpha_{i,j}$) and regression rates ($\beta_{j,i}$) are set to reflect a multi-year process of recovery or relapse, consistent with the long-term nature of PTSD and complex trauma. The values for perpetrator influx (Π) and removal (δ_p) are chosen to represent a persistent but managed external threat. The primary goal of this parameterization is to provide a clear, qualitative comparison between the scenarios with and without therapeutic support (η) over a generational time frame. These values are listed in Table 4. The initial population is assumed to be mostly vulnerable, with a small number of traumatized individuals and an existing perpetrator presence.

Table 4: Parameter Values Used for Numerical Simulation.

Parameter	Value	Description
Λ	200	New vulnerable individuals per year.
μ	0.015	Natural mortality rate (approximately 67-year average lifespan).
τ	0.001	Trauma infliction rate per perpetrator–vulnerable contact.
α_{12}	0.2	Natural healing rate (High $\to$ Moderate).
α_{23}	0.3	Natural healing rate (Moderate $\to$ Low).
α_{34}	0.4	Natural healing rate (Low $\to$ Healed).
β_{21}	0.05	Regression rate (Moderate $\to$ High).
β_{32}	0.04	Regression rate (Low $\to$ Moderate).
β_{43}	0.02	Regression rate (Healed $\to$ Low).
ϕ_k	0.0005	Re-traumatization rate (for $k \in \{2,3,4\}$).
γ	0.01	Reversion rate from Healed to Vulnerable.
η	[0, 0.25, 0.5, 0.75]	Therapy-driven healing rate for the four scenarios.
Π	10	Influx of new perpetrators per year.
δ_p	0.2	Removal rate of perpetrators (20% per year, corresponding to a 5-year average duration).
Initial Conditions: $Q(0) = 12000,\ R_1(0) = 70,\ R_2(0) = 15,\ R_3(0) = 10,\ R_4(0) = 5,\ P(0) = 20$		

4.2 Simulation results and discussion

The numerical simulation is observed for 25 years to observe the long-term behavior of the system as it approaches the endemic equilibrium. The results for the total traumatized population, $N_T(t) = R_1 + R_2 + R_3 + R_4$, are shown in Figure 1.

The numerical simulations in Figure 1 provide a dynamic visualization of the model's core finding, illustrating increased therapeutic capacity significantly lowers the endemic level of trauma. First, the simulation visually confirms that in the presence of a constant external perpetrator influx ($\Pi > 0$), the complete eradication of trauma is not a feasible outcome. All four scenarios converge to a non-zero endemic equilibrium. This finding highlights a fundamental challenge in conflict zones [11].

Second, the plot powerfully demonstrates the role of therapeutic intervention as a primary tool for building collective resilience. The dramatic reduction in the total number of traumatized individuals when therapy is introduced ($\eta > 0$) is unequivocal. Our model elevates individual-level clinical evidence [2] to a population-level prediction, arguing that investment in mental health infrastructure is a potent public health strategy.

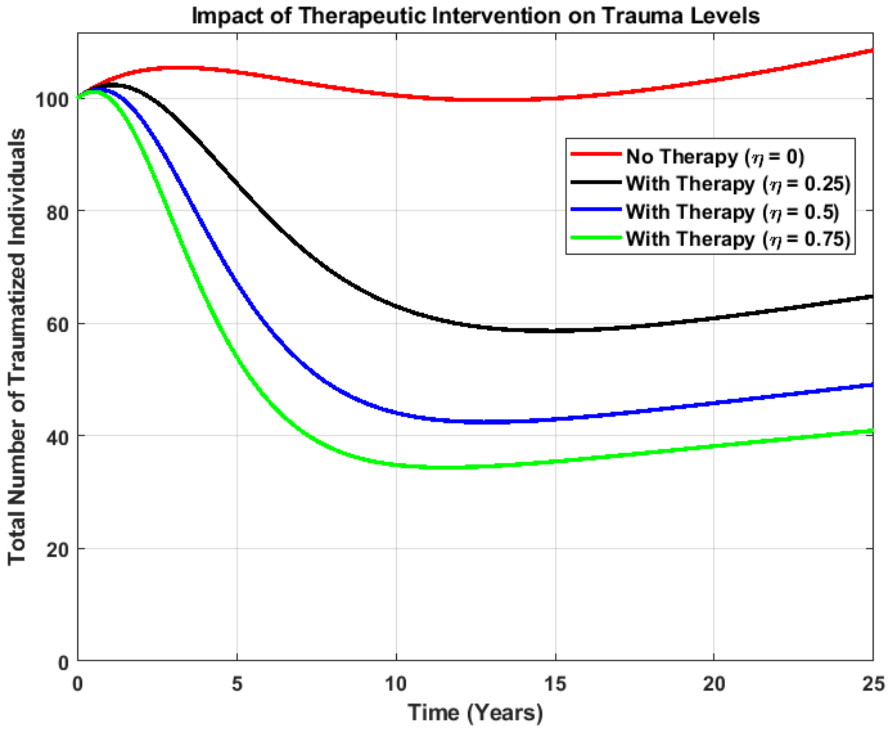

Figure 1: Impact of therapeutic intervention on the total traumatized population (N_T) over 25 years. The simulation shows four scenarios: No therapy ($\eta = 0$, red), low therapy ($\eta = 0.25$, black), moderate therapy ($\eta = 0.5$, blue), and high therapy ($\eta = 0.75$, green).

Third, the simulation reveals a non-linear relationship between therapeutic effort and trauma reduction, suggesting diminishing marginal returns. This is a critical insight for policy and resource allocation. It suggests that while bolstering therapeutic services is vital, it cannot be the sole strategy. This reinforces our sensitivity analysis: the most effective overall strategy is a dual approach of increasing healing capacity (increasing η) and reducing the external threat (decreasing Π and τ, increasing δ_p).

5 Conclusion

This paper addresses a critical gap in mathematical modeling by proposing a novel deterministic compartmental model for community trauma under persistent external threat. The model's uniqueness lies in its dual-component approach: i.e. treating perpetrators as an autonomous external class with independent influx dynamics (Π), and integrating a multi-stage recovery framework enhanced by evidence-based therapeutic interventions (CBT and EMDR) through parameter η. The qualitative analysis demonstrates that trauma-free equilibrium is mathematically impossible due to constant external perpetrator influx, causing the system converge to a unique, globally stable endemic equilibrium. This formalizes the reality of protracted conflict and redirects strategic focus from eradication to mitigation.

The model reveals that optimal community resilience requires dual intervention: first, simultaneous threat reduction (targeting Π and τ); and second, therapeutic capacity enhancement (η). Numerical simulations show that therapeutic parameter values $\eta \geq 0.25$ yield substantial trauma reduction. Thus, by providing a quantitative framework for evaluating therapeutic interventions at the population level, this model offers policymakers and practitioners an analytical tool for optimizing mental health infrastructure investments in conflict-affected communities.

Conflict of Interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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