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A mathematical approach on the dynamics of kidnapping with apprehended kidnappers

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Abstract

Kidnapping has emerged as a critical security challenge in Nigeria and other parts of the world. This study formulates a compartmental mathematical model that divides the population into vulnerable individuals $V(t)$, kidnappers $Q(t)$, hostages $H(t)$, and apprehended kidnappers $A(t)$. We introduce a Crime Propagation Number (C_{P_N}) to assess the risk of kidnapping. Analysis of the model shows that if $C_{P_N} < 1$ and $\lambda > \mu$, the kidnapping-free equilibrium is locally and globally asymptotically stable. Numerical simulations indicate that increasing the apprehension and rescue rates reduces the incidence of kidnapping.

Keywords and phrases: kidnapping; rescue operation; apprehension; ransom.

1 Introduction

Kidnapping has emerged as one of the most critical security challenges in Nigeria and other parts of the world over the past two decades. It disrupts social stability, instills fear, and undermines economic growth. The phenomenon has particularly worsened in sub-Saharan Africa due to unemployment, poverty, political instability, and weak law enforcement [1, 2]. Mathematical modeling provides a framework for policymakers to predict the dynamics of kidnapping, understand the roles of different sub-populations, and design effective control measures.

Kidnapping is a complex social phenomenon that undermines national security, economic growth, and public trust in governance. It has become increasingly prevalent in sub-Saharan Africa due to unemployment, poverty, and weak law enforcement mechanisms [2]. Unlike conventional crimes, kidnapping involves dynamic interactions among vulnerable populations, perpetrators, and state security agencies, making mathematical modeling a suitable tool for examining its propagation and control [3].

The intellectual roots of crime modeling trace back to Becker's (1968) Economic Theory of Crime, which posited that criminal behavior arises from rational choice under risk and reward. Subsequent models extended this framework using systems of differential equations to capture interactions among offenders, law enforcement, and communities [5]. In Nigeria, pioneering works on kidnapping dynamics emphasized socioeconomic drivers but did not rigorously formalize apprehension or hostage-rescue mechanisms. The novelty of recent approaches lies in their explicit separation of hostages as a distinct class and the incorporation of rescue-associated mortality [3].

Earlier models of kidnapping often aggregated victims into the "susceptible" population without distinguishing between hostages and the generally vulnerable population [6]. Such simplifications overlooked the unique risks associated with rescue operations, including hostage mortality and kidnapper casualties. The inclusion of explicit hostage compartments aligns with contemporary studies that highlight the policy relevance of understanding intervention outcomes [7].

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Despite these advances, contradictions persist in the literature. Some scholars argue that ransom payments may temporarily mitigate harm but paradoxically fuel long-term kidnapping by incentivizing perpetrators [8, 9]. Others emphasize deterrence through rapid apprehension and neutralization of kidnappers [10, 11]. However, empirical estimation of key parameters—such as actual apprehension rates—remains scarce due to underreporting and political sensitivities.

Previous studies have applied mathematical modeling techniques to criminology and security problems. For instance, [8] examined the social drivers of kidnapping in Nigeria [12, ?], while [3, 4] developed compartmental models focusing on the interactions among vulnerable populations, kidnappers, and victims. Other works, such as [14], emphasized the importance of incorporating rehabilitation and law enforcement controls.

The reviewed literature suggests that integrating socioeconomic determinants with compartments for hostages, kidnappers, and apprehended actors provides a more comprehensive understanding of kidnapping dynamics. The current model advances this agenda by incorporating policy-relevant parameters such as rescue and apprehension rates, making it both theoretically robust and practically applicable.

2 Methodology/Model formulation

We adopted compartmental modeling approach where the total human population $N(t)$, is divided into 4 classes namely, Vulnerable class, $V(t)$, kidnappers population $Q(t)$, hostages population, $H(t)$, and apprehended kidnappers Population $A(t)$) with

$$N(t) = V(t) + Q(t) + H(t) + A(t). \tag{1}$$

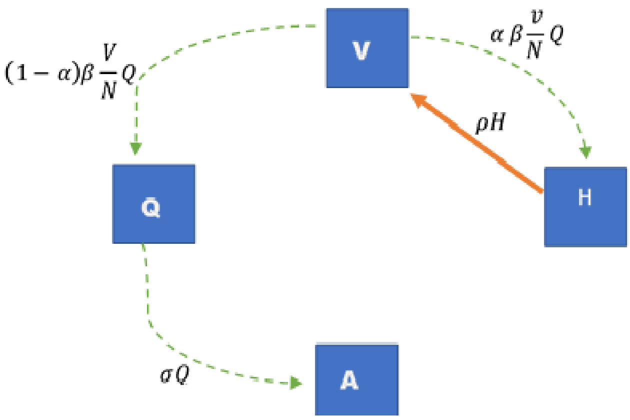

Figure 1: Schematic representation of kidnappers and vulnerable individuals interaction model.

2.0.1 Model equations

$$\frac{dV}{dt} = \lambda N + \rho H - \beta\frac{V}{N}Q - \mu V - \theta N^2, \tag{2}$$
$$\frac{dH}{dt} = \alpha\beta\frac{V}{N}Q - (\mu + \delta + \rho)H, \tag{3}$$
$$\frac{dQ}{dt} = (1-\alpha)\beta\frac{V}{N}Q - (\mu + \gamma + \sigma)Q, \tag{4}$$
$$\frac{dA}{dt} = \sigma Q - \mu A, \tag{5}$$
$$\frac{dN}{dt} = (\lambda - \mu)N - \delta H - \gamma Q - \theta N^2. \tag{6}$$

Table 1: Values of parameters used in simulation.

Parameter	Symbol	Value
Recruitment rate into the population	λ	0.5
Natural death rate	μ	0.02
Rescue rate of hostage	ρ	0.1
Recruitment rate into kidnapper	β	0.6
Saturation term coefficient	θ	0.0001
Fraction of kidnapping encounters producing hostage	α	0.6
Death rate of hostages during rescue	δ	0.05
Death rate of kidnappers during rescue	γ	0.04
Apprehension rate of kidnapper	σ	0.3

2.1 Model novelty and contribution

- Hostage compartment and rescue mortality:

  Most prior kidnapping models [3, 9] include vulnerable, kidnapper, and apprehended classes but do not always separate hostages as a distinct compartment with an explicit rescue flow and rescue-associated mortality terms (ρ, δ, γ). Our explicit modelling of rescue outcomes and associated mortality during rescue operations is a novel extension not commonly present in prior works.

- Integration of rescue dynamics

  The parameter ρ (rescue rate) and its interaction with δ (death during rescue) explicitly model the risks and successes of intervention efforts, which many crime models omit.

- Logistic total-population regulation:

  The density-dependent term θN^2 connects demographic realism to crime dynamics;. Including θ helps ensure boundedness and realistic long-term behavior.

- Policy-ready parameters: The parameters (ρ, σ) in this model are directly interpretable and estimateable from police/arrest/rescue data—making the model practically useful for intervention analysis.

3 Analysis of the model

3.1 Kidnapping-free equilibrium (KFE) and steady state solution

It can easily be shown from the system that the kidnaping free state is $(V, Q, H, A) = (V_0, 0, 0, 0)$. In the absence of kidnappers, $Q = H = 0$ and substituting this into the right hand side of (5) we obtain A = 0. Further substitution of the values of Q, H and A into (2) gives $V = V_0$. At the kidnapping free state, all humans are entirely susceptible and we obtain from (6) the following logistic equation, $\frac{dN}{dt} = r_1 N - \theta N^2$ With solution

$N(t) = \frac{KN_0}{(N_0 + (K - N_0)e^{-r_1 t}))}$

where $r_1 = \lambda - \mu$ and $V_0 = K = \frac{r_1}{\theta}$. As $t \to \infty, N(t) \to K$, which is the carrying capacity of the environment.

3.2 Positivity, existence and uniqueness of solution

The model is described in the domain $\Omega \in \mathbb{R}^5 = \{V, Q, H, A, N : V \geq 0, Q \geq 0, H \geq 0, A \geq 0, N = V + Q + H + A, N > 0\}$.

Suppose at Kidnapping Free Equilibruim, $KFE = X(0) = (V_0, Q_0, H_0, A_0, N_0) \in \Omega$ with $N_0 > 0, \lambda > \mu$ and with all parameters non-negative, then

Step 1: H, Q, A remain nonnegative. From the system,

$$\begin{aligned}\dot{H} &= \alpha\beta\frac{V}{N}Q - (\mu+\delta+\rho)H,\\ \dot{Q} &= (1-\alpha)\beta\frac{V}{N}Q - (\mu+\gamma+\sigma)Q,\\ \dot{A} &= \sigma Q - \mu A.\end{aligned}$$

Since $\alpha\beta\frac{V}{N}Q \geq 0$, $(1-\alpha)\beta\frac{V}{N}Q \geq 0$, and $\sigma Q \geq 0$ whenever $V, Q, N \geq 0$, we obtain

$$\dot{H} \geq -(\mu+\delta+\rho)H, \quad \dot{Q} \geq -(\mu+\gamma+\sigma)Q, \quad \dot{A} \geq -\mu A.$$

By Grönwall's inequality,

$$H(t) \geq H(0)e^{-(\mu+\delta+\rho)t} \geq 0, \quad Q(t) \geq Q(0)e^{-(\mu+\gamma+\sigma)t} \geq 0, \quad A(t) \geq A(0)e^{-\mu t} \geq 0.$$

Step 2: $N(t)$ stays strictly positive. The total population satisfies

$$\dot{N} = (\lambda-\mu)N - \delta H - \gamma Q - \theta N^2.$$

At, $KFE, H = Q = 0$ and $\dot{N} = 0$ so,

$$(\lambda-\mu)N - \theta N^2 = 0$$

$$N = \frac{(\lambda-\mu)}{\theta} > 0$$

Step 3: $V(t)$ is nonnegative. Since $N(t) = V(t)+H(t)+Q(t)+A(t)$ and we have shown $H(t), Q(t), A(t) \geq 0$ and $N(t) > 0$, it follows that

$$V(t) = N(t) - \big(H(t)+Q(t)+A(t)\big) \geq 0.$$

Combining Steps 1–3, we conclude that $V(t), H(t), Q(t), A(t) \geq 0$ and $N(t) > 0$ for all $t \geq 0$.

We note that the right-hand side of (2)–(6) are continuous with continuous partial derivatives, so solutions exist and are unique.

The model is therefore mathematically and criminologically well posed with solutions in Ω $\forall t \in [0, \infty)$.

3.3 Establishing the Crime Propagation Number, C_{P_N}

We consider the matrices F, M, and the control vector R:

$$F = \begin{bmatrix} 0 & \alpha\beta & 0 \\ 0 & (1-\alpha)\beta & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad M = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ -\alpha & 0 & \mu \end{bmatrix}, \qquad R = \begin{bmatrix} H \\ Q \\ A \end{bmatrix}. \tag{7}$$

Here, F represents the emergence of new kidnappers and kidnapped victims, M represents movement of kidnap cases within the various classes, and R is the kidnapping control vector. Define the composite parameters

$$a = \rho+\delta+\mu, \qquad b = \sigma+\gamma+\mu. \tag{8}$$

$$FM^{-1} = \begin{bmatrix} 0 & \dfrac{\alpha\beta}{a} & 0 \\ 0 & \dfrac{(1-\alpha)\beta}{b} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{9}$$

$$\mathrm{C_{P_N}} = \frac{(1-\alpha)\beta}{b} = \frac{(1-\alpha)\beta}{\mu+\gamma+\sigma}. \tag{10}$$

3.4 Local Stability Analysis at Kidnapping Free Equilibrium KFE

Theorem 3.1 (Local Stability Theorem). *Let the DFE be*

$$E = (V^*, H^*, Q^*, A^*, N^*) = (V^*, 0, 0, 0, N^*), \quad V^* = N^*.$$

The DFE E is locally asymptotically stable if $\lambda >\ mu$ and $\mathrm{C}_{P_N} < 1$; it is unstable if $\mathrm{C}_{P_N} > 1$.

Proof.
Step 1. Compute and evaluate Jacobian $J(t)$ matrix at KFE:

$$J = \begin{bmatrix} -\mu & \rho & -\beta & 0 & 2\mu-\lambda \\ 0 & -(\rho+\delta+\mu) & \alpha\beta & 0 & 0 \\ 0 & 0 & (1-\alpha)\beta-(\mu+\gamma+\sigma) & 0 & 0 \\ 0 & 0 & \sigma & -\mu & 0 \\ 0 & -\delta & -\gamma & 0 & -(\lambda-\mu) \end{bmatrix}.$$

Step 2. Characteristic polynomial and factorization. We form the polynomial

$$P(\lambda^*) = \det(J - \lambda^* I).$$

Expanding:

$$P(\lambda^*) = (-\mu-\lambda^*)\big(-(\rho+\delta+\mu)-\lambda^*\big)\big((1-\alpha)\beta-(\mu+\gamma+\sigma)-\lambda^*\big)(-\mu-\lambda^*)(-(\lambda-\mu)-\lambda^*) = 0.$$

Clearly,

$$\lambda^* = -\mu, \quad \lambda^* = -(\rho+\delta+\mu), \quad \lambda^* = -\mu, \quad \lambda^* = -(\lambda-\mu).$$

Step 3. Sign of eigenvalues and stability condition. We observe:

- The roots $\lambda^* = -\mu$, $\lambda^* = -(\rho+\delta+\mu)$, and $\lambda^* = -(\lambda-\mu)$ are strictly negative (parameters are nonnegative, and $\lambda > \mu$ ensures negativity).
- The remaining eigenvalue
$$\lambda^* = (1-\alpha)\beta - (\mu+\gamma+\sigma).$$

Recall:

$$\mathrm{C}_{P_N} = \frac{(1-\alpha)\beta}{\mu+\gamma+\sigma}.$$

Then $\lambda^* < 0 \iff \mathrm{C}_{P_N} < 1$.

Therefore, if $\lambda > \mu$ and $\mathrm{C}_{P_N} < 1$, all five eigenvalues are strictly negative. Hence the Jacobian evaluated at the KFE has all eigenvalues with negative real part, which implies the DFE is locally asymptotically stable.

If $\mathrm{C}_{P_N} > 1$, then $\lambda^* > 0$, so there exists an eigenvalue with positive real part and the KFE is unstable.

3.5 Global Stability Analysis at Kidnapping Free Equilibrium KFE

Theorem 3.2 (Global Stability Theorem). *If $\lambda > \mu$ and $\mathrm{C}_{P_N} < 1$, then the KFE is globally asymptotically stable in Ω.*

Proof. Let

$$\Delta = (1-\alpha)\beta - (\mu+\gamma+\sigma) = (1-\mathrm{C}_{P_N})(\mu+\gamma+\sigma) > 0, \quad C_{P_N} < 1.$$

Let $\phi > 0$, $\psi > 0$ such that

$$\alpha\phi\beta + \psi\sigma \ \le\ \frac{\Delta}{2}. \qquad (**)$$

Consider the Lyapunov Function

$$L(H,Q,A) = Q + \phi H + \psi A \ \geq 0, \quad L = 0 \iff H = Q = A = 0.$$

Using $0 \leq \frac{V}{N} \leq 1$, we compute:

$$\begin{aligned}
\dot{Q} &= \frac{(1-\alpha)\beta V}{N} - (\mu+\gamma+\sigma)Q \ \leq \ \big((1-\alpha)\beta - (\mu+\gamma+\sigma)\big)Q = -\Delta Q,\\
\dot{H} &= \frac{\alpha\beta V}{N}Q - (\mu+\delta+\rho)H \ \leq \ \alpha\beta Q - (\mu+\delta+\rho)H,\\
\dot{A} &= \sigma Q - \mu A.
\end{aligned}$$

Thus,

$$\begin{aligned}
\dot{L} &= \dot{Q} + \phi\dot{H} + \psi\dot{A}\\
&\leq \big[-\Delta + \phi\alpha\beta + \psi\sigma\big]Q - \phi(\rho+\delta+\mu)H - \psi\mu A\\
&\leq -\Delta Q - (\mu+\delta+\rho)H - \psi\mu A \ \leq 0,
\end{aligned}$$

by condition $(**)$.

The set where $\dot{L} = 0$ is

$$\mathcal{E} = \{(H,Q,A) : Q = 0,\ H = 0,\ A = 0\}.$$

By LaSalle's invariance principle,

$$(H(t), Q(t), A(t)) \to (0,0,0) \quad \text{as } t \to \infty$$

for all initial data in Ω.

Similarly, it follows that $V(t) = N(t) = N^*$.

Thus, every solution in Ω satisfies

$$(H,Q,A)(t) \to (0,0,0), \qquad V(t) = N(t) = N^*.$$

4 Numerical simulations/sensitivity analysis

4.0.1 Case A: Simulations without control mechanism. $C_{P_N} > 1$ (3.2727)

The parameter values used for the simulations below are as follows: $\lambda = 0.5; \mu = 0.02; \beta = 0.6; \alpha = 0.4; \rho = 0.1; \sigma = 0.05; \delta = 0.05; \gamma = 0.04; \theta = 0.0001$

4.0.2 Case B: Simulations with control mechanism , C_{P_N}<1 (0.6667)

The parameter values used for the simulations below are as follows: $\lambda = 0.5; \mu = 0.02; \beta = 0.6; \alpha = 0.6; \rho = 0.3; \sigma = 0.3; \delta = 0.05; \gamma = 0.04; \theta = 0.0001$

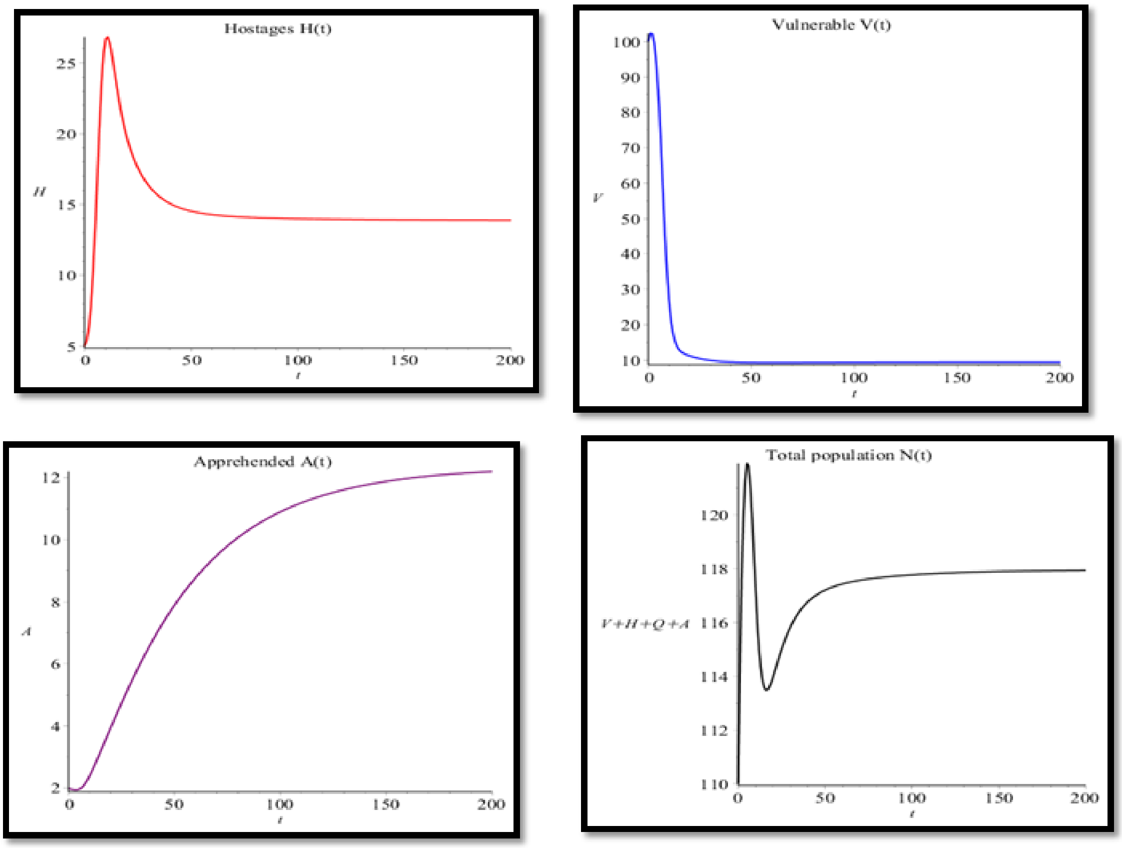

Figure 2: Simulations Without Control Mechanism

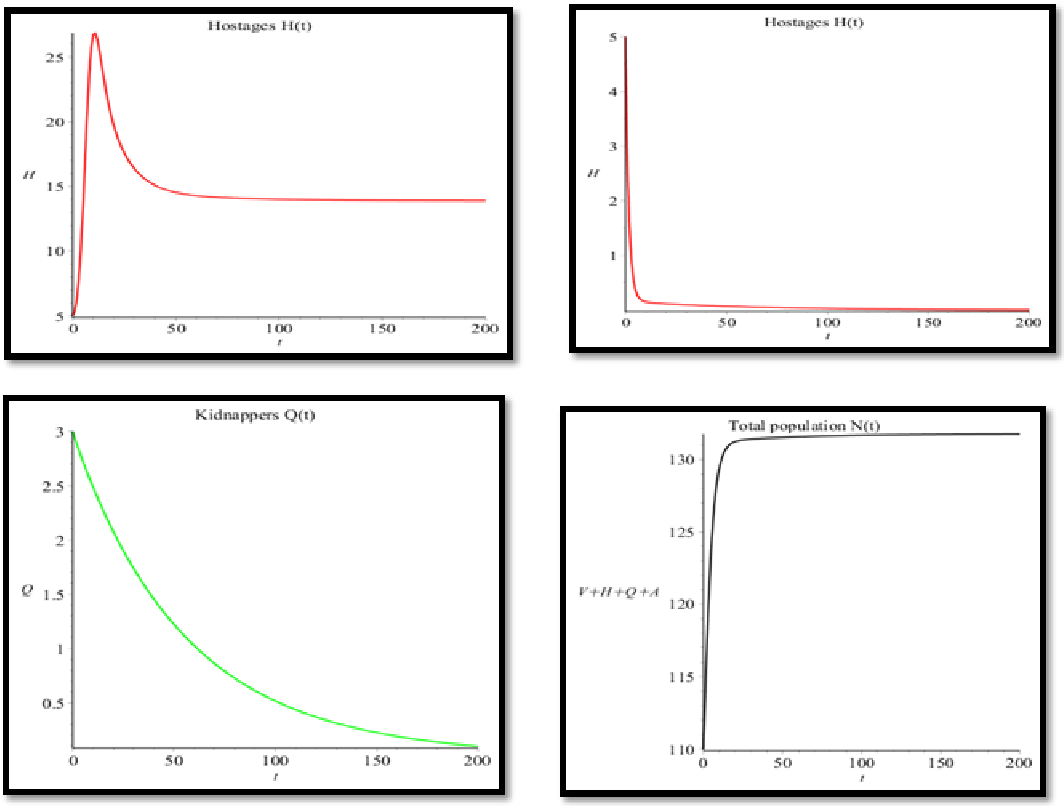

Figure 3: Simulations With Control Mechanism

5 Discussions and conclusion

5.1 Discussions

5.1.1 Implication of Model Parameters to Real Life

Table 2: Implication of Model Parameters to Real Life .

Parameter	Effect on C_{P_N}	Control / Intervention Strategy
β	Increases C_{P_N} when increased	Reduce contact between V and Q through community awareness campaigns, surveillance systems, and creation of safe zones.
α	Decreases C_{P_N} when increased	Strengthen rescue and intervention capacity so more incidents lead to hostages rather than increasing the kidnapper pool.
ρ	Decreases C_{P_N} when increased	Promote funding for anti-kidnapping squads and discourage ransom payment.

5.2 Conclusion

The analysis and simulations of the model shows that, kidnapping can be eradicated from the society, if the recruitment of kidnappers are discouraged through continuous apprehension and elimination of kidnappers by security agents.

To control the menace of kidnapping in the Society, ransom payment to secure release of kidnapped victims should be discouraged. Rather, there should be renewed efforts on the side of Government to recruit and train more security agents into the anti-kidnapping squad in order to apprehend more kidnappers.

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