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Medicolegal model for time of death of a homicide victim using computational approach

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Abstract

The study explores the medicolegal model for time of death of a homicide victim using computational approach. Matlab software using Runge-kutta ODE45 numerical scheme was used to simulate the results output which was analyzed as well. Three statements of problem were studied computationally and various time of death were predicted using step size $h = 1$, $h = 0.1$ and $h = 0.01$. The decaying rate parameter which was used in the construction of the predictive model equations for the four three statements of problem were computed as well with the values of beta1, beta2, beta3 and beta4 being stated as $0.0028, 0.6931, 0.1335,$ and 0.2877 respectively. The study achieved the following results: construction of predictive model equation for various statements of homicide victims' cases using secondary data, computation of the various decaying rate parameters, predictions of the time of dead using computational method.

Keywords and phrases: time of death; homicide victim; computational approach; medico-legal model; decaying rate parameters.

1 Introduction

One of the significant things in forensic science before undergoing a thorough inquiry was to determine the time of death [1]. Time passed after death remained a biggest issue for the forensic pathologist, whose dedication plays role in medical cases, as forensic experts often have to answer death questions in court [2]. Newton's law of cooling was generally used to estimate the time of death. According to newton's law of cooling, the rate of heat transferred from the body's surface to the surrounding fluid that is proportional to the temperature difference between the ground and the fluid [3]. Euler method is a mathematical solution for differential equation with a given initial value problem. The idea behind the Euler method was to combine several small line segments using the concept of local linearity to estimate the real curve [4]. The Euler method is both one and multi-step, together with the requirement for stability [5] the second method was Runge-kutta 4th order, which was common method in numerical analysis to approximate solutions of ordinary differential equations [6].

The work in [7] presented a simple BASIC computer program that enables solving of Marshall and Hoare's equation for the post-mortem cooling of bodies". [8] performed cooling experiments on dummies known as body-like cooling with sudden decrease and increase of ambient temperature in the order of 15^0C. [9] described a simple, reliable, and relatively accurate method for estimating the time since death. The method was based on the Triple-Exponential Formulae (TEF), which the authors devised for the first time in their study. [10] analyzed two murder cases in which available methods did not provide a sufficiently reliable estimate of the postmortem time. In both cases a study was performed to verify the statements of suspects. The authors developed a finite element computer model that simulates a human torso and its clothing. With this model, changes to the body and the environment were modeled; this was very relevant in one of the cases, as the body had been in the presence of a small fire. In both cases it was possible to falsify the statements of the suspects by improving the accuracy of

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the postmortem time estimate. The estimated postmortem time in both cases was within the range of Henssge's model. The standard deviation of the postmortem time estimate was 35 minutes in the first case and 45 minutes in the second case, compared to 168 minutes in Henssge's model. In conclusion, the authors noted that the model as presented can have additional value for improving the accuracy of the postmortem time estimate.

The work in [11] presented the temperature-oriented death time determination based on mathematical model curves of postmortem rectal cooling [12] presented a procedure for the postmortem interval estimation in the presence of a rapid increase of ambient temperature occurred during the cooling phase. 26 The resulting disturbance produced on the cooling curve is proved to obey a two exponential law and is removed from the theoretical or modified body temperature, which enables the estimation of the time since death by means of the standard Nomogram method. Also, the work in [13] studied estimation of time since death in the field of forensic medicine and analyzed some of the existing methods, compared obtained results to determine which method gives more precise results of the estimation of time since death. In [14] a systematic two-stage study was conducted in pigs to verify the models of postmortem body temperature decrease currently employed in forensic medicine. In [15] the authors presented a paper on time of death of victims found in cold water environment. In [16] the authors investigated the influence of variations in the environmental temperature, initial body core temperature, core temperature and time on the standard deviation of the most established model commonly used in forensic practice which was developed by Henssge to estimate the time since death. [17] mentioned that model-based methods play an important role in temperature-based death time determination. [18] noted that the most common method used in determining the estimated time since death in the early post-mortem phase is back calculation based on rectal temperature decrease.

Further, the work in [19] opined that Newton's law of cooling is generally limited to simple cases where the mode of energy transfer is convection, from a solid surface to a surrounding fluid in motion, and where the temperature difference is small, approximately less than 10^oC. The application spectrum was extended by examinations of different typical cooling conditions like, corpse in water, rapid change of ambient temperature [8]. [11] presented the temperature-oriented death time determination based on mathematical model curves of postmortem rectal cooling [12] presented a procedure for the postmortem interval estimation in the presence of a rapid increase of ambient temperature occurred during the cooling phase. 26 The resulting disturbance produced on the cooling curve is proved to obey a two exponential law and is removed from the theoretical or modified body temperature, which enables the estimation of the time since death by means of the standard Nomo-gram method.

2 Mathematical formulations

Model assumptions:

(a) Temperature of the surrounding is assumed to remain constant.

(b) Temperature of the body is the same as its surface temperature (assumption based on uniform cooling).

2.1 Mathematical equations

Newton's Model:

Newton's law of cooling based on the assumptions above is stated mathematically as

$$\frac{d\theta}{dt} = -\beta(\theta - \theta_s), \theta(0) = \theta_0 \tag{1}$$

where,
θ: Temperature of the cooling object at time t
t: time in hours since the first reading

θ_s: Temperature of surrounding medium (ambient temperature)
$beta$: Constant of proportionality

2.2 Method of solution of Newton's Equation (Model)

$$\frac{d\theta}{dt} = -\beta(\theta - \theta_s) \tag{2}$$

$$\frac{d\theta}{dt} + \beta\theta = \theta_s \tag{3}$$

Multiplying both sides by the integrating factor e^{kt}

$$\frac{d}{dt}\left[\theta(t)e^{kt}\right] = \beta\theta_s e^{kt} \tag{4}$$

Integrating both sides with respect to t

$$\int (\theta(t)e^{kt})' dt = \int \beta\theta_s e^{kt} \tag{5}$$

$$(\theta(t)e^{kt}) = \theta_s e^{kt} + C \tag{6}$$

$$\theta(t) = \theta_s + Ae^{-kt} \tag{7}$$

$$\text{At } t = 0, \theta(0) = \theta_0 \tag{8}$$

$$A = \theta_0 - \theta_s \tag{9}$$

$$\theta(t) = \theta_s + (\theta_0 - \theta_s)e^{-kt} \tag{10}$$

Using the following condition to obtain β
At $t = 0$, $\theta(0) = 130^oC$
At $t = 4$, $\theta(4) = 100^oC$
$\theta_s = 70$
Hence obtain the particular solution $\theta(t)$

$$\begin{aligned} \theta(t) &= 70 + (130 - 70)e^{-kt} \\ \theta(t) &= 70 + 60e^{-kt} \end{aligned}$$

At t = 4, $\theta(4) = 100$

$$\begin{aligned} 100 &= 70 + 60e^{-4\beta} \\ 100 - 70 &= 60e^{-4\beta} \\ 30 &= 60e^{-4\beta} \\ e^{-4\beta} &= \frac{30}{60} \\ e^{-4\beta} &= \frac{1}{2} \end{aligned}$$

Taking the $\log_e$ of both sides

$$\begin{aligned}
\log_e e^{-4\beta} &= \log\frac{1}{2} \\
-4\beta \log_e e &= \log\frac{1}{2} \\
\beta &= \frac{\log\frac{1}{2}}{-4} = 0.17329
\end{aligned}$$

Hence equation (10) becomes

$$\theta(t) = 70 + 60e^{-0.17329t}$$

2.3 Application to statement of problem

The required predictive model equation that is used in this study is stated as:

$$\frac{d\theta}{dt} = -\beta(\theta - \theta_s),\ \theta(0) = \theta_0 > 0$$

Statement Problem One (1):

A homicide victim was found by police personnel at 10.40 pm and the body temperature at the exact time was 94.4^oF. The body was located in a room where the room temperature is kept constant at 70^oF. Not long after the death, the body temperature was decreasing, and assumed that the victim's temperature was 98.6^oF at the time of death [20]. After 90 minutes, the body temperature was recorded at 89^oF. Find the required predicting model.

Method of solution

$$\frac{d\theta}{dt} = -\beta(\theta - \theta_s),$$
$$\theta(0) = 98.6$$

Where, $\theta_s = 70$
To evaluate β we use the solution map

$$\theta(t) = \theta_s + (\theta_0 - \theta_s)e^{-\beta t}$$

Here θ is the temperature on arrival of expert.
Substituting θ_0 and $\theta_s = 70$

$$\begin{aligned}
\theta(t) &= 70 + (94.4 - 70)e^{-\beta t} \\
\theta(t) &= 70 + 24.4e^{-\beta t}
\end{aligned}$$

At $t = 90$ minute, $\theta(90) = 89$

$$\begin{aligned}
90 &= 70 + 24.4e^{-90(\beta)} \\
89 - 70 &= 24.4e^{-90(\beta)} \\
e^{-90(\beta)} &= \frac{19}{24.4} = 0.778688524
\end{aligned}$$

Taking the natural log of both sides

$$
\begin{aligned}
\log_e e{-}90\beta &= \log_e(0.778688524) \\
-90\beta &= -0.250144153 \\
\beta &= \frac{-0.250144153}{-90} \\
\beta &= 0.002779379488
\end{aligned}
$$

Thus, the required predicting model will be

$$
\frac{d\theta}{dt} = -0.002779379488(\theta - 70)
$$

$$
\theta(0) = \theta_0 = 98.6
$$

Statement Problem Two (2):

A homicide victim was found in a room that is kept at a constant temperature of 70^oF. At 12 noon, the forensic experts arrived and measured the temperature of the dead body. It was 80^oF at that time one hour later, the temperature reduced to 75^oF. Assuming that the victim's temperature was 98.6^oF just before death, determine the time of death.

Method of solution

$$
\theta(t_\alpha = 80, \theta_s = 70
$$

$$
\theta(t_\alpha + 1) = 75, \theta(0) = 98.6
$$

Using $\frac{d\theta}{dt} = -\beta(\theta - \theta_s)$

$$
\theta(0) = \theta_0 = 98.6
$$

And the solution map

$$
\theta(t) = \theta_s + (\theta_0 - \theta_s)e^{-\beta t}
$$

At $t = t_\alpha, \theta(t_\alpha) = 80$

$$
\begin{aligned}
80 &= 70 + (98.6 - 70)e^{-\beta t_\alpha} \\
10 &= 28.6e^{-\beta t_\alpha} \\
e^{-\beta t_\alpha} &= \frac{10}{28.6} = 0.349650349
\end{aligned}
$$

At $t = t_\alpha + 1, \theta(t_\alpha + 1) = 75$

$$
\begin{aligned}
75 &= 70 + (98.6 - 70)e^{-\beta(t_\alpha+1)} \\
5 &= 28.6e^{-\beta t_\alpha} \cdot e^{-\beta} \\
5 &= 28.6\left[\frac{10}{28.6}\right]e^{-\beta} \\
e^{-\beta} &= \frac{5}{10} = 0.5
\end{aligned}
$$

Taking the natural log of both sides

$$
\begin{aligned}
-\beta &= -0.69314718 \\
\beta &= 0.69314718 \text{ (hours)}
\end{aligned}
$$

Thus, the predicting equation is stated as:

$$
\frac{d\theta}{dt} = -0.69314718(\theta - 70)
$$
$$
\theta(0) = \theta_0 = 98.6
$$

Statement Problem Three (3)

A man shot dead at Place Dadar, Mumbai (Date: 12 Oct 2018). The Detective arrives at the scene at 10 : 23 pm. immediately; the temperature of the body is taken and is found to be 80^oF. The detective finds that the room temperature has been constant for 68^oF. From the crime scene evidence is obtained. Again the temperature of the body is taken and found to be 78.5^oC. This last temperature reading was taken exactly one hour after the first one.

Method of solution

$$
\theta(t_\alpha) = 80, \theta_s = 68^oF
$$
$$
\theta(t_\alpha + 1) = 78.5, \theta(0) = 98.6
$$

Using $\frac{d\theta}{dt} = -\beta(\theta - \theta_s)$

$$
\theta(0) = \theta_0 = 90
$$

the solution map:

$$
\theta(t) = \theta_s + (\theta_0 - \theta_s)e^{-\beta t}
$$

At $t = t_\alpha, \theta(t_\alpha) = 80$

$$
\begin{aligned}
80 &= 68 + (98.6 - 68)e^{-\beta t_\alpha} \\
12 &= 30.6e^{-\beta t_\alpha} \\
e^{-\beta t_\alpha} &= \frac{12}{30.6} = 0.392156862
\end{aligned}
$$

At $t = t_\alpha + 1, \theta(t_\alpha + 1) = 78.5$

$$
\begin{aligned}
78.5 &= 68 + (98.6 - 68)e^{-\beta(t_\alpha + 1)} \\
10.5 &= 30.6e^{-\beta t_\alpha} \cdot e^{-\beta} \\
10.5 &= 30.6\left[\frac{12}{30.6}\right]e^{-\beta} \\
e^{-\beta} &= \frac{10.5}{12} = 0.875
\end{aligned}
$$

Taking the natural log of both sides

$$
\begin{aligned}
-\beta &= -0.133531392 \\
\beta &= 0.133531392 \text{ (inhours)}
\end{aligned}
$$

Thus, the predicting equation is stated as:

$$\frac{d\theta}{dt} = -0.1335392(\theta - 68)$$
$$\theta(0) = \theta_0 = 98.6$$

Statement Problem Four (4)

At 2 o'clock in the afternoon, a house help (Mr. Musa) reported discovering the dead body of his master, chief Benson in the chief's personal wine bar. At 4 O'clock, the forensics expert arrived and measured the temperature of the body. It was 90 degrees as of that time. One hour later, the body cooled to 80 degrees. It was also noted that the wine bar was maintained at temperature of 50 degrees. Should Musa be arrested for murder?

3 Results and discussion

Table 1: Estimating time of death for three case scenario for time interval in hours using step size, $h = 1.0$

Time	Temp. Scenario 2	Temp. Scenario 3	Temp. Scenario 4
0	98.6000	98.6000	98.2000
1.0000	84.3004	94.7750	86.1500
2.0000	77.1502	91.4281	77.1125
3.0000	73.5751	88.4996	70.3344
4.0000	71.7876	85.9372	65.2508
5.0000	70.8938	83.6950	61.4381
6.0000	70.4470	81.7331	58.5786
7.0000	70.2235	80.0165	56.4339
8.0000	70.1118	78.5144	54.8254
9.0000	70.0559	77.2001	53.6191

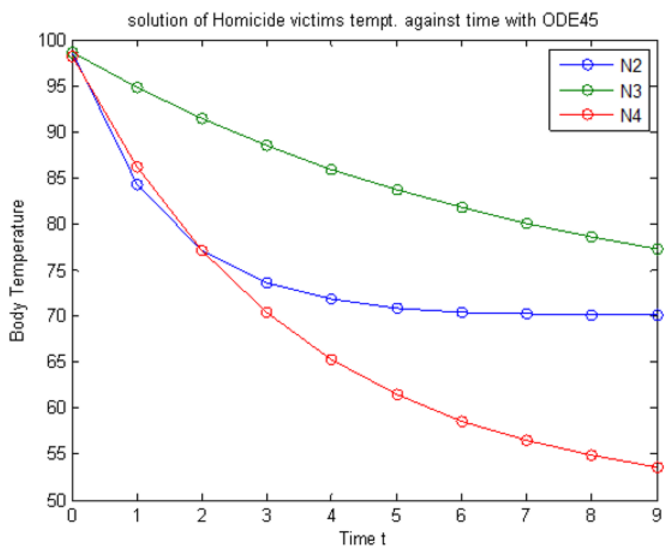

Figure 1: Solution trajectory of homicide victims body temperature against time using step size $h = 1$

Table 2: Estimating time of death for three case scenario for time interval in hours using **step size, h=0.1**

Time	Temp. Scenario 2	Temp. Scenario 3	Temp. Scenario 4
0	98.6000	98.6000	98.2000
0.1000	96.6847	98.1941	96.8331
0.2000	94.8977	97.7936	95.5050
0.3000	93.2303	97.3984	94.2146
0.4000	91.6747	97.0085	92.9607
0.5000	90.2232	96.6237	91.7424
0.6000	88.8690	96.2440	90.5587
0.7000	87.6051	95.8694	89.4085
0.8000	86.4255	95.4997	88.2909
0.9000	85.3250	95.1349	87.2050

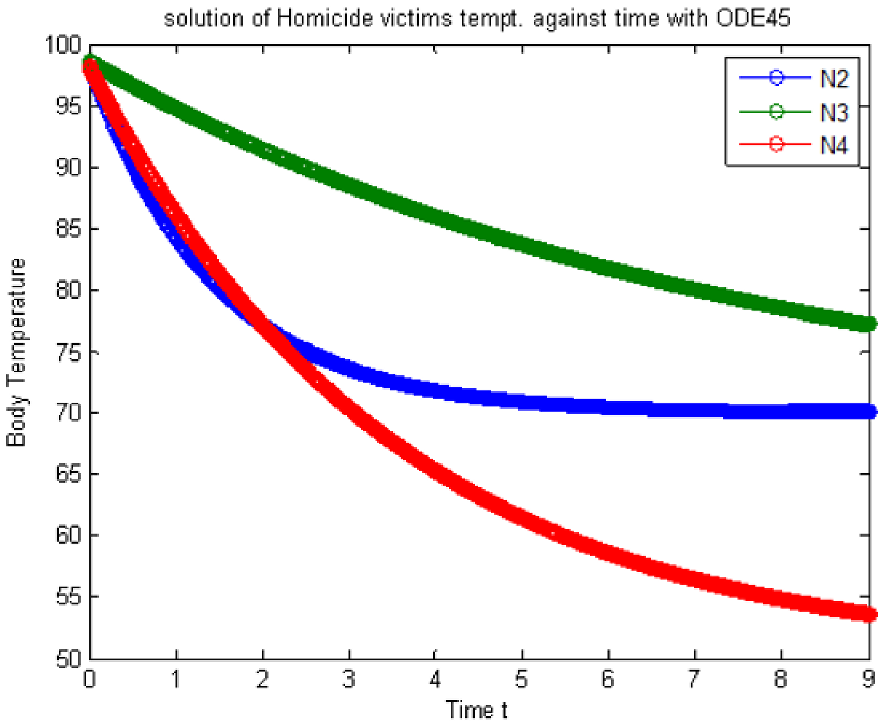

Figure 2: Solution trajectory of homicide victims body temperature against time using step size $h = 0.1$

3.1 Discussion of results

Table 1 shows the monitoring data base for estimating the time of death for 3 case scenario which are statement problem two (2) three (3) *and* four (4) for time interval $0(1)9$ in hours using step size $h = 1.0$. Fig 1 which summarizes the solution trajectory of the homicide victims body temperature against time for the 3 case scenario using the same step size $h = 1.0$ at time interval $0(1)9$.

Table 2 shows that monitoring data base for estimating the time of death for the case 3 scenario which are statement problem Two (2), three (3) and Four (4) for time interval $0(0.01)7.05$ in hours using step size $h = 0.01$.

From Table 1; when all model parameter values are fixed for statement problem Two (2) represented as N2, statement problem three (3) represented as N3 and statement problem four N4 at time interval $0(1.0)9$ in hours with a step size $h = 1.0$, we observe that on the base time here called the initial condition (IC) of the homicide victim temperature N2(IC)$=$ 98.60 unit, N3(IC) $=$ 98.6 unit and N4(IC) $=$ 98.2 unit. As time increases monotonically down the trend from one (1) hour after the IC with a step size of $h = 1.0$ up to nine (9) hours, we observe a monotone decrease in the various coordination of the homicide victim temperature.

Furthermore, the N2 coordinate records 84.3004 unit one hour after death and converges to 70.0559 unit nine (9) hours after death. The temperature interval of the N2 coordinate indicates that the time of death indicator of the homicide victim occurs within the time interval $1(1.0)2$ with temperature interval $84.3004 - 77.1502$ unit which the actual time of death can be predicted using a further reduced step size for proper interpolation since the temperature of the homicide victim on the time of arrival of the forensic expert is 80 unit which lies between $84.3004 - 77.1502$ unit.

Similarly, N3 coordinate records 94.7750 unit one (1) hour after death and converges at 77.2001 unit nine (9) hours after death. The temperature interval of the N3 coordinate indicates that the time of death indicator of the homicide victim occurs within the time interval $6(1.0)8$ with temperature interval $81.7331 - 78.5144$ unit which the actual time of death can be predicted using a further reduced step size for proper interpolation since the temperature of the homicide victim on the time of arrival of the forensic expert is 80 which lies between $81.7331 - 78.5144$ unit.

More so, N4 coordinate records 86.1500 unit one (1) hour after death and converges at 53.6191 unit nine (9) hours after death. The temperature interval of the N4 coordinate indicates that the time of death indicator of the homicide victim occurs within the time interval $0(1.0)1.0$ with temperature interval $98.20 - 86.10$ unit which the actual time of death can be predicted using a further reduced step size for proper interpolation since the temperature of the homicide victim on the time of arrival of the forensic expert is 90 unit which lies between $98.20 - 86.10$ unit.

4 Conclusion

This study has achieved the following: Construction of predictive model equation for various statements of problem of homicide victims' cases using secondary data, Predictions of the time of dead with applications to the various statements of problem as case studies using computational method.

5 Recommendations

This information as seen on the monitoring database of a homicide victim will be useful in predicting the time of death of any case scenario using it's mathematical accuracy and precision which can be tendered in any court of jurisdiction and can be used as evidence against an homicides suspect as well as a vital information and proper planning in building medico-legal practice monitoring database.

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