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On impact of experimental time of mangrove forest relative abundance in the absence of Artisinal hydrocarbon effect

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Abstract

An impact study of experimental time of mangrove forest relative abundance in the absence of artisanal hydrocarbon effect has been investigated. The analytical solutions of tree growth models were derived and used to predict the impact of experimental time of mangrove forest relative abundance through computational method using MATLAB ODE45. Three different mangrove sites in the Niger delta and considered as natural mangrove forest vegetation sites in the absence of artisanal hydrocarbon. It is observed that on the based day of our experimental time called the initial condition, the relative abundance of the three sites was recorded as 0.01 units, same values each. Furthermore, from the tenth month up to the one hundred and twentieth month, the data base result shows a monotonic increasing pattern in the relative abundance of the three coordinates as 0.0246 units on the tenth month to a saturating value of 0.2427 units on the one hundred and twentieth months. These observations show a strong increase in the numerical strength of the relative abundance of the second coordinate followed by the first coordinate with a slow increase in numerical strength and a strong increase pattern in the third coordinate which converges earlier to its saturated value.

Keywords and phrases: Relative abundance; artisanal hydrocarbon; experimental time; computational method; mangrove forest.

1 Introduction

1.1 Background of the study

Mangroves are dicotyledonous shrubs or trees growing along tidal mudflats and on shallow coastal water consisting of mono specific patches or belts [1]. They are important as hatchery and nursery habitat for juveniles of fish and a large amount of organisms like insects, reptiles, amphibians, birds and mammals live in this habitat [2]. Mangrove trees supply timber for construction, firewood, charcoal, fishing poles, pulp and tannin [3]. Many parts of the mangrove can also be utilized as medicines [4]. Mangrove forests are the world's most productive ecosystems. They enrich coastal waters, yield commercial forest products, protect coastlines, and support coastal fisheries. The forests are major blue carbon systems, storing considerable amounts of carbon in marine sediments, thus becoming important regulators of climate change [5]. Measuring and managing ecosystems based on their carbon stocks, particularly under the umbrella of reducing emissions from deforestation and degradation may also benefit biodiversity research and conservation [6].

Relative abundance refers to the proportion of a particular mangrove species within a given area or population compared to other species in the same area. Nigeria has extensive mangrove forests in the coastal region of the Niger Delta. Lagos, a coastal area in Nigeria has a total expanse of 42.20 square kilometers. Within this area, specifically 3.13 square kilometers, are designated as forest reserves. This portion within the forest reserves represents 7.42% of the overall area of Lagos. Essentially, this signifies that a small but significant part of Lagos is designated as a forest reserve, contributing to the

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preservation and conservation of the region's natural habitat and ecosystems [7]. In the Edo/Delta region of Nigeria, the mangrove forest area covers a substantial $3,470.32$ square kilometers. Within this extensive area, 143.75 square kilometers of mangroves are located within the designated forest reserves. This signifies that approximately 4.14% of the total mangrove area is officially protected within these forest reserves. Having a portion of the mangrove area within forest reserves is vital for their preservation and sustainable management. This helps maintain the ecological balance and the divers array of flora and fauna that depend on mangrove ecosystems [7]. In the Rivers/Bayelsa region of Nigeria, the mangrove forest area spans a vast $5,435.96$ square kilometers. Within this extensive expanse, 90.62 square kilometers of mangroves are designated within forest reserves. This indicates that approximately 1.67% of the total mangrove area is safeguarded within these specific forest reserves [7].

In Cross River/Akwa Ibom region, the mangrove forest spans an expansive area of 721.86 square kilometers, and within this expanse, 67.19 square kilometers fall under the protection of forest reserves accounting for a notable 9.31%. Altogether, considering all regions, the total mangrove forest area amounts to an impressive $9,723.14$ square kilometers, with 304.69 square kilometers nestled within designated forest reserves [7].

The artisanal form of crude oil refining characterized by its small scale and limited technology infrastructure has emerged as a major environmental in several regions around the world. One of the primary concerns associated with artisanal crude oil refining is the uncontrolled release of hydrocarbons and other toxic by-products into the surrounding soil, water bodies and atmosphere [8]. The predominant degradation of petroleum hydrocarbons within the environment are primarily driven by indigenous bacterial populations. Numerous studies confirmed the occurrence of active microorganisms in petroleum oil-contaminated areas such as oil reservoirs and oil spills [9]. Artesanal crude oil refining sites, commonly referred to as 'illegal' or 'bush' refineries, are prevalent in developing countries especially in regions with abundant oil reserves but limited regulatory oversight and enforcement mechanism [8].

2 Mathematical formulations

2.1 Mangrove tree growth model

The structure and conceptual basis of the growth rate model formulation are based on the classical Botkin model [10]. The complex dynamical system of ordinary differential equation of first order considered the diameter at breast height (dbh) as a function of time and established a relationship with tree Height (H).

The mathematical growth equation is stated as follows:

$$\frac{d}{dt}(dbh) = G_{opt}.MUL\left[\frac{dbh(1-dbh\frac{H}{D_{\max}H_{\max}})}{2b_1+(3b_2.dbh)-(4b_{3.dbh^2})}\right] \tag{1}$$

$$dbh(0) = dbh_0 \succ 0$$

Here,

$$H = b_1 + b_2 \cdot dbh - b_3 \cdot dbh^2 \tag{2}$$

$$\begin{aligned} b_2 &= \frac{2(H_{max} - b_1)}{D_{max}} \\ b_3 &= \frac{H_{max} - b_1}{D_{max}} \\ Biom &= a_1 \cdot dbh^{c1} \end{aligned}$$

Dbh	represent diameter at breast height (cm)
H	represents tree height
Hmax	represents species specific maximum values of tree height
Dmax	represent species specific maximum values of tree diameter
b1, b2, b3	represents estimating charactering the relationship or contributions (enhancing or inhibit) between the tree height H and diameter at breast height, dbh.
Gopt	represents the species specific growth rate under optimal environment condition.
A1 and c1	represent scaling parameters for calculating tree biomass (Biom) from dbh value
MUL	represents the growth multiplier depending on environmental conditions and tree competition.

Where

In this study, the range of Mul lies between 0 and 1 inclusive which comprises of the influencing factors salinity, ground elevation, tree density and heavy metal population. In the model formulation, the inner physiological differentiations among trees of the same species are omitted and assume that mangrove structure and productivity are governed by environmental factors and intra competition coefficients (interactions).

Hence, we consider a mathematical model for a given age; trees of the same species will attain the same size under optimal conditions for tree growth which is stated as follows due to the form of multiplier function for salinity and density factors:

$$g_i(x) = \frac{1 - ao_i}{1 + e^{di}(tri - x_i)} + ao_i \tag{3}$$

In this model formulation, we state as follows: $g_i(x)$ represents multiplier function for salinity and density factors (tree growth)

Tri	Represents the threshold Value
Dl	Represents the slope of the environment
ao_i	Represent the asymptotic value of the tree growth where (index) represents salinity and density factors.

The multiplier function is determined by the threshold value, the slope and the asymptotic value ensuring test the growth rate remains at a small level even at high values of salinity or tree density.

2.2 Model solution

$$\frac{d(D)}{dt} = \left[\frac{G.D(1-\frac{DH}{D_m H_m})}{2b_1+(3b_2.D)-4b_3.D^2})\right] \tag{4}$$

where $G = Gopt.MuL$

$$\begin{aligned} b_2 &= \frac{2(H_m - b_1)}{D_m} \\ b_3 &= \frac{H_m - b_1}{D_m^2} \\ H &= b_1 + b_2 D - b_3 D^2 \end{aligned}$$

Set $a = H_m - b_1$, so that
$b_2 = \frac{2a}{D_m}, b_3 = \frac{a}{D_m^2}, b_1 = H_m - a$

So that

$$\begin{aligned} 2b_1 + 3b_2D + 4b_3D^2 &= 2b_1 + 6_a\frac{D}{D_m} - \frac{4aD^2}{D^2m} \\ &= 2b_1 + 6_a\frac{D}{D_m} - 4a\left(\frac{D}{D_m}\right)^2 \\ 2b_1 + 3b_2D + 4b_3D^2 &= 2(b_1 + 3_a\left(\frac{D}{D_m}\right) - 2a\left(\frac{D}{D_m}\right)^2 \end{aligned}$$

$$\begin{aligned} H &= b_1 + b_2D - b_3D^2 = b_1 + 2_a\frac{D}{D_m} - a\frac{D^2}{D_m^2} \\ H &= b_1 + 2_a\left(\frac{D}{D_m}\right) - a\left(\frac{D}{D_m}\right)^2 \end{aligned}$$

By letting

$$\begin{aligned} \log_e\frac{D}{D_m} &= x \\ \frac{D}{D_m} &= e^x; D = D_me^x \\ dD &= D_me^xdx \end{aligned}$$

On substituting into equation (4), we have

$$\frac{2b_1 + 2b_2D + 4b_3D^2dD}{D(1 - \frac{DH}{D_mH_m})} = Gdt$$

Inputting the replacement, we have

$$\begin{aligned} \frac{2(b_1 + 3a(\frac{D}{D_m}) - 2a(\frac{D}{D_m})^2)dD}{D(1 - (\frac{D}{D_m)} \cdot (\frac{H}{H_m}))} &= Gdt \\ \frac{2(b_1 + 3a(\frac{D}{D_m}) - 2a(\frac{D}{D_m})^2)dD}{D(1 - (\frac{D}{D_m})) \cdot \frac{(b_1 + 2a(\frac{D}{D_m}) - a(\frac{D}{D_m})^2)}{H_m}} &= Gdt \end{aligned}$$

Substituting $\frac{D}{D_m} = e^x; D = D_me^x; dD = D_me^xdx$
We have,

$$\begin{aligned} \frac{2(b_1 + 3ae^x - 2a(e^x)^2)D_me^xdx}{D_me^x(1 - \frac{e^x}{H_m}(b_1 + 2ae^x - a(e^x)^2))} &= Gdt \\ \frac{2(b_1 + a(3e^x - 2e^{2x})dx}{H_m - (b_1e^x + 2ae^{2x} - ae^{2x})} &= Gdt \\ \frac{2(b_1 + a(3e^x - 2e^{2x}))dx}{a + b_1 - e^x(b_1 + a(2e^x - e^{2x}))} &= Gdt \\ \frac{2(b_1 + a(3e^x - 2e^{2x}))dx}{a + b_1 - e^x(b_1 + a(2e^x - e^{2x}))} &= Gdt \end{aligned}$$

For the purpose of predicting the impact of artisanal refining on mangrove forest vegetation and to reflect the regrowth pattern (re-sprouting capacity) that will be observed in the experimental study, a simple model with mangrove species being characterized by different growth rates, competition coefficient and sensitivities will be considered following [11] and [12] competition models with flexible behavior and species-specific carrying capacity.

In the model formulation, the growth rate of all species is affected (with different sensitivities) by resource level and by available space (species and total space) as well as in increased mortality due to human activities. The system dynamics model is a set of first order ordinary differential equation (ODE) following [12] which is stated as follows:

$$\frac{dB_i}{d_t} = g_i B_i W_i (1 - \frac{B_i}{M_i})(1 - \frac{B_{tot}}{B_{max}}) - (d_i + h_i) \cdot B_i \tag{5}$$

$B_i(0) = B_0 > 0$

Where:

$\frac{dB_i}{d_t}$	Represents the rate of change of mangrove vegetation with respect to time
B_i	Represents mangrove vegetation over time (cover percentage)
i	Represents species index (i = 1 species 1, i = 2, species II, I = 3 species III)
W_i	Represents the growth resource function
g_i	Represents growth rate parameters for each species
M_i	Represents maximal biomass for each species
B_{tot}	Represents sum of all species biomass
B_{max}	Represents max biomass of all species which is the system carrying capacity
D_i	Represents death rate of each species due to natural factor
h_i	Represents extra loss of plant species biomass due to hydrocarbon from artisanal refining

For simplicity, species differ on in their growth rate parameters (g_i), the growth resource effects W_i and species carrying capacity

M_i. The relative cover, palatability values which is the protein content of the leaves as well as the re-sprouting capacity after the effect of hydrocarbon, plants densities as per individual stems per m^2 and the mortality rate due to the hydrocarbon will be modeled under the same post hazard conditions such as plant densities and post hazard treatments.

2.3 Numerical iterations mathematical preliminaries

Following [13], when numerical solutions to initial value problems (IVPs) are required that cannot be obtained by analytical means, it is necessary to use numerical methods. From the numerical methods that exist in solving initial value problems, we have only considered the popular fourth-order Runge-Kutta method in this study as part of the mathematical preliminaries. The mathematical structure and the theoretical definitions of this method are presented as follows: The fourth-order Runge-Kutta (R-K) method is an accurate and flexible method based on a Taylor series approximation to the function $f(x, y)$ in the initial value problem:

$$\frac{dy}{dx} = f(x, y) \tag{6}$$

Subject to the initial condition $y(x_0) = y_0$

The increment h in x may be changed at each step, but is usually kept constant so that after n steps, we have

$$x_n = x_0 + nh \tag{7}$$

The Runge-Kutta algorithm for the determination of the approximation y_{n+1} to $y(x_{n+1})$ is:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{8}$$

Where,

$$\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1) \\ k_3 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2) \\ k_4 &= hf(x_{n+1}, y_n + k_3) \end{aligned}$$

The local error involved in the determination of y_{n+1} from y_n is $0(h^5)$
The above method can be extended to find solution to a system of differential equations such as

$$\frac{dy}{dx} = f(x, y, z) \tag{9}$$

$$\frac{dz}{dx} = g(x, y, z) \tag{10}$$

Subject to the initial condition

$$y(x_0) = y_0 \text{ and } z(x_0) = z_0 \tag{11}$$

These are the types of equations considered by this study which consists of a system of two first order nonlinear differential equations.

At the *nth* integration step, using a step of length h, the Runge-Kutta Algorithm for the system takes the form

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{12}$$

$$z_{n+1} = z_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \tag{13}$$

Where,

$$\begin{aligned} k_1 &= hf(x_n, y_n, z_n) \\ k_2 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}K_1) \\ k_3 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2, z_n + \frac{1}{2}K_2) \\ k_4 &= hf(x_n + h, y_n + k_3, z_n + K_3) \end{aligned}$$

and

$$\begin{aligned} K_1 &= hg(x_n, y_n, z_n) \\ K_2 &= hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1, z_n + \frac{1}{2}K_1) \\ K_3 &= hg(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2, z_n + \frac{1}{2}K_2) \\ K_4 &= hg(x_n + h, y_n + k_3, z_n + K_3) \end{aligned}$$

As with the Runge-Kutta method, the local error involved in the determination of y_{n+1} from y_n and Z_{n+1} from Z_n is

$$0(h^5)0(h^5) \tag{14}$$

It is a good numerical analysis practice that in the event of a complex dynamical system that can not admit an analytic solution for impact and sensitivity analysis on our mangrove forest vegetation: we have to adopt an alternative method to study the qualitative behavior of the impact of hydrocarbon from artisanal refining on mangrove forest biodiversity. This challenging environmental problem will be tackled computationally using MATLAB ODE45 numerical scheme.

$g1 = 0.2; w1 = 0.5; m1 = 0.5; Bm1 = 1; d1 = 0.1; h1 = 0.2;$

$g2 = 0.4; w2 = 0.5; m2 = 0.5; Bm2 = 1; d2 = 0.1; h2 = 0.1;$

$g3 = 0.38; w3 = 0.5; m3 = 0.2; Bm3 = 1; d3 = 0.1; h3 = 0.3;$

3 Results and discussion

3.1 Results

Table 1: Impact of Experimental Time of Mangrove Forest Relative Abundance in the Absence of Artisanal Hydrocarbon Effect

Example	Time (Months)	Site1 RA (N21)	Site2 RA (N22)	Site3 RA (N23)
1	0	0.0100	0.0100	0.0100
2	10.000	0.0246	0.0580	0.0471
3	20.000	0.0500	0.1888	0.1138
4	30.000	0.0800	0.3201	0.1572
5	40.0000	0.1086	0.3952	0.1764
6	50.0000	0.1342	0.4342	0.1854
7	60.0000	0.1569	0.4556	0.1902
8	70.0000	0.1768	0.4681	0.1929
9	80.0000	0.1941	0.4759	0.1946
10	90.0000	0.2091	0.4810	0.1957
11	100.0000	0.2220	0.4845	0.1965
12	110.0000	0.2331	0.4870	0.1970
13	120.0000	0.2427	0.4888	0.1974

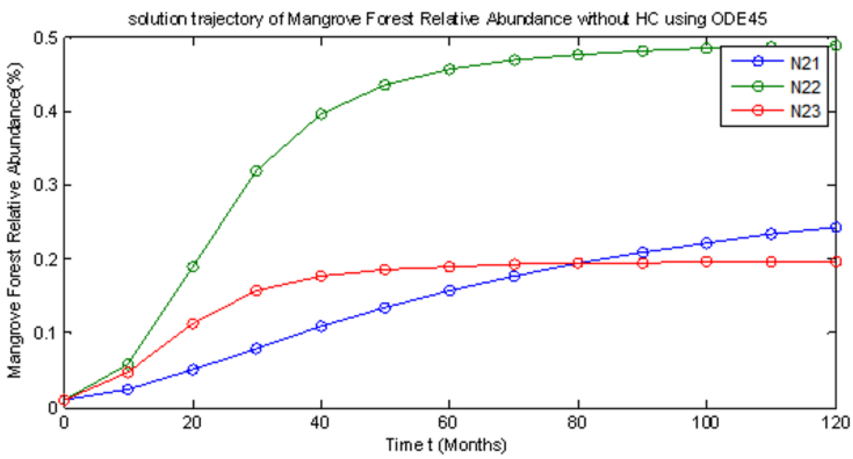

Figure 1: Solution Trajectory of Mangrove Forest Relative Abundance without HC using ODE45

3.2 Discussion of results

In studying the impact of experimental time of mangrove forest relative abundance in the absence of hydrocarbon for a time interval of 0(10)120 months when all model parameter values are fixed, three different mangrove sites which are N21, N22 and N23 in the Niger delta and considered as natural mangrove forest vegetation sites in the absence of artisanal hydrocarbon. From the numerical result obtained, we observed that on the based day of our experimental time here called the initial condition, the relative abundance of the first sites N21 N22 and N23 recorded 0.01 units same values each. Furthermore, from the tenth (10^{th}) month up to the one hundred and twentieth (120^{th}) months, the data base result shows a monotonic increasing pattern in the relative abundance in the coordinates of N21, N22 and N23 of which the value of the coordinates of N21 was recorded as 0.0246 units on the tenth (10^{th}) month to a saturating value of 0.2427 units on the one hundred and twentieth (120^{th}) months. In the same scenario, the same observation is noted on the N22 and N23 coordinates as the value on the N22 coordinates is recorded 0.0580 units on the tenth (10^{th}) month to a saturating value of 0.488 units on the one hundred and twentieth (120^{th}) months as well as N23 coordinates having 0.0471 units on the tenth (10^{th}) month to a saturating value of 0.1974 units on the one hundred and twentieth (120^{th}) months. These observations show a strong increase in the numerical strength of the relative abundance of the N22 coordinates followed by N21 coordinates with a slow increase in numerical strength and a strong increase pattern in the N23 coordinates which converges earlier to its saturated value.

4 Conclusions

These observations show a strong increase in the numerical strength of the relative abundance of the second coordinate followed by the first coordinate with a slow increase in the numerical strength and a strong increase pattern in the third coordinate which converges earlier to its saturated value.

4.1 Recommendations

This study would recommend the following: Characterization of mangrove forest relative abundance and artisanal hydrocarbon in selected crude oil impacted sites, impact of illegally refined crude oil residue on soil microorganism and modeling the effects of artisanal crude oil refining activities on soil microorganisms.

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