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Global stability analysis of endemic equilibrium point of mathematical model for the transmission and control of Zika Virus disease dynamics

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Abstract

We constructed Goh Volterra type of Lyapunov function from the transformed model of our formulated mathematical model for the transmission and control of zika virus disease dynamics. Applying the La Salle's invariance principle to the time derivative of the constructed Lyapunov function confirms the existence of global asymptotic stability of unique endemic equilibrium point of the model. The model is solved using differential transformation method based on two different levels of controls. The Graphical simulations of some vital solutions of the model equations showed the impact of the various control measures incorporated into the model. Moreover, the sensitivity analysis of the sensitive parameters is carried out using the effective reproduction number of the model equations. Various characteristics of the vital state variables and the sensitive parameters provide forecasting premises for any possible outbreak of zika virus disease for effective control measure or containment. This study provides a suitable guide for the containment or control measure of any possible outbreak of zika virus disease. Therefore, with strict adherence to the various measures incorporated into the model equations, the outbreak of zika virus disease can be controlled whenever the effective reproduction number is reduced to less than unity.

Keywords: Endemic equilibrium point, global asymptotic stability; sensitive parameters; Zika Virus disease.

1 Introduction

Zika virus (ZIKV) is a microscopic infectious agent which is a member of the family called Flaviviridae and the genus called Flavivirus that causes zika virus fever in human and microcephaly (smaller head than normal) to the infants of infected pregnant women as in [3]. Zika virus has similar characteristic with Dengue and Chikungunya described in [16]. About 80% of the infectious individuals are asymptomatic in any outbreak of zika virus disease; and some symptoms of the

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infection are similar to the mild form of Dengue, Chikungunya and that of Malaria fever. The incubation period of zika infection is between three to twelve days. The illness is generally mild with symptoms not lasting more than a week, and it rarely kill. Differential or co –clinical diagnosis with Dengue, Chikungunya and Malaria could be considered for an infected individual. To detect Zika virus, a blood or tissue sample from the first week of infection is subjected to sophisticated molecular testing. The four major routes of transmission of zika virus are: through female Aedes aegypti mosquitoes' bite as in[11], [8], and [13]; through sexual transmission as in [5] and [9] while [10] worked on the effectiveness of condom in preventing sexual transmission of zika infection; Vertical transmission from infected pregnant women to their fetuses as in [22], during pregnancy and effects on childhood development, [17] showed the potential of zika virus transmission through blood transfusion; [19], worked on the risk of microcephaly induced by zika virus, while [23] showed the possible risk factor of zika virus for autism spectrum disorder. The aim of this study is to show that the unique endemic equilibrium point obtained in [3] is shown to be globally asymptotically stable, in order to provide a valid guide for the control of any possible outbreak of zika virus disease. This is achieved by using the Goh Volteral type of Lyapunov function, constructed from our transformed model and then showed that the time derivative of the function is less or equal to zero. Applying La Salle's invariance principle shows that the unique endemic equilibrium point is globally asymptotically stable. Moreover, the Graphical simulations of some solutions of the model equations are obtained and the sensitivity analysis of the sensitive parameters is carried out using the effective reproduction number of the model equations.

2 Model Formulation

As in [3], the population of the study involves humans and mosquitoes respectively. Human population is divided into female population and male population. SEIR framework is adopted for the state variables (Susceptible – Expose – Infected – Removed), while SI (Susceptible – Infectious) framework is used for the state variables of the mosquito population.

2.1 Model Equations

$$\dot{S}_1 = \theta_1\omega_1\Lambda_1 - \frac{S_1}{N_1}\{\alpha_1\phi_3 I_3 + \alpha_{21}\phi_{21}I_{21}(1-\epsilon_c\,\tau_c) + \alpha_{22}\phi_{22}I_{22}(1-\epsilon_c\,\tau_c)\} - \mu_1 S_1 \qquad (1)$$

$$\dot{E}_1 = (1-\theta_1)\omega_1\Lambda_1 + \frac{S_1}{N_1}\{\alpha_1\phi_3 I_3 + \alpha_{21}\phi_{21}I_{21}(1-\epsilon_c\,\tau_c) + \alpha_{22}\phi_{22}I_{22}(1-\epsilon_c\,\tau_c)\} - (\gamma_1 + \sigma_{11} + \sigma_{12} + \mu_1)E_1 \qquad (2)$$

$$\dot{I}_{11} = \sigma_{11}E_1 - (\gamma_{11} + \mu_1)I_{11} \qquad (3)$$

$$\dot{I}_{12} = \sigma_{12}E_1 - (\gamma_{12} + \mu_1)I_{12} \qquad (4)$$

$$\dot{R}_1 = (1-\omega_1)\Lambda_1 + \gamma_1 E_1 + \gamma_{11}I_{11} + \gamma_{12}I_{12} - \mu_1 R_1 \qquad (5)$$

$$\dot{S}_3 = \Lambda_3 - \frac{S_3}{N_3}(\alpha_3^{11}\phi_{11}I_{11} + \alpha_3^{12}\phi_{12}I_{12} + \alpha_3^{21}\phi_{21}I_{21} + \alpha_3^{22}\phi_{22}I_{22}) - (\mu_3 + \delta)S_3 \qquad (6)$$

$$\dot{I}_3 = \frac{S_3}{N_3}(\alpha_3^{11}\phi_{11}I_{11} + \alpha_3^{12}\phi_{12}I_{12} + \alpha_3^{21}\phi_{21}I_{21} + \alpha_3^{22}\phi_{22}I_{22}) - (\mu_3 + \delta)I_3 \qquad (7)$$

$$\dot{S}_2 = \theta_2\omega_2\Lambda_2 - \frac{S_2}{N_2}\{\alpha_2\phi_3I_3 + \alpha_{11}\phi_{11}I_{11}(1-\in_c \tau_c) + \alpha_{12}\phi_{12}I_{12}(1-\in_c \tau_c)\} - \mu_2S_2 \qquad (8)$$

$$\dot{E}_2 = (1-\theta_2)\omega_2\Lambda_2 + \frac{S_2}{N_2}\{\alpha_2\phi_3I_3 + \alpha_{11}\phi_{11}I_{11}(1-\in_c \tau_c) + \alpha_{12}\phi_{12}I_{12}(1-\in_c \tau_c)\} - (\gamma_2 + \sigma_{21} + \sigma_{22} + \mu_2)E_2 \qquad (9)$$

$$\dot{I}_{21} = \sigma_{21}E_2 - (\gamma_{21} + \mu_2)I_{21} \qquad (10)$$

$$\dot{I}_{22} = \sigma_{22}E_2 - (\gamma_{22} + \mu_2)I_{22} \qquad (11)$$

$$\dot{R}_2 = (1-\omega_2)\Lambda_2 + \gamma_2E_2 + \gamma_{21}I_{21} + \gamma_{22}I_{22} - \mu_2R_2 \qquad (12)$$

2.2 Model Variables and Parameters

The state variables and parameters of the model equations (1) – (12) are defined in Table 1 and Table 2.

Table 1: Description of variables of the model

Variable	Interpretation
S_1, S_2	Susceptible female, male compartment
E_1, E_2	Exposed female, male compartment
I_{11}, I_{21}	Symptomatic female, male compartment
I_{12}, I_{22}	Asymptomatic female, male compartment
R_1, R_2	Removed female, male compartment
S_3, I_3	Mosquitoes without virus, mosquitoes with virus

Table 2: Description of parameters of system (1) – (12)

Parameter	Interpretation
$\Lambda_1, \Lambda_2, \Lambda_3$	Recruitment rate of females, males and mosquitoes
ω_1, ω_2	Proportion of female and male births without microcephaly
$(1-\omega_1), (1-\omega_2)$	Proportion of female and male births with microcephaly
θ_1, θ_2	Proportion of susceptible female and male births without microcephaly
$(1-\theta_1), (1-\theta_2)$	Proportion of exposed female and male births without microcephaly
μ_1, μ_2, μ_3	Natural death rate of females, males and mosquitoes
δ	Death rate of mosquitoes due to insecticides
α_1, α_2	Transmission rate of ZIKV through mosquito bite to the susceptible females and males
$\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}$	Transmission rate of ZIKV through sex from symptomatic and asymptomatic female and males to susceptible females and males
ϕ_3	Is measuring the reduction in effectiveness of mosquito activities in transmitting ZIKV by creating non conducive environment for the mosquitoes including the use of air conditioner
$\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}$	Is measuring the reduction in effectiveness of sexual transmission through adherence to the preventive instructions
$(1-\in_c \tau_c)$	Reflects the impact of condom usage which is enhanced by public campaign (efficacy and compliance) on sexual transmission where $0 < \in_c, \tau_c < 1$
$\alpha_3^{11}, \alpha_3^{12}, \alpha_3^{21}, \alpha_3^{22}$	Transmission rate of ZIKV from symptomatic and asymptomatic females and males to the susceptible mosquitoes
$\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}$	Progression rate of ZIKV from exposed females and males to the symptomatic and asymptomatic compartment
γ_1, γ_2	Recovery rate from exposed females and males to the removed

	compartment
$\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}$	Recovery rate from symptomatic and asymptomatic females and males to the removed compartment

2.3 The Transformed Model

In order to obtain condition for the global stability, we transformed equations (1) – (12) from standard incidence function to bilinear incidence function and using the substitutes in (26) and (27) to obtain equations (13) – (24).

$$\dot{S}_1 = \theta_1\omega_1\Lambda_1 - S_1(\overline{\beta_1}I_3 + \overline{\beta_2}I_{21} + \overline{\beta_3}I_{22}) - \mu_1 S_1 \tag{13}$$

$$\dot{E}_1 = (1-\theta_1)\omega_1\Lambda_1 + S_1(\overline{\beta_1}I_3 + \overline{\beta_2}I_{21} + \overline{\beta_3}I_{22}) - k_1 E_1 \tag{14}$$

$$\dot{I}_{11} = \sigma_{11}E_1 - k_2 I_{11} \tag{15}$$

$$\dot{I}_{12} = \sigma_{12}E_1 - k_3 I_{12} \tag{16}$$

$$\dot{R}_1 = (1-\omega_1)\Lambda_1 + \gamma_1 E_1 + \gamma_{11}I_{11} + \gamma_{12}I_{12} - \mu_1 R_1 \tag{17}$$

$$\dot{S}_3 = \Lambda_3 - S_3(\overline{\beta_4}I_{11} + \overline{\beta_5}I_{12} + \overline{\beta_6}I_{21} + \overline{\beta_7}I_{22}) - k_4 S_3 \tag{18}$$

$$\dot{I}_3 = S_3(\overline{\beta_4}I_{11} + \overline{\beta_5}I_{12} + \overline{\beta_6}I_{21} + \overline{\beta_7}I_{22}) - k_4 I_3 \tag{19}$$

$$\dot{S}_2 = \theta_2\omega_2\Lambda_2 - S_2(\overline{\beta_8}I_3 + \overline{\beta_9}I_{11} + \overline{\beta_{10}}I_{12}) - \mu_2 S_2 \tag{20}$$

$$\dot{E}_2 = (1-\theta_2)\omega_2\Lambda_2 + S_2(\overline{\beta_8}I_3 + \overline{\beta_9}I_{11} + \overline{\beta_{10}}I_{12}) - k_5 E_2 \tag{21}$$

$$\dot{I}_{21} = \sigma_{21}E_2 - k_6 I_{21} \tag{22}$$

$$\dot{I}_{22} = \sigma_{22}E_2 - k_7 I_{22} \tag{23}$$

$$\dot{R}_2 = (1-\omega_2)\Lambda_2 + \gamma_2 E_2 + \gamma_{21}I_{21} + \gamma_{22}I_{22} - \mu_2 R_2 \tag{24}$$

$$\left.\begin{array}{c} N_1 = S_1 + E_1 + I_{11} + I_{12} + R_1 \\ N_3 = S_3 + I_3 \\ N_2 = S_2 + E_2 + I_{21} + I_{22} + R_2 \end{array}\right] \tag{25}$$

where,
$$\left.\begin{array}{l}\overline{\beta_1}=\frac{\beta_1\mu_1}{\Lambda_1},\overline{\beta_2}=\frac{\beta_2\mu_1}{\Lambda_1},\overline{\beta_3}=\frac{\beta_3\mu_1}{\Lambda_1},\overline{\beta_4}=\frac{\beta_4\mu_3}{\Lambda_3},\overline{\beta_5}=\frac{\beta_5\mu_3}{\Lambda_3}\\ \overline{\beta_6}=\frac{\beta_6\mu_3}{\Lambda_3},\overline{\beta_7}=\frac{\beta_7\mu_3}{\Lambda_3},\overline{\beta_8}=\frac{\beta_8\mu_2}{\Lambda_2},\overline{\beta_9}=\frac{\beta_9\mu_2}{\Lambda_2},\overline{\beta_{10}}=\frac{\beta_{10}\mu_2}{\Lambda_2}\end{array}\right] \tag{26}$$

$$\left.\begin{array}{c}\beta_1=\alpha_1\phi_3,\beta_2=\alpha_{21}\phi_{21}(1-\in_c\tau_c),\beta_3=\alpha_{22}\phi_{22}(1-\in_c\tau_c),\beta_4=\alpha_3^{11}\phi_{11},\beta_5=\alpha_3^{12}\phi_{12},k_6=\gamma_{21}+\mu_2\\ \beta_6=\alpha_3^{21}\phi_{21},\beta_7=\alpha_3^{22}\phi_{22},\beta_8=\alpha_2\phi_3,\beta_9=\alpha_{11}\phi_{11}(1-\in_c\tau_c),\beta_{10}=\alpha_{12}\phi_{12}(1-\in_c\tau_c),k_7=\gamma_{22}+\mu_2\\ k_1=\gamma_1+\sigma_{11}+\sigma_{12}+\mu_1,k_2=\gamma_{11}+\mu_1,k_3=\gamma_{12}+\mu_1,k_4=\mu_3+\delta,k_5=\gamma_2+\sigma_{21}+\sigma_{22}+\mu_2\end{array}\right] \tag{27}$$

3 Global Stability Analysis

A Goh –Volterra type of Lyapunov function is constructed from equations (14) – (24) as in [12] and [18]. Then the constructed Lyapunov function is given by

$$G=\left[\begin{array}{c}\{S_1-S_1^{**}-S_1^{**}\ln\frac{S_1}{S_1^{**}}+E_1-E_1^{**}-E_1^{**}\ln\frac{E_1}{E_1^{**}}+A_1(I_{11}-I_{11}^{**}-I_{11}^{**}\ln\frac{I_{11}}{I_{11}^{**}})+B_1(I_{12}-I_{12}^{**}-I_{12}^{**}\ln\frac{I_{12}}{I_{12}^{**}})+\\ S_3-S_3^{**}-S_3^{**}\ln\frac{S_3}{S_3^{**}}+B_3(I_3-I_3^{**}-I_3^{**}\ln\frac{I_3}{I_3^{**}})+S_2-S_2^{**}-S_2^{**}\ln\frac{S_2}{S_2^{**}}+E_2-E_2^{**}-E_2^{**}\ln\frac{E_2}{E_2^{**}}+\\ A_2(I_{21}-I_{21}^{**}-I_{21}^{**}\ln\frac{I_{21}}{I_{21}^{**}})+B_2(I_{22}-I_{22}^{**}-I_{22}^{**}\ln\frac{I_{22}}{I_{22}^{**}})\}\end{array}\right.$$

The time derivative of the Lyapunov function, G now becomes

$$\dot{G}=\left[\begin{array}{c}\{(\dot{S}_1-\frac{S_1^{**}}{S_1}\dot{S}_1)+(\dot{E}_1-\frac{E_1^{**}}{E_1}\dot{E}_1)+A_1(\dot{I}_{11}-\frac{I_{11}^{**}}{I_{11}}\dot{I}_{11})+B_1(\dot{I}_{12}-\frac{I_{12}^{**}}{I_{12}}\dot{I}_{12})+(\dot{S}_3-\frac{S_3^{**}}{S_3}\dot{S}_3)+B_3(\dot{I}_3-\frac{I_3^{**}}{I_3}\dot{I}_3)+\\ (\dot{S}_2-\frac{S_2^{**}}{S_2}\dot{S}_2)+(\dot{E}_2-\frac{E_2^{**}}{E_2}\dot{E}_2)+A_2(\dot{I}_{21}-\frac{I_{21}^{**}}{I_{21}}\dot{I}_{21})+B_2(\dot{I}_{22}-\frac{I_{22}^{**}}{I_{22}}\dot{I}_{22})\}\end{array}\right.$$

Substitute equations (13) –(16), (18), (19), (20) –(23) in the time derivative of the Lyapunov function

and simplifying gives

$$\dot{G} = \left[\begin{array}{l} \{2\mu_1 S_1^{**} - \mu_1 S_1 - \frac{S_1^{**^2}}{S_1}(\overline{\beta_1} I_3^{**} + \overline{\beta_2} I_{21}^{**} + \overline{\beta_3} I_{22}^{**}) - \mu_1 \frac{S_1^{**^2}}{S_1} + 2k_1E_1^{**} - k_1 \frac{E_1^{**^2}}{E_1} + \frac{E_1^{**}}{E_1} S_1^{**}(\overline{\beta_1} I_3^{**} + \overline{\beta_2}I_{21}^{**} + \overline{\beta_3} I_{22}^{**}) - \\ \frac{E_1^{**}}{E_1} S_1(\overline{\beta_1}I_3 + \overline{\beta_2}I_{21} + \overline{\beta_3}I_{22}) - \frac{I_{11}^{**}}{I_{11}} A_1\sigma_{11}E_1 + A_1k_2I_{11}^{**} - \frac{I_{12}^{**}}{I_{12}} B_1\sigma_{12}E_1 + B_1k_3I_{12}^{**} + B_4k_4I_3^{**} + 2k_4S_3^{**} + \\ S_3^{**}(\overline{\beta}_4 I_{11}^{**} + \overline{\beta}_5 I_{12}^{**} + \overline{\beta}_6 I_{21}^{**} + \overline{\beta}_7 I_{22}^{**}) - \frac{S_3^{**^2}}{S_3}(\overline{\beta}_4 I_{11}^{**} + \overline{\beta}_5 I_{12}^{**} + \overline{\beta}_6 I_{21}^{**} + \overline{\beta}_7 I_{22}^{**}) - \frac{I_3^{**}}{I_3} B_3S_3(\overline{\beta}_4 I_{11} + \overline{\beta}_5 I_{12} + \overline{\beta}_6 I_{21} + \overline{\beta}_7 I_{22}) + \\ 2\mu_2S_2^{**} - \mu_2S_2 - \frac{S_2^{**^2}}{S_2}(\overline{\beta}_8 I_3^{**} + \overline{\beta}_9 I_{11}^{**} + \overline{\beta}_{10} I_{12}^{**}) - \mu_2\frac{S_2^{**^2}}{S_2} + 2k_5E_2^{**} - k_5\frac{E_2^{**^2}}{E_2} + \frac{E_2^{**}}{E_2}S_2^{**}(\overline{\beta}_8 I_3^{**} + \overline{\beta}_9 I_{11}^{**} + \overline{\beta}_{10} I_{12}^{**}) - \\ \frac{E_2^{**}}{E_2} S_2(\overline{\beta_8} I_3 + \overline{\beta_9} I_{11} + \overline{\beta_{10}} I_{12}) - \frac{I_{21}^{**}}{I_{21}} A_2\sigma_{21}E_2 + A_2k_6I_{21}^{**} - \frac{I_{22}^{**}}{I_{22}}B_2\sigma_{22}E_1 + B_2k_7I_{22}^{**} - k_4S_3 - k_4\frac{S_3^{**^2}}{S_3}\} \end{array} \right.$$

Using the following assumptions:
$S_1 \le S_1^{**}, E_1 \le E_1^{**}, I_{11} \le I_{11}^{**}, I_{12} \le I_{12}^{**}, S_2 \le S_2^{**}, E_2 \le E_2^{**}, I_{21} \le I_{21}^{**}, I_{22} \le I_{22}^{**}, S_3 \le S_3^{**}, k_1, k_5 \le 1,$
$\beta_8 > \overline{\beta_8} > \overline{\beta_1}, \beta_8 > \overline{\beta_9}, \beta_8 > \overline{\beta_{10}}, k_1 = k_5 \ge k_4$ and since the arithmetic mean exceeds the geometric mean, the following inequalities from $\dot{G}$ hold:

$$\mu_1 S_1^{**}(2 - \frac{S_1}{S_1^{**}} - \frac{S_1^{**}}{S_1}) \le 0, \frac{E_1^{**}}{E_1} S_1^{**}(\overline{\beta_1} I_3^{**} + \overline{\beta_2} I_{21}^{**} + \overline{\beta_3} I_{22}^{**})(1 - \frac{S_1^{**}}{S_1}) \le 0,$$

$$2k_1E_1^{**}[2 - \frac{E_1^{**}}{E_1} - \frac{S_1}{E_1k_1}(\overline{\beta_1} I_3^{**} + \overline{\beta_2} I_{21}^{**} + \overline{\beta_3} I_{22}^{**})] < 2k_1E_1^{**}[2 - \frac{E_1^{**}}{E_1} - \frac{S_1}{E_1k_1}\frac{(I_3^{**} + I_{21}^{**} + I_{22}^{**})}{\theta_2\omega_2}\frac{\theta_2\omega_2\beta_8}{k_4}],$$

$$2k_1E_1^{**}[2 - \frac{E_1^{**}}{E_1} - \frac{S_1}{E_1}\frac{(I_3^{**} + I_{21}^{**} + I_{22}^{**})}{\theta_2\omega_2}\frac{\theta_2\omega_2\beta_2}{k_4}] = 2k_1E_1^{**}[2 - \frac{E_1^{**}}{E_1} - \frac{S_1}{E_1}\frac{(I_3^{**} + I_{21}^{**} + I_{22}^{**})}{\theta_2\omega_2} R_e] < 0,$$

$$A_1\sigma_{11}E_1^{**}(1 - \frac{I_{11}^{**}E_1}{I_{11}E_1^{**}}) \le 0, B_1\sigma_{12}E_1^{**}(1 - \frac{I_{12}^{**}E_1}{I_{12}E_1^{**}}) \le 0, k_4S_3^{**}(2 - \frac{S_3}{S_3^{**}} - \frac{S_3^{**}}{S_3}) \le 0,$$

$$A_2\sigma_{21}E_2^{**}(1 - \frac{I_{21}^{**}E_2}{I_{21}E_2^{**}}) \le 0, B_2\sigma_{22}E_2^{**}(1 - \frac{I_{22}^{**}E_2}{I_{22}E_2^{**}}) \le 0, \mu_2S_2^{**}(2 - \frac{S_2}{S_2^{**}} - \frac{S_2^{**}}{S_2}) \le 0,$$

$$(\overline{\beta_1}S_1^{**} + \overline{\beta_8}S_2^{**})I_3^{**}[1 - \frac{I_3^{**}(\overline{\beta_4}I_{11} + \overline{\beta_5}I_{12} + \overline{\beta_6}I_{21} + \overline{\beta_7}I_{22})}{I_3(\overline{\beta_4}I_{11}^{**} + \overline{\beta_5}I_{12}^{**} + \overline{\beta_6}I_{21}^{**} + \overline{\beta_7}I_{22}^{**})}] \le 0, S_3^{**}(\overline{\beta_4}I_{11}^{**} + \overline{\beta_5}I_{12}^{**} + \overline{\beta_6}I_{21}^{**} + \overline{\beta_7}I_{22}^{**})(1 - \frac{S_3^{**}}{S_3}) \le 0,$$

$$\frac{E_2^{**}}{E_2}S_2^{**}(\overline{\beta_8}I_3^{**} + \overline{\beta_9}I_{11}^{**} + \overline{\beta_{10}}I_{12}^{**})(1 - \frac{S_2^{**}}{S_2}) \le 0,$$

$$2k_5E_2^{**}[2 - \frac{E_2^{**}}{E_2} - \frac{S_2}{E_2k_5}(\overline{\beta_8}I_3^{**} + \overline{\beta_9}I_{11}^{**} + \overline{\beta_{10}}I_{12}^{**})] < 2k_5E_2^{**}[2 - \frac{E_2^{**}}{E_2} - \frac{S_2}{E_2}\frac{(I_3^{**} + I_{11}^{**} + I_{12}^{**})}{\theta_2\omega_2}\frac{\theta_2\omega_2\beta_2}{k_4}],$$

$$2k_5E_2^{**}[2 - \frac{E_2^{**}}{E_2} - \frac{S_2}{E_2}\frac{(I_3^{**} + I_{11}^{**} + I_{12}^{**})}{\theta_2\omega_2}\frac{\theta_2\omega_2\beta_2}{k_4}] = 2k_5E_2^{**}[2 - \frac{E_2^{**}}{E_2} - \frac{S_2}{E_2}\frac{(I_3^{**} + I_{11}^{**} + I_{12}^{**})}{\theta_2\omega_2} R_e] < 0.$$

Thus, $\dot{G} \le 0$ for $R_e > 1$. Hence G is a Lyapunov function. It follows by LaSalle's invariance principle that every solution to the equations of the model (13) – (24) at the endemic equilibrium point ε_E is globally asymptotically stable as explained in [14], in the stability of the dynamical system.

4 Model Solution

Differential transformation method is used in solving the differential equations (15), (16), (19), (22) and (23) as used in [21] and [1]. If the model equation $q(t)$ be a function, then it can be expanded in Taylor series about a point t = 0 as $q(t)=\sum_{k=0}^{\infty}\frac{t^k}{k}\left[\frac{d^kq}{dt^k}\right]_{t=0}$ (28)

$$where, q(t)=\{s_1(t),e_1(t),i_{11}(t),i_{12}(t),r_1(t),s_3(t),i_3(t),s_2(t),e_2(t),i_{21}(t),i_{22}(t),r_2(t)\}$$

The differential transformation of $q(t)$ is defined as $Q(t)=\frac{1}{k!}\left[\frac{d^kq}{dt^k}\right]_{t=0}$ (29)

$$where, Q(t)=\{S_1(t),E_1(t),I_{11}(t),I_{12}(t),R_1(t),S_3(t),I_3(t),S_2(t),E_2(t),I_{21}(t),I_{22}(t),R_2(t)\}$$

The inverse differential transform is $q(t)=\sum_{k=0}^{\infty}t^kQ(t)$ (30)

With the non-negative initial condition and parameters, the recurrence relation of the variables in the program (31) is used for the solutions of the model equations

$$\left.\begin{array}{c} I_{11_{k+1}}=\frac{1}{k+1}[\sigma_{11}E_{1_k}-k_2I_{11_k}] \\ I_{12_{k+1}}=\frac{1}{k+1}[\sigma_{12}E_{1_k}-k_3I_{12_k}] \\ I_{3_{k+1}}=\frac{1}{k+1}\{\sum_{m=0}^{k}S_{3_m}(\overline{\beta_4}I_{11_{k-m}}+\overline{\beta_5}I_{12_{k-m}}+\overline{\beta_6}I_{21_{k-m}}+\overline{\beta_7}I_{22_{k-m}})-k_4I_{3_k}\} \\ I_{21_{k+1}}=\frac{1}{k+1}[\sigma_{21}E_{2_k}-k_6I_{21_k}] \\ I_{22_{k+1}}=\frac{1}{k+1}[\sigma_{22}E_{2_k}-k_7I_{22_k}] \end{array}\right] \quad (31)$$

to obtain the respective values of the vital state variables, when k = 0, k=1, k=2 and k=3. Using the demographic and epidemic data of zika virus disease outbreak of Cape Verde in 2016 as premises for the initial values for the model solution, subject to two conditions of 99% and 70% level of the effectiveness at which mosquitoes transmit zika virus.

Table 3: Solutions of System when k = 0

Symbol	**Case one value**	**Case two value**
I_{11_1}	1006.4	1006.4
I_{12_1}	1955.0	1955.0
I_{3_1}	-14511.6	-19057.6
I_{21_1}	998.1	998.1
I_{22_1}	1937.1	1937.1

Table 4: Solutions of System when k = 1

Symbol	**Case one value**	**Case two value**
I_{11_2}	-561.5	-564.8
I_{12_2}	-1571.9	-1580.4
I_{3_2}	5709.8	5975.7
I_{21_2}	-558.8	-558.6
I_{22_2}	-1563.3	-1563.0

Table 5: Solutions of System when k = 2

Symbol	Case one value	Case two value
I_{11_3}	165.6	166.8
I_{12_3}	513.8	517.3
I_{3_3}	-2785.1	-2004.5
I_{21_3}	212.0	205.0
I_{22_3}	630.9	613.1

Table 6: Solutions of System when k = 3

Symbol	Case one value	Case two value
I_{11_4}	-48.6	-47.2
I_{12_4}	-147.1	-143.6
I_{3_4}	1074.6	734.9
I_{21_4}	-49.4	-48.6
I_{22_4}	-154.2	-144.2

The following solutions: (32) – (36) for the case one with the condition of 99%, and (37) – (41) for the case two with the condition of 70% when applying the inverse differential transform are obtained

4.1 Model Solution for Case One

$$i_{11}(t) = 761 + 1006.4t - 280.8t^2 + 55.2t^3 - 12.2t^4 \tag{32}$$

$$i_{12}(t) = 3043 + 1955t - 785t^2 + 171.3t^3 - 36.8t^4 \tag{33}$$

$$i_3(t) = 250,000 - 14,511.6t + 2,854.9t^2 - 928.4t^3 + 268.7t^4 \tag{34}$$

$$i_{21}(t) = 755 + 998.1t - 279.4t^2 + 70.7t^3 - 12.4t^4 \tag{35}$$

$$i_{22}(t) = 3021 + 1937.1t - 781.7t^2 + 210.3t^3 - 38.6t^4 \tag{36}$$

4.2 Model Solution for Case Two

$$i_{11}(t) = 761 + 1006.4t - 282.4t^2 + 55.6t^3 - 11.8t^4 \tag{37}$$

$$i_{12}(t) = 3043 + 1,955t - 790.2t^2 + 172.4t^3 - 35.9t^4 \tag{38}$$

$$i_3(t) = 250,000 - 19,057.6t + 2,987.9t^2 - 668.2t^3 + 183.7t^4 \tag{39}$$

$$i_{21}(t) = 755 + 998.1t - 279.3t^2 + 68.3t^3 - 12.2t^4 \tag{40}$$

$$i_{22}(t) = 3021 + 1,937.1t - 781.5t^2 + 204.4t^3 - 36.1t^4 \tag{41}$$

5 Graphical Simulation of the Solutions of the Model Equations

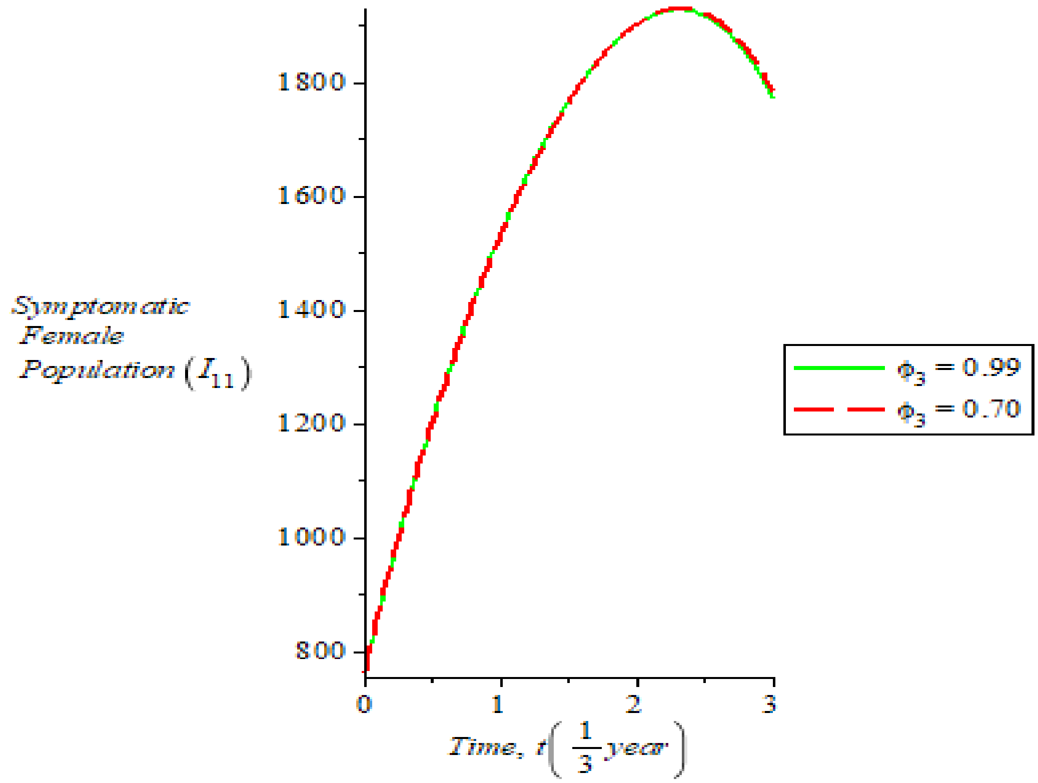

Figure 1: Effect of Reduced Rate in the effectiveness of Mosquito in Transmitting Zika Virus on the Symptomatic Female Population

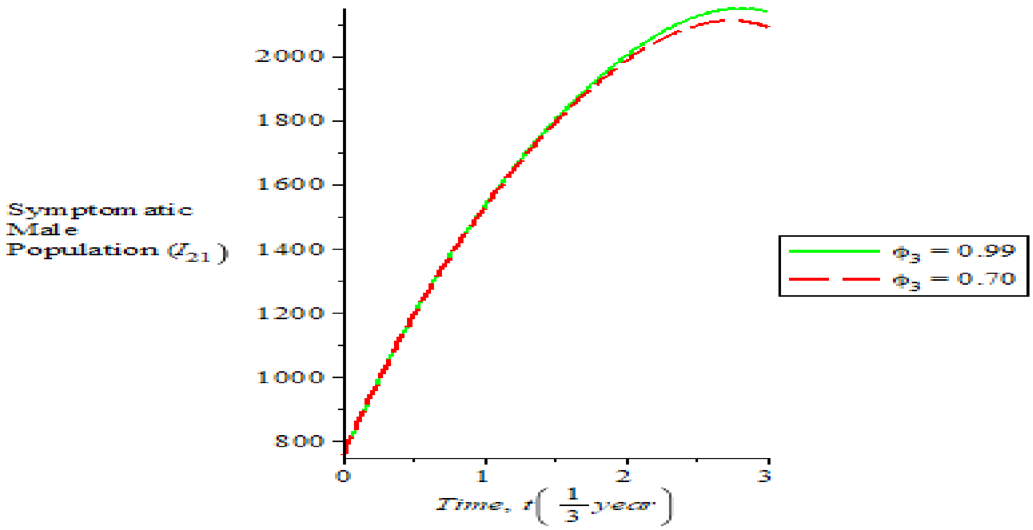

Figure 2: Effect of Reduced Rate in the effectiveness of Mosquito in Transmitting Zika Virus on the Symptomatic Male Population

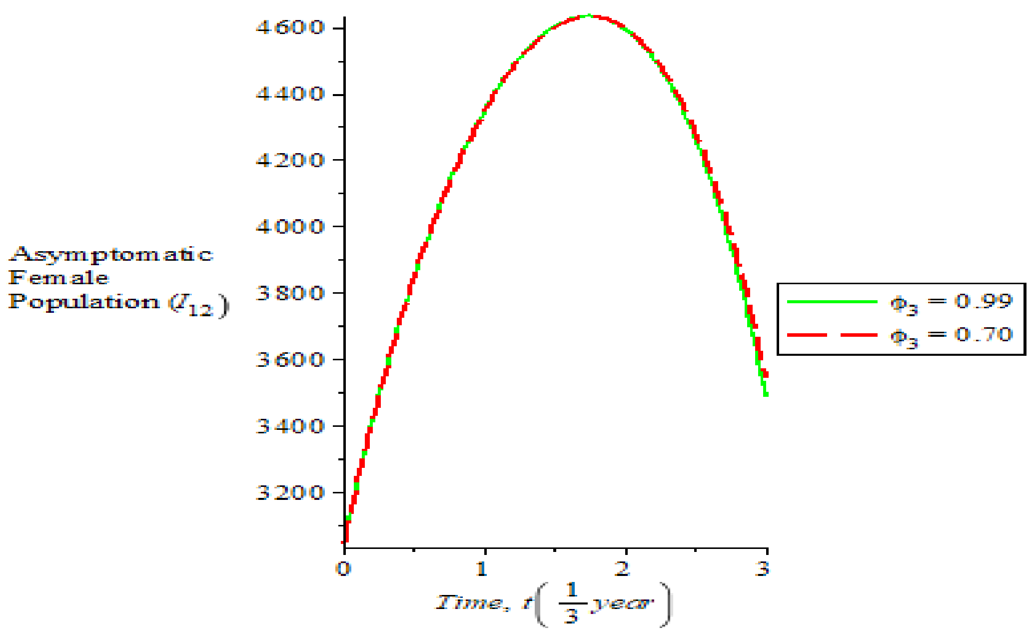

Figure 3: Effect of Reduced Rate in the effectiveness of Mosquito in Transmitting Zika Virus on the Asymptomatic Female Population

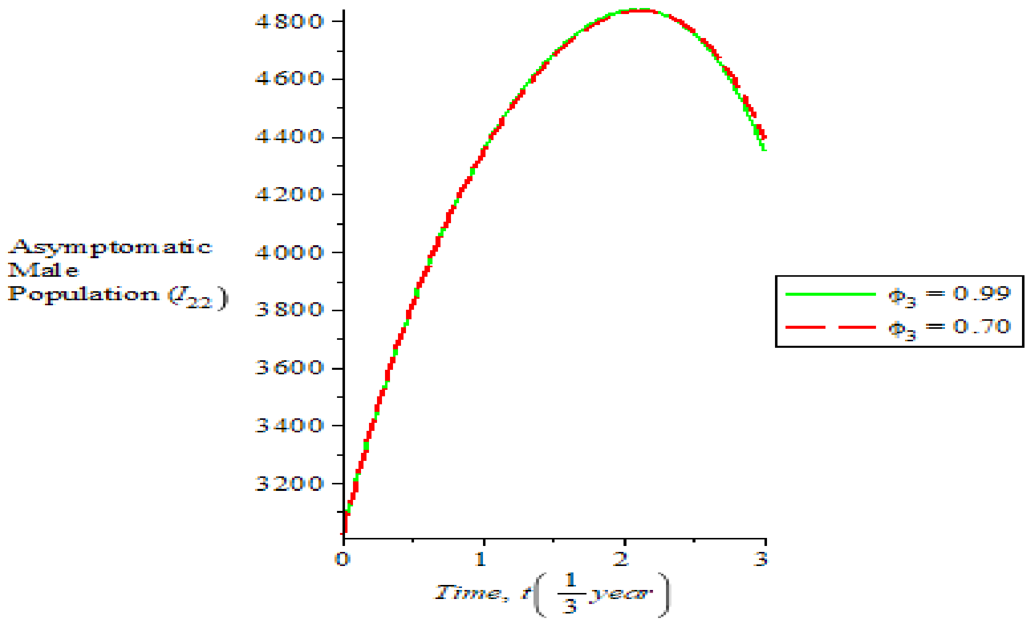

Figure 4: Effect of Reduced Rate in the effectiveness of Mosquito in Transmitting Zika Virus on the Asymptomatic Male Population.

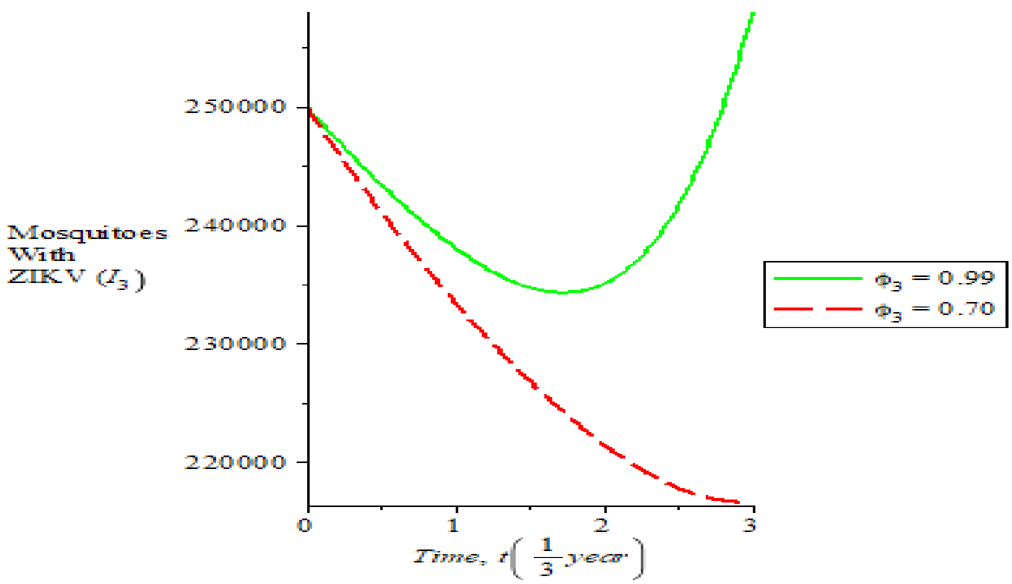

Figure 5: Effect of Reduced Rate in the effectiveness of Mosquitoes in Transmitting Zika Virus on the Population of Mosquitoes with Zika Virus.

6 Sensitivity Analysis of the Effective Reproduction Number

To bring out the importance of the model parameters towards a sound prediction, sensitivity analysis is carried out in a similar approach to the techniques used in [6], [7], [15] and [2]. The sensitivity index of the system is given by $S_{\rho}^{R_0} = \frac{\partial R_0}{\partial \rho} x \frac{\rho}{R_0}$ using the derived effective reproduction number in [3] as given by

$$R_e = \frac{\theta_2 \omega_2 \alpha_2 \phi_3}{\mu_3 + \delta}; \rho = \{\theta_2 = 0.9997, \omega_2 = 0.9982, \alpha_2 = 0.33, \phi_3 = 0.99, \mu_3 = 0.067, \delta = 0.05\} \Rightarrow R_e = 2.78644$$

The sensitive parameters' values are derived from reported cases of zika virus outbreak in Cape (Capo) –Verde in 2016 as in [20] and [24]

Table 7: Sensitivity Indices for the Sensitive Parameters Using the Basic (Effective) Reproduction Number (R_e)

Parameter	Sensitivity index
θ_2	1.000000000
ω_2	1.000000000
ϕ_3	0.9999999999
α_2	0.9999999999
μ_3	-567.1910080
δ	-567.1910080

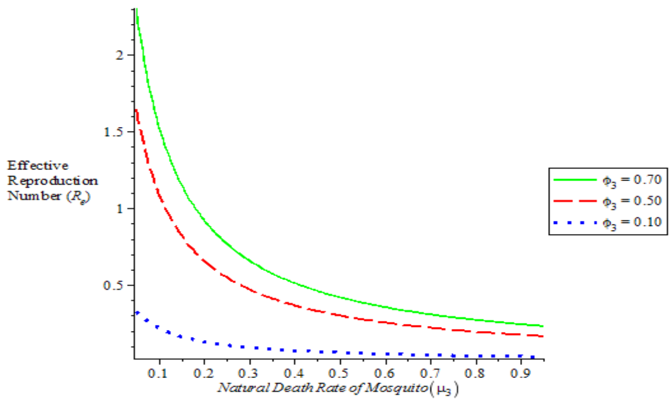

Figure 6: Effect of natural death rate of mosquitoes on the effective reproduction number.

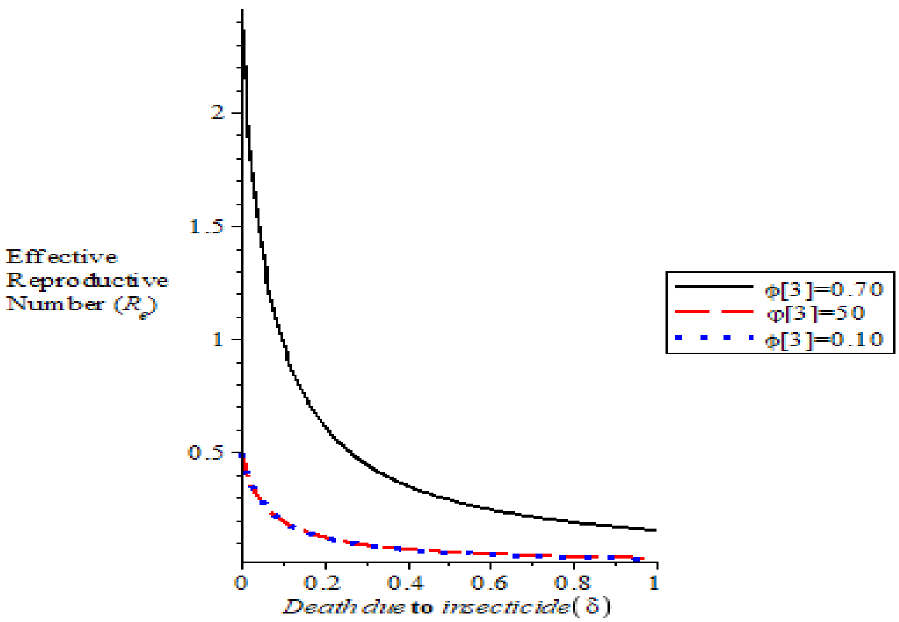

Figure 7: Effect of Death of Mosquitoes due to insecticide on the effective reproduction number

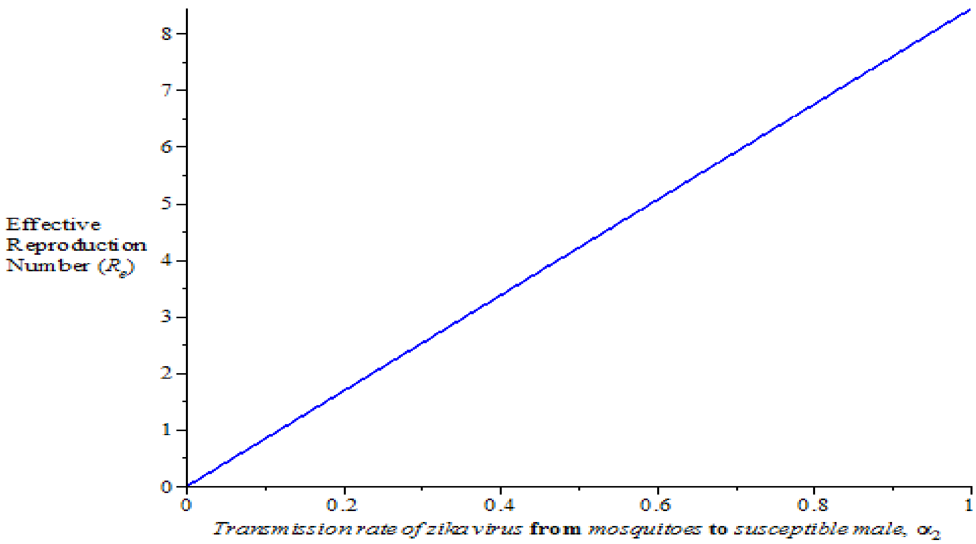

Figure 8: Effect of transmission rate of Zika Virus through mosquito bite on the effective reproduction number

7 Results and Discussion

(i) The time derivative of the constructed Lyapunov function shown to be less than or equal to zero implies that the endemic equilibrium point of the model (13) –(24) is globally asymptotically stable.

(ii) Figures 1 - 4 initially show upward trends as a result of onward progression from the human exposed compartment including females and males, to the infetious compartment (symptomatic and asymptomatic) till peaks of the curves are attained. The direction of the graphs changed from upward to downward because the exposed population is exhausted as a result of reduction in the effectiveness of mosquitoes in transmitting zika virus.

(iii) Figure 5 shows that 70% control of reduction in the effectiveness of mosquitoes in transmitting Zika Virus is more effective in reducing the population of mosquitoes with Zika Virus than 99% control.

(iv) Figures 6 and 7 show that the death of mosquitoes, either natural or through application of insecticides reduces the size of effective reproduction number of the model which implies reduction in the transmission of zika virus. Moreover, if the values of the parameters with negative sensitivity indices increase then the transmission will fall, while increase in the variable with positive sensitivity indices as in Figure 8 will increase the disease but reduction in the variable will make the disease to fall and make the disease to die out if sustained.

8 Conclusion

It is observed that the endemic equilibrium point of the model is globally asymptotically stable when applying LaSalle's invariance principle to the time derivative of Goh Volterral type of Lyapunov function that is shown to be less or equal to zero. The epidemiological implication of the global stability is that the inevitable fate of the outbreak is attained and stabilized irrespective of starting situation. If at the start of the outbreak the number of the infected individual is less than the eventual endemic equilibrium point of the model, then the number of infected individuals will be on the increase until the equilibrium point is attained. On the other hand if the start of the outbreak is higher than the endemic equilibrium point of the model then with strict adherence to the various measures and controls incorporated into the model the size of infections will fall to the equilibrium point of model, which is the inevitable fate of the outbreak. Moreover if the effort is sustained and the effective reproduction number is made to reduce to less than unity, then the outbreak will be brought under control and the disease will die out.

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References

[1] Abdel–Halim Hassan, I. H. (2008): Application to differential transformation for solving systems of differential equations. Journal of Applied Mathematical Modeling, 32(12), 2552 – 2559. https://doi.org/10.1016/*j.apm*.2007.09.025

[2] Abdulrahman, S., Akinwande, N. I., Awojoyegbe, O. B and Abubakar, U. Y. (2013) Sensitivity analysis of the parameters of a mathematical model of Hepatitis B virus transmission, Universal Journal of Applied Mathematics*,* 1(4), 230 – 240.

[3] Adeyemo, K. A., Akinwande, N. I., Abdulrahman, S., and Kuta, F.A. (2018) Formulation of a mathematical model for the transmission and control of zika virus fever dynamics. J. Appl. Sci. Environ. Manage. Vol.22(10) 1363 -1368. DOI:https://dx.doi.org/10.4314/jasem.v22i10.14

[4] Adeyemo, K. A. Akinwande, N.I. and Aye, P. O. (2018) Stability analysis of disease free equilibrium point for transmission and control of zika virus fever dynamics, International Journal of Science Technology*,* 6(11), 34 -40.

[5] Agusto, F. B., Bewick, S. Fagan, W.F. (2017): Mathematical model for Zika virus dynamics with sexual transmission route. Ecological Complexity*,* 29:61-81. http://dx.doi.org/10.1016/j.ecocom.2016.12.007

[6] Ariola, I and Hyman, J. (2007): Forward and adjoint sensitivity analysis with applications in Dynamical system, Lecture Notes in Linear Algebra and Optimization.

[7] Chitnis, N., Hyman J. M., and Cushing, J. M. (2008): Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model, Bulletin of Mathematical Biology, doi: 10.1007/s11538 – 008 – 9299 – 0.

[8] Funk, S., Kucharski, A. J., Camacho, L. Y. Eggo, A. and Edmunds, W. J. (2016) Comparative analysis of Dengue and Zika outbreaks reveals the differences by setting and virus. PLOS Neglected Tropical Disease, 108(12): e0005173. doi:10.1371/journalpntd.0005173.e0005173.ecollection2016Dec

[9] Gao, D. Lou, Y. He, D. Porce, T. C. Kuang, Y. Chowell, G. Ruan, S. (2016) Prevention and control of Zika a mosquito- borne and sexually transmitted disease: A Mathematical analysis, Scientific Reports, 6:28070. Doi: 10.1038/srep28070.

[10] Holmes, K. K., Levine, R. and Weaver, M. (2004): Effectiveness of condoms in preventing sexually transmitted infections. Bulleting of the World Health Organization, 82, 454 – 461.

[11] Isea, R and Lonngren, K. E. (2016) A preliminary mathematical model for the dynamic transmission of Dengue, Chikungunya and Zika, American Journal of Modern Physics and Application. 3(2): 11-15.

[12] Korobeinikov, A. (2004): Lyapunov functions and global properties for SEIR and SEIS Epidemic models, Mathematics Applied in Medicine and Biology. 1: 1 – 9.

[13] Kucharski, A. J. Funk, S. Eggo, R. M. M. Mallet, H. P. Edmunds, W. J. Nilles E. J. (2016) transmission dynamics of zika virus in island populations: A modeling analysis of the 2013-2014 French Polynesia outbreaks. doi: http//dx.org/10.1101/038588

[14] LaSalle, J. and Lefschetz, S. (1976) The stability of the dynamical system, SIAM, Philadelphia

[15] Mikuchi, M. A. (2012) Sensitivity analysis of the basic reproduction number and other qualities for infectious disease models. An *M.Sc. Thesis*, Colorado state University Fort Collins, Colorado, Spring. 116

[16] Musso, D. Cao - Lormeau, V. M. Gubler. D. J. (2015) Zika virus following the path of Dengue and Chikunguny*a*, The Lancent, 386(9990): 243-244.

[17] Musso, D., Nhan, T., Robin, E. Bierlaire, D. Zisou, K. Shan, Y. A. Cao- Lormeau, V. M. Broult, J. (2014 Potential for Zika virus transmission through blood transfusion demonstrated during an outbreak in French Polynesia November 2013 to February 2014, Eurosurveillance, 19: 20761

[18] Okuonghae, D. and Korobeinikov, A. (2007) Dynamics of tuberculosis: the effect of direct observation therapy strategy (DOTS) in Nigeria. Math. Model of Nat. Phenon.*,* 2(1): 99 – 111.

[19] Pizza, D. M. M. Loaiza, A. M. Montoya, J.F. A. Manrique, O. A. Sossa, V. A. Munoz, C. A. A. Raigosa, S. Contreras, H. M Contreras, I. P. Perea M. E. C and Enriquez, M. A. (2016) A model for the risk of microcephaly induced by the Zika Virus (ZIKV), Open Journal of Modeling and Simulation*,* 4: 109 - 117. http://dx.doi.org/10.4236/ojmsi.2016.43010.

[20] Rodrigues, F. N. (2018) Epidemiology of Zika Virus Outbreak in the Cabo Verde Islands, West Africa, PLOS Currents, doi: 10.1371/current outbreaks.19433be4d007451c691f138e1e67e8c.

[21] Soltanalizadeh, B (2012). Application of differential transformation method for solving a fourth order parabolic partial differential equations. International Journal of Pure and Applied Mathematics, 78 (3): 299 – 308.

[22] Subissi, L. Dub, T. Besnard, M. Mariteragi –Helle, T. Nhan, T. Lutringer –Magnin, D. and Mallet, H. (2018) Zika virus infection during pregnancy and effects on early childhood development, French Polynesia, 2013 -2016, Emerging Infectious Disease, 24(10), 1850-1858, https://dx.doi.org/10.3201/eid2410.172079

[23] Vianna, P. Gomes, J.D.A. Boquett, J. A Fraga, L.R. Schuch, J.B., Vianna, F.S.L and Schuler –Faccine, L. (2018) Zika virus as a possible risk factor for autism spectrum disorder,

Neuroimmunmodulation, 25(5-6): 320 -327.
[24] World Population Prospects (2018): Cape–Verde population, 2018(Demographics, Maps, Graphs). World populationreview.com/countries/cape –verde –population/ July 11 2019.