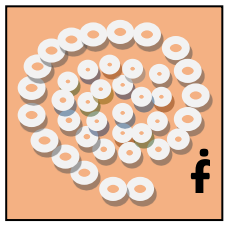

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Numerical investigations on Dengue fever model through singular and non-singular fractional operators

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Abstract

Untreated Dengue fever is a big issue today because it is caused by the Dengue virus which is considered a highly dangerous and deadly viral infection. In other words, the purpose of this research is to formulate new fractional derivative model that can explain what leads to Dengue fever outbreaks. Mathematical techniques along with the information found here play an important role in helping society overcome some diseases. Its basic aim is to describe how the recommended Arbitrary-order Dengue fever disease model works and the methods used to create computerized versions of it. It represents the likelihood that the virus will be spread to a general population. The research explored Dengue fever using three types of arbitrary-order derivative operators: Caputo with power law, Caputo–Fabrizio with exponential decay and Atangana–Baleanu with a Mittag–Leffler function. The arbitrarily ordered system is studied using fixed-point theory, we examined the basic reproduction number and its stability as well. Systematic studies are done to determine the numerical solutions of Dengue fever using three different types of numerical approaches. A study for numerical simulation looked at the influence of and on the spread of Dengue fever. It was shown that vaccinating more people and decreasing contact rates lead to less Dengue fever in human populations.

Keywords and phrases: existence and uniqueness of solutions; fractional; Dengue fever model; numerical simulation.

1 Introduction

The dengue virus is typically the reason behind dengue fever which is referred to as a vector-borne disease. Parts of the Flavivirus family are the DENV 1 to DENV4 serotypes. Most countries encounter this disease as a health risk Apart from the mentioned regions, the Americas, subtropics, Eastern Mediterranean, Africa and above all, the Western Pacific and Southeast Asia, are where dengue is the most widespread [1, 2]. With dengue fever, mosquitoes are responsible for the second-highest

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death toll and have infected over 390 million people worldwide [1, 2, 3].According to the report published in 2012, dengue fever is a threat to 100 or more nations [4]. Many mosquitoes such as Aedes and mostly the Aegypti, can pass the dengue virus to people. About dengue fever, mild illness or even death may happen and those who recover usually do so within one to two weeks of having the fever[5]. A few people who contract dengue may experience dengue shock syndrome (DSS) or hemorrhagic fever (DHF).[6] WHO reports that dengue hemorrhagic fever (DHF) cases are increasing every year [7].

Carriers of the dengue virus (the female mosquito) spread the disease by biting people [8]. The virus is caught when a mosquito takes blood from someone who has malaria and then passes it on to more healthy people. Also, a person who recovers from one DENV serotype can be partially or totally immune to other serotypes [6]. So far, there is no proven treatment for dengue virus except giving fluids and also some old forms of treatments [9]. Treatment is still unavailable to people infected with dengue virus and an effective vaccine for preventing dengue is yet to be offered. WHO has some recommendations on developing a vaccine for the dengue virus, but currently there is no commercially available vaccine against it [10]. In 2015, an analyst pointed out that Mexico was the first country to develop the dengue vaccine [10].

By increasing the order of derivatives or integrals, calculus of arbitrary-order derivatives enters the field. Because of its practical and real-world benefits, fractional calculus (FC) became widely used in applied mathematics. Just as the operators in mathematical analysis do, arbitrary-order operators can better represent how hereditary and memory are seen in real cases. In fields of engineering, physics, bio-mathematics, as well as signal processing, arbitrary-order derivatives are very useful [11, 12, 13, 14, 15, 16, 17, 18, 19].

FC includes many directly computed derivatives and these are broadly classified as singular and non-singular. Most of the time, you will encounter Caputo and Riemann–Liouville operators [20] which use a power law as their kernel. These days, power laws are widely used to describe and model various fields. Previously, the proof of power laws depended on the existence of a single non-zero nth derivative; this is changing because, lately, power laws have been successfully applied using kernels made of many non-zero derivatives[21, 22]. Since it uses a kernel of the exponential law, the Caputo–Fabrizio (CF) derivative is frequently applied in fractional modeling [22]. Another usual choice for real problems is the Atangana–Baleanu (AB) fractional operator which works with a Mittag–Leffler-type kernel with non-singular and non-local features [23]. A number of studies have shown that variable-order derivative operators are important in handling mathematical models with Gaussian, exponential and power kernels [23, 24, 25, 26, 27, 28, 29]. There are times when solving a problem exactly is very difficult. Because of this situation, researchers are naturally more interested in exploring fractional operators and using numbers to solve problems. A number of methods have been introduced to address arbitrary-order differential equations (DEs) [30, 31, 32, 33, 16]. Biologists have recently given more focus to studying infectious diseases with fractional operators and a number of outstanding studies have been published such as works cited in [34, 35, 36, 37, 38, 39].

A mathematical model of Dengue fever was formulated in this paper using fractional derivative operators and three kinds of kernels[1].. The fractional operators include their approximate values which are obtained through suggested numerical ways. The main points of the rest of the work are gathered as follows: Section 2 introduces the mathematical setup for variable-order arbitrary derivatives. It concisely explains what classical Dengue fever looks like. Afterward, part of the paper focuses on classical systems calculated with a variable Caputo derivative and proposes a solution algorithm for the Caputo equation. In Sections 3.3 and 3.4 of the Dengue fever study, the variable-order derivative from Caputo–Fabrizio is applied and the necessary conditions for the existence and uniqueness of solutions are studied as well as a numerical method. We examine in Section 4 Dengue fever disease by using the AB operator. You can see the results in graph form in Section 5. Paramount outcomes are detailed in Section 6.

2 Model formulation

Those with Dengue fever are mainly separated into eight groups according to the integer-order model. Susceptible individuals(S_h), Exposed individuals (E_D),Dengue fever infected individuals (I_D), individuals on Dengue fever treatment (T_D),Vaccinated human population against Dengue fever (V),Recovered human population from Dengue fever (R_D),susceptible vectors (S_V), and infected vectors(I_V).

Susceptible humans (S_h) to Dengue fever are recruited at the rate of (Λ_h) , The group decreases due to the natural death rate (μ_h) among humans and vaccination rate (τ_2),the rate (α) with which recovered individuals become vulnerable to re-infection is why the infection can spread, and vaccine failure rate(τ_1) , Several elements influencing the variable are the effectiveness of Dengue in infecting its hosts plus changes in the number of vectors in the area (β_D, β_V) respectively.
Consequently, we established the dynamics of susceptible individuals as follows:

$$\frac{dS_h}{dt} = \pi_h + \alpha R_D + \tau_1 V - \lambda_D S_h - (\tau_2 + \mu_h) S_h.$$

Exposed humans to Dengue fever (E_D): The population encounter more new cases which causes the infection rate to increase at the rate of (β_D, β_V) respectively. The rate of exposed humans becoming fully infected with Dengue fever determines the decrease of this class (ω) and by natural death rate (μ_h).

The dynamics of this population is hereby presented as:

$$\frac{dE_D}{dt} = \lambda_D S_h - (\omega + \mu_h) E_D.$$

Dengue fever Infected humans (I_D) : Numbers keep increasing because people with Dengue fever continue to pass the disease to others at the rate of(ω), Natural death causes the human population size to decrease (μ_h) , Tropical diseases such as dengue fever, led to human deaths at given death rates (δ_1) , and treatment rate of infected humans (ρ) .
The dynamics of this population is hereby presented as:

$$\frac{dI_D}{dt} = \omega E_D - (\rho + \delta_1 + \mu_h) I_D.$$

The growth of humans who receive Dengue fever treatment (T_D) depends on the number of infected individuals receiving treatment at rate (ρ), The population numbers reduces as people recover from their Dengue fever infections due to treatment at the rate of (σ) The diseases induced death rate of humans on treatment and natural death rate are (δ_2) and (μ_h) respectively.
The dynamics of this population is hereby presented as:

$$\frac{dT_D}{dt} = \rho I_D - (\sigma + \delta_2 + \mu_h) T_D.$$

Vaccinated human population against Dengue fever (V) : The number of people who can get Dengue fever rises when vaccination rates (τ_2) decrease, The population reduces in the course of natural fatalities (μ_h) and waning rate of vaccine (τ_1) respectively. The dynamics of this population is hereby presented as:

$$\frac{dV}{dt} = \tau_2 S_h - (\tau_1 + \mu_h) V.$$

Recovered human population from Dengue fever (R_D) : More people were living with Dengue through determining the recovery rates (σ) of those receiving treatment, The number of recovered humans decreases due natural death occurs at(μ_h) , and the rate of re-susceptibility among humans as well as affect the time needed to relapse into the infection state (α) respectively.
The dynamics of this population is hereby presented as:

$$\frac{dR_D}{dt} = \sigma T_D - (\alpha + \mu_h) R_D.$$

Susceptible vector population (S_V) are recruited at the rate of (Λ_V) , The group decreases due to the rate at which vectors bite humans at the rate of (β_V) and natural death rate among vector population (μ_V). The dynamics of this population is hereby presented as:

$\frac{dS_V}{dt} = \pi_V - \lambda_V S_V - \mu_V S_V$.

Infected vector population (I_V): The group increases due to the rate at which vectors bite humans at the rate of (β_V) , the group also decreases due to the rate at which vectors died due to an attempt to bite humans(δ_V) and natural death rate among vector population (μ_V). The dynamics of this population is hereby presented as:

$\frac{dI_V}{dt} = \lambda_V S_V - (\delta_V + \mu_h) I_V$.

2.1 Model flow diagram

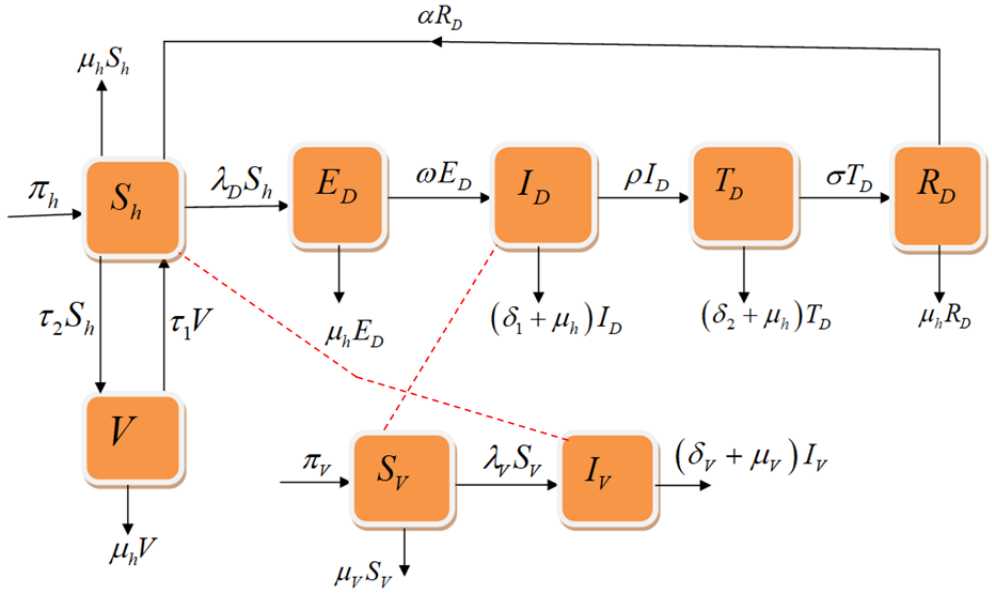


Figure 1: model schematic flow diagram.

2.2 Model equation

$$\left.\begin{aligned}
\frac{dS_h}{dt} &= \pi_h + \alpha R_D + \tau_1 V - \lambda_D S_h - (\tau_2 + \mu_h) S_h, \\
\frac{dE_D}{dt} &= \lambda_D S_h - (\omega + \mu_h) E_D, \\
\frac{dI_D}{dt} &= \omega E_D - (\rho + \delta_1 + \mu_h) I_D, \\
\frac{dT_D}{dt} &= \rho I_D - (\sigma + \delta_2 + \mu_h) T_D, \\
\frac{dV}{dt} &= \tau_2 S_h - (\tau_1 + \mu_h) V, \\
\frac{dR_D}{dt} &= \sigma T_D - (\alpha + \mu_h) R_D, \\
\frac{dS_V}{dt} &= \pi_V - \lambda_V S_V - \mu_V S_V, \\
\frac{dI_V}{dt} &= \lambda_V S_V - (\delta_V + \mu_h) I_V.
\end{aligned}\right\} \quad (1)$$

where $\lambda_D = \frac{q\beta_D I_V}{N_h}$ and $\lambda_V = \frac{\beta_V I_D}{N_h}$.

2.3 Disease free equilibrium point

Disease-free equilibrium is a point where by all people in the population are free of illness symptoms. At DFE, $(S_h \neq 0, E_D = 0, I_D = 0, T_D = 0, V \neq 0, R_D = 0, S_V \neq 0, I_V = 0)$

$$(S_h, E_D, I_D, T_D, V, R_D, S_V, I_V) = \left(\frac{(\tau_1 + \mu_h)\pi_h}{\mu_h(\tau_2 + \tau_1 + \mu_h)}, 0, 0, 0, \frac{\tau_2 \pi_h}{\mu_h(\tau_2 + \tau_1 + \mu_h)}, 0, \frac{\pi_V}{\mu_V} 0 \right). \quad (2)$$

2.4 Dengue fever basic reproduction number

Basic Reproduction number, measures all the new infections that occur when someone with the disease infects others. Next generation method $R_0^D = \rho FV^{-1}$ is applied to determine by using F as a non-negative matrix with other transition terms V and determining the largest Eigen value ρ.

$$F = \begin{pmatrix} 0 & 0 & 0 & \frac{b\beta_D(\tau_1+\mu_h)}{(\tau_2+\tau_1+\mu_h)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{\beta_V\Lambda_V\mu_h}{\Lambda_h\mu_V} & 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} P_2 & 0 & 0 & 0 \\ -\omega & P_3 & 0 & 0 \\ 0 & -\rho & P_4 & 0 \\ 0 & 0 & 0 & P_8 \end{pmatrix}$$

We now obtain:

$$R_0^D = \frac{\sqrt{\Lambda_h\mu_V P_2P_3\left(\tau_2+\tau_1+\mu_h\right)P_8\beta_V\Lambda_V\mu_h\omega b\beta_D\left(\tau_1+\mu_h\right)}}{\Lambda_h\mu_V P_2P_3\left(\tau_2+\tau_1+\mu_h\right)P_8}. \tag{3}$$

This is the largest Eigen value.

2.5 Dengue fever endemic equilibrium points

Endemic Equilibrium signifies the permanent existence of Dengue fever among human population. At endemic equilibrium point,

$(S_h \neq 0, E_D \neq 0, I_D \neq 0, T_D \neq 0, V \neq 0, R_D \neq 0, S_V \neq 0, I_V \neq 0)$.

$$\left.\begin{aligned} S_h^* &= -\frac{P_6P_5P_4P_3P_2\pi_h}{\alpha\omega\rho\sigma P_5\lambda_D - P_1P_2P_3P_4P_5P_6 - P_2P_3P_4P_5P_6\lambda_D + P_2P_3P_4P_6\tau_1\tau_2}, \\ E_D^* &= -\frac{\pi_h P_3P_4P_5P_6\lambda_D}{\alpha\omega\rho\sigma P_5\lambda_D - P_1P_2P_3P_4P_5P_6 - P_2P_3P_4P_5P_6\lambda_D + P_2P_3P_4P_6\tau_1\tau_2}, \\ I_D^* &= -\frac{\omega\pi_h P_4P_5P_6\lambda_D}{\alpha\omega\rho\sigma P_5\lambda_D - P_1P_2P_3P_4P_5P_6 - P_2P_3P_4P_5P_6\lambda_D + P_2P_3P_4P_6\tau_1\tau_2}, \\ T_D^* &= -\frac{\omega\pi_h P_5P_6\lambda_D\rho}{\alpha\omega\rho\sigma P_5\lambda_D - P_1P_2P_3P_4P_5P_6 - P_2P_3P_4P_5P_6\lambda_D + P_2P_3P_4P_6\tau_1\tau_2}, \\ V^* &= -\frac{\tau_2P_6P_4P_3P_2\pi_h}{\alpha\omega\rho\sigma P_5\lambda_D - P_1P_2P_3P_4P_5P_6 - P_2P_3P_4P_5P_6\lambda_D + P_2P_3P_4P_6\tau_1\tau_2}, \\ R_D^* &= -\frac{\rho\lambda_D P_5\pi_h\sigma\omega}{\alpha\omega\rho\sigma P_5\lambda_D - P_1P_2P_3P_4P_5P_6 - P_2P_3P_4P_5P_6\lambda_D + P_2P_3P_4P_6\tau_1\tau_2}, \\ S_V^* &= \frac{\pi_V(\rho\lambda_D P_5\sigma\omega + \omega P_5P_6\lambda_D\rho + \omega P_4P_5P_6\lambda_D + P_2P_3P_4P_5P_6 + P_2P_3P_4P_6\tau_2 + P_3P_4P_5P_6\lambda_D)}{\omega\rho\sigma P_5P_7\lambda_D + \omega\rho P_5P_6P_7\lambda_D + \beta_V\omega P_4P_5P_6\lambda_D + \omega P_4P_5P_6P_7\lambda_D + P_2P_3P_4P_5P_6P_7 + P_2P_3P_4P_6P_7\tau_2 + P_3P_4P_5P_6P_7\lambda_D}, \\ I_V^* &= \frac{\lambda_D P_6P_5P_4\omega\beta_V\pi_V}{P_8(\omega\rho\sigma P_5P_7\lambda_D + \omega\rho P_5P_6P_7\lambda_D + \beta_V\omega P_4P_5P_6\lambda_D + \omega P_4P_5P_6P_7\lambda_D + P_2P_3P_4P_5P_6P_7 + P_2P_3P_4P_6P_7\tau_2 + P_3P_4P_5P_6P_7\lambda_D)}. \end{aligned}\right\} \tag{4}$$

Substituting into the force of infection $\lambda_D = \frac{q\beta_D I_V}{N_h}$ and $\lambda_V = \frac{\beta_V I_D}{N_h}$ we have:

$Q_1\lambda + Q_2 = 0.$

Where,

$$Q_1 = P_6P_5P_4\beta_V\begin{pmatrix} \alpha b\omega\rho\sigma P_5\pi_V\beta_D - bP_2P_3P_4P_5P_6\pi_V\beta_D + \omega\rho\sigma P_5P_7P_8\pi_h \\ +\omega\rho P_5P_6P_7P_8\pi_h + \omega P_4P_5P_6P_7P_8\pi_h + \omega P_4P_5P_6P_8\pi_h\beta_V \\ +P_3P_4P_5P_6P_7P_8\pi_h \end{pmatrix},$$

$$Q_2 = P_6P_5P_4\beta_V\left(1 - \left(R_0^D\right)^2\right). \tag{5}$$

Where $\begin{aligned} &P_1 = (\tau_2+\mu_h), P_2 = (\omega+\mu_h), P_3 = (\rho+\delta_1+\mu_h), P_4 = (\sigma+\delta_2+\mu_h), \\ &P_5 = (\tau_1+\mu_h), P_6 = (\alpha+\mu_h), P_7 = \mu_V, (\delta_V+\mu_h). \end{aligned}$

It means that model (1) has a stable equilibrium point for endemic infections.

3 Preliminaries Of FC (Fractional Caputo)

We will outline here the essentials of both singular and non-singular operators as required for this work [40, 41, 42, 43, 44, 45, 46].

Definition 2.1:[20]. The Liouville–Caputo equation for arbitrary-order $\eta \in [0,1)$ derivative with variable-order explains the function in detail as:

$$ {}_0^c D_t^{\eta} Y(t) = \frac{1}{\Gamma(1-\eta)} \int_0^t (t-\alpha)^{-\sigma} \frac{d}{dt} Y(\alpha)\, d\alpha, \tag{6} $$

Definition 2.2:The Caputo–Fabrizio (CF) derivative without singular kernel of a function $Y(t)$ of variable-order $\eta \in [0,1)$ is defined as:

$$ {}_0^{CF} D_t^{\eta} Y(t) = \frac{N(\eta)}{(1-\eta)} \int_0^t \exp\left[-\eta \frac{(t-\alpha)}{(1-\eta)}\right] \frac{d}{dt} Y(\alpha)\, d\alpha, \tag{7} $$

Where $N(\eta)$ has $N(0) = N(1)$ and $Y \in H'(0,T)$, $T > 0$.

Definition 2.3:[23]. For function $Y(t)$, the arbitrary-order integral with variable-order $\eta \in [0,1)$ is described as:

$$ {}_0^{CF} I_t^{\eta}(t)\, Y(t) = \frac{2(1-\eta)}{(2-\eta) N(\eta)} Y(t) + \frac{2\eta}{(2-\eta) N(\eta)} \int_0^t Y(\alpha)\, d\alpha. \tag{8} $$

Definition 2.4:[21] The Atangana-Baleanu (AB) arbitrary-order operator of variable-order $\eta \in [0,1)$ in Caputo sytle is written as:

$$ {}_0^{ABC} D_t^{\eta} Y(t) = \frac{AB(\eta)}{(1-\eta)} \int_0^t \varepsilon_{\eta} \left[-\eta \frac{(t-\alpha)^{\eta}}{(1-\eta)}\right] \frac{d}{dt} Y(\alpha)\, d\alpha. \tag{9} $$

Mittag–Leffler function is where ε_{η} , as defined by:
$\varepsilon_{\eta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\eta k+1)}, \eta \in c,\ R(\eta) > 0, z \in c,$ and $AB(\eta) = 1 - \eta + \frac{\eta}{\Gamma(\eta)}$ is normalization function.
This explains arbitrary-order integral related to the AB derivative is that:

$$ {}_0^{AB} I_t^{\eta} Y(t) = \frac{(1-\eta)}{AB(\eta)} Y(t) + \frac{\eta}{AB(\eta)\Gamma(\eta)} \int_0^t (t-\alpha)^{\eta-1} Y(\alpha)\, d\alpha. \tag{10} $$

3.1 Dengue fever fractional mathematical model

$$ \left.\begin{aligned} D_t S_h(t) &= \pi_h + \alpha R_D + \tau_1 V - \lambda_D S_h - (\tau_2 + \mu_h) S_h, \\ D_t E_D(t) &= \lambda_D S_h - (\omega + \mu_h) E_D, \\ D_t I_D(t) &= \omega E_D - (\rho + \delta_1 + \mu_h) I_D, \\ D_t T_D(t) &= \rho I_D - (\sigma + \delta_2 + \mu_h) T_D, \\ D_t V(t) &= \tau_2 S_h - (\tau_1 + \mu_h) V, \\ D_t R_D(t) &= \sigma T_D - (\alpha + \mu_h) R_D, \\ D_t S_V(t) &= \pi_V - \lambda_V S_V - \mu_V S_V, \\ D_t I_V(t) &= \lambda_V S_V - (\delta_V + \mu_h) I_V. \end{aligned}\right\} \tag{11} $$

With the initial conditions:

$$ S_h(0) = S_{h0}(t), E_D(0) = E_{D0}(t), I_D(0) = I_{D0}(t), T_D(0) = T_{D0}(t), V(0) = V_0(t), $$
$$ R_D(0) = R_{D0}(t), S_V(0) = S_{V0}, I_V(0) = I_{V0}(t). $$

With our non-linear model of Dengue fever, described by five ODEs as [48, 49] not only makes it easier to see the transmission of the disease in the population, but also converting an integer-order model into an arbitrary-order model helps us understand it better. For our research, we make a change to model (11) by substituting the classical time derivative (D_t) using Caputo $\left({}_0^c D_t^{\eta}\right)$, $\left({}_0^{CF} D_t^{\eta}\right)$ and $\left({}_0^{ABC} D_t^{\eta}\right)$. arbitrary-order derivatives. To cover a different perspective, Liouville–Caputo arbitrary-order derivatives are used to outline Dengue fever in this section. In [34], the Caputo variant uses a

predictor–corrector scheme which is an improved form of the ABM technique. Dengue fever, as used in the context, is described as the following:

$$\left.\begin{aligned} {}_0^cD_t^\eta S_h(t) &= \pi_h + \alpha R_D + \tau_1 V - \lambda_D S_h - (\tau_2 + \mu_h) S_h, \\ {}_0^cD_t^\eta E_D(t) &= \lambda_D S_h - (\omega + \mu_h) E_D, \\ {}_0^cD_t^\eta I_D(t) &= \omega E_D - (\rho + \delta_1 + \mu_h) I_D, \\ {}_0^cD_t^\eta T_D(t) &= \rho I_D - (\sigma + \delta_2 + \mu_h) T_D, \\ {}_0^cD_t^\eta V(t) &= \tau_2 S_h - (\tau_1 + \mu_h) V, \\ {}_0^cD_t^\eta R_D(t) &= \sigma T_D - (\alpha + \mu_h) R_D, \\ {}_0^cD_t^\eta S_V(t) &= \pi_V - \lambda_V S_V - \mu_V S_V, \\ {}_0^cD_t^\eta I_V(t) &= \lambda_V S_V - (\delta_V + \mu_h) I_V. \end{aligned}\right\} \tag{12}$$

In the current section, we outline Dengue fever using an alternative way that includes CF differential form. Variable-order systems are defined by using the theories of fixed points. The suggested system was solved approximately by using an ABM numerical method in two steps. A mathematical expression of dengue fever disease is given with the framework of CF, using a derivative kernel that follows an exponential law below:

$$\left.\begin{aligned} {}_0^{CF}D_t^\eta S_h(t) &= \pi_h + \alpha R_D + \tau_1 V - \lambda_D S_h - (\tau_2 + \mu_h) S_h, \\ {}_0^{CF}D_t^\eta E_D(t) &= \lambda_D S_h - (\omega + \mu_h) E_D, \\ {}_0^{CF}D_t^\eta I_D(t) &= \omega E_D - (\rho + \delta_1 + \mu_h) I_D, \\ {}_0^{CF}D_t^\eta T_D(t) &= \rho I_D - (\sigma + \delta_2 + \mu_h) T_D, \\ {}_0^{CF}D_t^\eta V(t) &= \tau_2 S_h - (\tau_1 + \mu_h) V, \\ {}_0^{CF}D_t^\eta R_D(t) &= \sigma T_D - (\alpha + \mu_h) R_D, \\ {}_0^{CF}D_t^\eta S_V(t) &= \pi_V - \lambda_V S_V - \mu_V S_V, \\ {}_0^{CF}D_t^\eta I_V(t) &= \lambda_V S_V - (\delta_V + \mu_h) I_V. \end{aligned}\right\} \tag{13}$$

3.2 Numerical technique for Caputo derivative

The main points of a predictor–corrector type numerical algorithm are given below.
Take this basic differential equation as an example:

$$\begin{cases} {}_0^C D_t^\eta Y(t) = f(t, Y(t)), 0 \le t \le T, \\ Y^{(k)}(0) = Y_0^{(k)}, k = 0, 1, 2, \ldots m-n. \end{cases} \tag{14}$$

$$Y(t) = \sum_{k=0}^{[\eta]-1} Y_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(\eta)} \frac{\int_0^t f(\alpha, Y(\alpha)) d\alpha}{(t-\alpha)^{1-\eta}}, \tag{15}$$

$h = \frac{T}{N}$, $t_n = nh$, $(n = 0, 1, 2, ..., N)$.

Thus, the equation (14) can be discretized as follows:

$$Y_n(t_{n+1}) = \sum_{k=0}^{[\eta]-1} \frac{t_{n+1}^k}{k!} Y_0^{(k)} + \frac{h^\eta}{\Gamma(\eta+2)} \left[f(t_{n+1}, Y_n^p(t_{n+1})) + \sum_{j=0}^{n} a_j, n+1, f(t_j, Y_h(t_j)) \right],$$

Where $Y_n^p(t_{n+1})$ is calculated by the arbitrary Adams–Bashforth technique is:

$$Y_h^p(t_{n+1}) = \sum_{k=0}^{[\eta]-1} \frac{t_{n+1}^k}{k!} Y_0^{(k)} + \frac{1}{\Gamma(\eta)} \sum_{j=0}^{n} \Delta_j, n+1, f(t_j, Y_h(t_j)),$$

Where

$a_{j,n+1} = \begin{cases} n^{\eta+1}-(n-\eta)(n+1)^n, j=0 \\ (n-j+2)^{n+1}+(n-j)^{n+1}-2(n-j+2)^{n+1}, 1\leq j\leq n, \\ 1, \quad j=n+1, \end{cases}$ and $\Delta_{j,n+1} = \frac{h^\eta}{\eta}(n+1-j)^{\eta}-(n-j)^{\eta},\ 0 \leq j \leq n,\ j = 1,2,3,...,n.$

3.3 Predictor–corrector scheme on proposed Dengue fever model

According to the suggested variable-order model of Dengue fever disease, the predictor–corrector method is used to solve it when the derivative is taken to any arbitrary order. The method described treats system (13) as a collection of discrete equations.

$$\left.\begin{aligned}
S_{h(n+1)} &= S_h(0) + \tfrac{h^\eta}{\Gamma(\eta+2)}\left[F_1\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, v^p_{(n+1)}, \\ T^p_{D(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad \sum_{n=0}^{n} a_{j,n+1}\left[F_1(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj})\right], \\
E_{D(n+1)} &= E_D(0) + \tfrac{h^\eta}{\Gamma(\eta+2)}\left[F_2\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, v^p_{(n+1)}, \\ T^p_{D(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad \sum_{n=0}^{n} a_{j,n+1}\left[F_2(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj})\right], \\
I_{D(n+1)} &= I_D(0) + \tfrac{h^\eta}{\Gamma(\eta+2)}\left[F_3\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, v^p_{(n+1)}, \\ T^p_{D(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad \sum_{n=0}^{n} a_{j,n+1}\left[F_3(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj})\right], \\
T_{D(n+1)} &= T_D(0) + \tfrac{h^\eta}{\Gamma(\eta+2)}\left[F_4\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, v^p_{(n+1)}, \\ T^p_{D(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad \sum_{n=0}^{n} a_{j,n+1}\left[F_4(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj})\right], \\
V_{(n+1)} &= V(0) + \tfrac{h^\eta}{\Gamma(\eta+2)}\left[F_5\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, v^p_{(n+1)}, \\ T^p_{D(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad \sum_{n=0}^{n} a_{j,n+1}\left[F_5(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj})\right], \\
R_{D(n+1)} &= R_D(0) + \tfrac{h^\eta}{\Gamma(\eta+2)}\left[F_6\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, v^p_{(n+1)}, \\ T^p_{D(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad \sum_{n=0}^{n} a_{j,n+1}\left[F_6(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj})\right], \\
S_{V(n+1)} &= S_V(0) + \tfrac{h^\eta}{\Gamma(\eta+2)}\left[F_7\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, v^p_{(n+1)}, \\ T^p_{D(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad \sum_{n=0}^{n} a_{j,n+1}\left[F_7(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj})\right], \\
I_{V(n+1)} &= I_V(0) + \tfrac{h^\eta}{\Gamma(\eta+2)}\left[F_8\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, v^p_{(n+1)}, \\ T^p_{D(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad \sum_{n=0}^{n} a_{j,n+1}\left[F_8(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj})\right].
\end{aligned}\right\} \tag{16}$$

The predator form $S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)}$, are suggested as follows:

$$\left.\begin{aligned}
S_{h^p_{(n+1)}} &= S_h(0) + \tfrac{1}{\Gamma(\eta)} \sum_{j=0}^{n} \Delta_{j,n+1} \left[F_1\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
E_{D^p_{(n+1)}} &= E_D(0) + \tfrac{1}{\Gamma(\eta)} \sum_{j=0}^{n} \Delta_{j,n+1} \left[F_2\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
I_{D^p_{(n+1)}} &= I_D(0) + \tfrac{1}{\Gamma(\eta)} \sum_{j=0}^{n} \Delta_{j,n+1} \left[F_3\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
T_{D^p_{(n+1)}} &= T_D(0) + \tfrac{1}{\Gamma(\eta)} \sum_{j=0}^{n} \Delta_{j,n+1} \left[F_4\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
V^p_{(n+1)} &= V(0) + \tfrac{1}{\Gamma(\eta)} \sum_{j=0}^{n} \Delta_{j,n+1} \left[F_5\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
R_{D^p_{(n+1)}} &= R_D(0) + \tfrac{1}{\Gamma(\eta)} \sum_{j=0}^{n} \Delta_{j,n+1} \left[F_6\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
S_{V^p_{(n+1)}} &= S_V(0) + \tfrac{1}{\Gamma(\eta)} \sum_{j=0}^{n} \Delta_{j,n+1} \left[F_7\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
I_{V^p_{(n+1)}} &= I_V(0) + \tfrac{1}{\Gamma(\eta)} \sum_{j=0}^{n} \Delta_{j,n+1} \left[F_8\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right].
\end{aligned}\right\} \tag{17}$$

Where

$$\left.\begin{aligned}
F_1(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V) &= \pi_h + \alpha R_D + \tau_1 V - \lambda_D S_h - (\tau_2 + \mu_h) S_h, \\
F_2(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V) &= \lambda_D S_h - (\omega + \mu_h) E_D, \\
F_3(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V) &= \omega E_D - (\rho + \delta_1 + \mu_h) I_D, \\
F_4(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V) &= \rho I_D - (\sigma + \delta_2 + \mu_h) T_D, \\
F_5(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V) &= \tau_2 S_h - (\tau_1 + \mu_h) V, \\
F_6(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V) &= \sigma T_D - (\alpha + \mu_h) R_D, \\
F_7(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V) &= \pi_V - \lambda_V S_V - \mu_V S_V, \\
F_8(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V) &= \lambda_V S_V - (\delta_V + \mu_h) I_V.
\end{aligned}\right\} \tag{18}$$

3.4 Existence and uniqueness of variable-order Dengue fever disease system

This section ponders whether the variable-order arbitrary-order Dengue fever disease system (3.3) has unique solutions and how to verify their existence. So, here we alter model (3.3) into the following integral equation:

$$\left.\begin{aligned}
S_h(t) - S_h(0) &= {}_0^{CF}I_t^{\eta}\left[\pi_h + \alpha R_D + \tau_1 V - \lambda_D S_h - (\tau_2 + \mu_h) S_h\right], \\
E_D(t) - E_D(0) &= {}_0^{CF}I_t^{\eta}\left[\lambda_D S_h - (\omega + \mu_h) E_D\right], \\
I_D(t) - I_D(0) &= {}_0^{CF}I_t^{\eta}\left[\omega E_D - (\rho + \delta_1 + \mu_h) I_D\right], \\
T_D(t) - T_D(0) &= {}_0^{CF}I_t^{\eta}\left[\rho I_D - (\sigma + \delta_2 + \mu_h) T_D\right], \\
V(t) - V(0) &= {}_0^{CF}I_t^{\eta}\left[\tau_2 S_h - (\tau_1 + \mu_h) V\right], \\
R_D(t) - R_D(0) &= {}_0^{CF}I_t^{\eta}\left[\sigma T_D - (\alpha + \mu_h) R_D\right], \\
S_V(t) - S_V(0) &= {}_0^{CF}I_t^{\eta}\left[\pi_V - \lambda_V S_V - \mu_V S_V\right], \\
I_V(t) - I_V(0) &= {}_0^{CF}I_t^{\eta}\left[\lambda_V S_V - (\delta_V + \mu_h) I_V\right].
\end{aligned}\right\} \tag{19}$$

simplifying, the kernel is define as follows:

$$\left.\begin{aligned} X_1(t,S_h) &= \pi_h + \alpha R_D + \tau_1 V - \lambda_D S_h - (\tau_2+\mu_h) S_h, \\ X_2(t,E_D) &= \lambda_D S_h - (\omega+\mu_h) E_D, \\ X_3(t,I_D) &= \omega E_D - (\rho+\delta_1+\mu_h) I_D, \\ X_4(t,T_D) &= \rho I_D - (\sigma+\delta_2+\mu_h) T_D, \\ X_5(t,V) &= \tau_2 S_h - (\tau_1+\mu_h) V, \\ X_6(t,R_D) &= \sigma T_D - (\alpha+\mu_h) R_D, \\ X_7(t,S_V) &= \pi_V - \lambda_V S_V - \mu_V S_V, \\ X_8(t,I_V) &= \lambda_V S_V - (\delta_V+\mu_h) I_V. \end{aligned}\right\} \tag{20}$$

Applying the arbitrary integral on Equation (19), we obtain the following:

$$\left.\begin{aligned} S_h(t) - S_h(0) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_1(t,S_h) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_1(\eta,S_h)d\eta, \\ E_D(t) - E_D(0) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_2(t,E_D) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_2(\eta,E_D)d\eta, \\ I_D(t) - I_D(0) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_3(t,I_D) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_3(\eta,I_D)d\eta, \\ T_D(t) - T_D(0) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_4(t,T_D) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_4(\eta,T_D)d\eta, \\ V(t) - V(0) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_5(t,V) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_5(\eta,V)d\eta, \\ R_D(t) - R_D(0) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_6(t,R_D) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_6(\eta,R_D)d\eta, \\ S_V(t) - S_V(0) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_7(t,S_V) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_7(\eta,S_V)d\eta, \\ I_V(t) - I_V(0) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_8(t,I_V) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_8(\eta,I_V)d\eta. \end{aligned}\right\} \tag{21}$$

Theorem 3.1 : The kernels $X_1, X_2, X_3, X_4, X_5, X_6, X_7 \, and \, X_8$ in Equation (20) will holds the Lipschitz and contraction condition, if satisfy the following inequality:

$0 \leq (a_1 d_2 + \leq a_2 d_3) < 1.$

Proof. We begin with kernelX_1. Consider S_h and S_h^* as the two functions of the kernel, then we have:

$$\begin{aligned} \|X_1(t,S_h) - X_1(t,\, S_h^*)\| &= \|-(a_1 E_D + a_2 I_D)(S_h, -S_h^*)\|, \\ &\leq (a_1 \|E_D\| + a_2 \|I_D\|) \|(S_h - S_h^*)\|, \end{aligned}$$

$$\leq (a_1 d_2 + a_2 d_3) \|(S_h - S_h^*)\|, \tag{22}$$

suppose, $\gamma_1 = (a_1 d_2 + a_2 d_3)$, Here, we note that: $\|S_h\| = d_1, \|E_D\| = d_2, \|I_D\| = d_3, \|T_D\| = d_4, \|V\| = d_5, \|R_D\| = d_6, \|S_h\| = d_7, \|I_h\| = d_8$. are the bounded functions, where $d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$ are some positive constant. We obtain:

$\|X_1(t,S_h) - X_1(t,S_h^*)\| \leq \gamma_1 \|S_h - S_h^*\|.$

Consequently, that formula means the kernels X_1 satisfy the Lipschitz condition $0 \leq \gamma_1 < 1$. yet keep the contraction condition We can also write the other kernels' expressions as:
We can also write the other kernels' expressions as:

$$\left.\begin{aligned} \|X_2(t,E_D) - X_2(t,E_D^*)\| &\leq \gamma_2 \|E_D - E_D^*\|, \\ \|X_3(t,I_D) - X_2(t,I_D^*)\| &\leq \gamma_3 \|I_D - I_D^*\|, \\ \|X_4(t,T_D) - X_4(t,T_D^*)\| &\leq \gamma_4 \|T_D - T_D^*\|, \\ \|X_5(t,V) - X_5(t,V^*)\| &\leq \gamma_5 \|V - V^*\|, \\ \|X_6(t,R_D) - X_6(t,R_D^*)\| &\leq \gamma_6 \|R_D - R_D^*\|, \\ \|X_7(t,S_V) - X_7(t,S_V^*)\| &\leq \gamma_7 \|S_V - S_V^*\|, \\ \|X_8(t,I_V) - X_8(t,I_V^*)\| &\leq \gamma_8 \|I_V - I_V^*\|. \end{aligned}\right\} \tag{23}$$

Now, we introduce the recursive formula using (21), given as:

$$\left.\begin{aligned}
S_{hn}(t) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_1\left(t,S_{h(n-1)}\right)+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_1\left(\eta,S_{h(n-1)}\right)d\eta,\\
E_{Dn}(t) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_2\left(t,E_{D(n-1)}\right)+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_2\left(\eta,E_{D(n-1)}\right)d\eta,\\
I_{Dn}(t) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_3\left(t,I_{D(n-1)}\right)+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_3\left(\eta,I_{D(n-1)}\right)d\eta,\\
T_{Dn}(t) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_4\left(t,T_{D(n-1)}\right)+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_4\left(\eta,T_{D(n-1)}\right)d\eta,\\
V_n(t) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_5\left(t,V_{(n-1)}\right)+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_5\left(\eta,V_{(n-1)}\right)d\eta,\\
R_{Dn}(t) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_6\left(t,R_{D(n-1)}\right)+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_6\left(\eta,R_{D(n-1)}\right)d\eta,\\
S_{Vn}(t) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_7\left(t,S_{V(n-1)}\right)+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_7\left(\eta,S_{V(n-1)}\right)d\eta,\\
I_{Vn}(t) &= \tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_8\left(t,I_{V(n-1)}\right)+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_8\left(\eta,I_{V(n-1)}\right)d\eta.
\end{aligned}\right\} \tag{24}$$

The Initial values in the recursive formula listed above are suggested as:

$$S_h(0)=S_{h0}(t), E_D(0)=E_{D0}(t), I_D(0)=I_{D0}(t), T_D(0)=T_{D0}(t), V(0)=V_0(t),$$
$$R_D(0)=R_{D0}(t), S_V(0)=S_{V0}, I_V(0)=I_{V0}(t).$$

Taking away each successive term, the suggested formula is:

$$\left.\begin{aligned}
\psi_n(t) &= S_{hn}(t)-S_{h(n-1)}(t)=\tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_1\left(t,S_{h(n-1)}\right)-X_1\left(t,S_{h(n-2)}\right)\\
&\quad+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_1\left(\eta,S_{h(n-1)}\right)-X_1\left(\eta,S_{h(n-2)}\right)d\eta,\\
\zeta_n(t) &= E_{Dn}(t)-E_{D(n-1)}(t)=\tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_2\left(t,E_{D(n-1)}\right)-X_2\left(t,E_{D(n-2)}\right)\\
&\quad+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_2\left(\eta,E_{D(n-1)}\right)-X_2\left(\eta,E_{D(n-2)}\right)d\eta,\\
\chi_n(t) &= I_{Dn}(t)-I_{D(n-1)}(t)=\tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_3\left(t,I_{D(n-1)}\right)-X_3\left(t,I_{D(n-2)}\right)\\
&\quad+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_3\left(\eta,I_{D(n-1)}\right)-X_3\left(\eta,I_{D(n-2)}\right)d\eta,\\
\varphi_n(t) &= T_{Dn}(t)-T_{D(n-1)}(t)=\tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_4\left(t,T_{D(n-1)}\right)-X_4\left(t,T_{D(n-2)}\right)\\
&\quad+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_4\left(\eta,T_{D(n-1)}\right)-X_4\left(\eta,T_{D(n-2)}\right)d\eta,\\
\vartheta_n(t) &= V_n(t)-V_{(n-1)}(t)=\tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_5\left(t,V_{(n-1)}\right)-X_5\left(t,V_{(n-2)}\right)\\
&\quad+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_5\left(\eta,V_{(n-1)}\right)-X_5\left(\eta,V_{(n-2)}\right)d\eta,\\
\omega_n(t) &= R_{Dn}(t)-R_{D(n-1)}(t)=\tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_6\left(t,R_{D(n-1)}\right)-X_6\left(t,R_{D(n-2)}\right)\\
&\quad+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_6\left(\eta,R_{D(n-1)}\right)-X_6\left(\eta,R_{D(n-2)}\right)d\eta,\\
\beta_n(t) &= S_{Vn}(t)-S_{V(n-1)}(t)=\tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_7\left(t,S_{V(n-1)}\right)-X_7\left(t,S_{V(n-2)}\right)\\
&\quad+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_7\left(\eta,S_{V(n-1)}\right)-X_7\left(\eta,S_{V(n-2)}\right)d\eta,\\
\nu_n(t) &= I_{Vn}(t)-I_{V(n-1)}(t)=\tfrac{2(1-\eta)}{(2-\eta)N(\eta)}X_8\left(t,I_{V(n-1)}\right)-X_8\left(t,I_{V(n-2)}\right)\\
&\quad+\tfrac{2\eta}{(2-\eta)N(\eta)}\int_0^t X_8\left(\eta,I_{V(n-1)}\right)-X_8\left(\eta,I_{V(n-2)}\right)d\eta.
\end{aligned}\right\} \tag{25}$$

where

$$\begin{aligned}
&S_h(t)=\sum_{i=1}^{n}\psi_i(t), E_D(t)=\sum_{i=1}^{n}\zeta_i(t), I_D(t)=\sum_{i=1}^{n}\chi_i(t), T_D(t)=\sum_{i=1}^{n}\varphi_i(t), V(t)=\sum_{i=1}^{n}\vartheta_i(t),\\
&R_D(t)=\sum_{i=1}^{n}\omega_i(t), S_V(t)=\sum_{i=1}^{n}\beta_i(t), I_V(t)=\sum_{i=1}^{n}\nu_i(t).
\end{aligned} \tag{26}$$

Using the norm onto successive inequalities gives:

$$\|\psi_n(t)\| = \left\|S_{hn}(t) - S_{h(n-1)}(t)\right\| = \left\| \begin{array}{l} \frac{2(1-\eta)}{(2-\eta)N(\eta)} X_1\left(t, S_{h(n-1)}\right) - X_1\left(t, S_{h(n-2)}\right) \\ + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_1\left(\eta, S_{h(n-1)}\right) - X_1\left(\eta, S_{h(n-2)}\right) d\eta \end{array} \right\|,$$

Now, to apply the triangle inequality, the previous equation becomes:

$$\begin{aligned} \left\|S_{hn}(t) - S_{h(n-1)}(t)\right\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)} \left\|X_1\left(t, S_{h(n-1)}\right) - X_1\left(t, S_{h(n-2)}\right)\right\| \\ + \frac{2\eta}{(2-\eta)N(\eta)} \left\|\int_0^t X_1\left(\eta, S_{h(n-1)}\right) - X_1\left(\eta, S_{h(n-2)}\right)\right\| d\eta, \end{aligned}$$

The kernel satisfies the Lipschitz condition and that means we get the following as a result:

$$\left\|S_{hn}(t) - S_{h(n-1)}(t)\right\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_1 \left\|S_{h(n-1)} - S_{h(n-2)}\right\| + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_1 \int_0^t \left\|S_{h(n-1)} - S_{h(n-2)}\right\| d\eta.$$

Thus, we get:

$$\|\psi_n(t)\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_1 \|\psi_{n-1}(t)\| + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_1 \int_0^t \|\psi_{n-1}(\eta)\| d\eta. \tag{27}$$

Similarly, we can obtain the following results:

$$\left.\begin{array}{r} \|\zeta_n(t)\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_2 \|\zeta_{n-1}(t)\| + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_2 \int_0^t \|\zeta_{n-1}(\eta)\| d\eta, \\ \|\chi_n(t)\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_3 \|\chi_{n-1}(t)\| + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_3 \int_0^t \|\chi_{n-1}(\eta)\| d\eta, \\ \|\varphi_n(t)\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_4 \|\varphi_{n-1}(t)\| + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_4 \int_0^t \|\varphi_{n-1}(\eta)\| d\eta, \\ \|\vartheta_n(t)\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_5 \|\vartheta_{n-1}(t)\| + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_5 \int_0^t \|\vartheta_{n-1}(\eta)\| d\eta, \\ \|\omega_n(t)\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_6 \|\omega_{n-1}(t)\| + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_6 \int_0^t \|\omega_{n-1}(\eta)\| d\eta, \\ \|\beta_n(t)\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_7 \|\beta_{n-1}(t)\| + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_7 \int_0^t \|\beta_{n-1}(\eta)\| d\eta, \\ \|\nu_n(t)\| \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_8 \|\nu_{n-1}(t)\| + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_8 \int_0^t \|\nu_{n-1}(\eta)\| d\eta. \end{array}\right\} \tag{28}$$

For this reason, showing the existence of a solution requires us to work with its results.

Theorem 3.2 : A coupled solution for the fractional Dengue fever disease system (13) exists, if there exists, some t_0 such that:

$$\frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_1 + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_1 t_0 < 1.$$

Proof. We obtained the inequalities below by repeatedly using Equations (27) and (28), assuming that $S_h(t), E_D(t), I_D(t), T_D(t), V(t), R_D(t), S_V, I_V(t)$ kernels are Lipschitzian and functions are bounded.

$$\left.\begin{array}{r} \|\psi_n(t)\| \leq \|S_h(0)\| \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_1 + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_1 t\right]^n, \\ \|\zeta_n(t)\| \leq \|E_D(0)\| \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_2 + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_2 t\right]^n, \\ \|\chi_n(t)\| \leq \|I_D(0)\| \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_3 + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_3 t\right]^n, \\ \|\varphi_n(t)\| \leq \|T_D(0)\| \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_4 + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_4 t\right]^n, \\ \|\vartheta_n(t)\| \leq \|V(0)\| \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_5 + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_5 t\right]^n, \\ \|\omega_n(t)\| \leq \|R_D(0)\| \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_6 + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_6 t\right]^n, \\ \|\beta_n(t)\| \leq \|S_V(0)\| \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_7 + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_7 t\right]^n, \\ \|\nu_n(t)\| \leq \|I_V(0)\| \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_8 + \frac{2\eta}{(2-\eta)N(\eta)}\gamma_8 t\right]^n, \end{array}\right\} \tag{29}$$

In order to show that the above function is the special solutions of the system (13), we assume that:

$$\left.\begin{array}{r} S_h(t)-S_h(0)=S_{hn}(t)-U_n(t), \\ E_D(t)-E_D(0)=E_{Dn}(t)-V_n(t), \\ I_D(t)-I_D(0)=I_{Dn}(t)-Y_n(t), \\ T_D(t)-T_D(0)=T_{Dn}(t)-K_n(t), \\ V(t)-V(0)=V_n(t)-L_n(t), \\ R_D(t)-R_D(0)=R_{Dn}(t)-J_n(t), \\ S_V(t)-S_V(0)=S_{Vn}(t)-W_n(t), \\ I_V(t)-I_V(0)=I_{Vn}(t)-X_n(t). \end{array}\right\} \tag{30}$$

So, we have:

$$\|U_n(t)\| = \left\| \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_1(t,S_h) - X_1\left(t,S_{h(n-1)}\right) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_1(\eta,S_h) - X_1\left(\eta,S_{h(n-1)}\right) d\eta \right\|,$$

$$\|U_n(t)\| \leq \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} \left\| X_1(t,S_h) - X_1\left(t,S_{h(n-1)}\right)\right\| + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t \left\| X_1(\eta,S_h) - X_1\left(\eta,S_{h(n-1)}\right)\right\| d\eta,$$

$$\|U_n(t)\| \leq \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} \gamma_1 \left\| \left(S_h - S_{h(n-1)}\right)\right\| + \tfrac{2\eta}{(2-\eta)N(\eta)} \gamma_1 \int_0^t \left\| \left(S_h - S_{h(n-1)}\right)\right\| d\eta t,$$

We followed as:

$$\|U_n(t)\| \leq \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)} \gamma_1 + \frac{2\eta}{(2-\eta)N(\eta)} \gamma_1 t \right]^{n+1} B, \tag{31}$$

at t_0, we have

$$\|U_n(t)\| \leq \left[\frac{2(1-\eta)}{(2-\eta)N(\eta)} \gamma_1 + \frac{2\eta}{(2-\eta)N(\eta)} \gamma_1 t_0 \right]^{n+1} B. \tag{32}$$

Applying the limit on Equation (32), as $n \to \infty$, , then $\|U_n(t)\| \to 0$.

Similarly, we can prove that:

$$\|V_n(t)\| \to 0, \|Y_n(t)\| \to 0, \|K_n(t)\| \to 0, \|K_n(t)\| \to 0, \|L_n(t)\| \to 0,$$
$$\|J_n(t)\| \to 0, \|W_n(t)\| \to 0, \|X_n(t)\| \to 0.$$

Consequently, there exists a solution to the system of equations (13). We will now examine how solutions us can provide are one of a kind.

Theorem 3.3 : The arbitrary-order Dengue fever disease system (13) has a unique solution if hold the following inequality:

$$\left(1 - \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} \gamma_1 + \tfrac{2\eta}{(2-\eta)N(\eta)} \gamma_1 t\right) > 0.$$

Proof. We assume that $[S_h^*(t), E_D^*(t), I_D^*(t), T_D^*(t), V^*(t), R_D^*(t), S_V^*(t), I_V^*(t)]$ is another form of solutions of variable-order system (13). Then:

$$S_h(t) - S_h^*(t) = \tfrac{2(1-\eta)}{(2-\eta)N(\eta)} X_1(t,S_h) - X_1(t,S_h^*) + \tfrac{2\eta}{(2-\eta)N(\eta)} \int_0^t X_1(\eta,S_h) - X_1(\eta,S_h^*) d\eta,$$

using norm on above equation, we have:

$$S_h(t) - S_h^*(t) \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)} \|X_1(t,S_h) - X_1(t,S_h^*)\| + \frac{2\eta}{(2-\eta)N(\eta)} \int_0^t \|X_1(\eta,S_h) - X_1(\eta,S_h^*)\| d\eta. \tag{33}$$

As, the kernel satisfied the Lipschitz condition, thus we have:

$S_h(t) - S_h^*(t) \leq \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_1 \|S_h(t) - S_h^*(t)\| + \frac{2\eta}{(2-\eta)N(\eta)}\int_0^t \gamma_1 t \|S_h(t) - S_h^*(t)\|$.
by re-arrangement, we get:

$$S_h(t) - S_h^*(t)\left(1 - \frac{2(1-\eta)}{(2-\eta)N(\eta)}\gamma_1 - \frac{2\eta}{(2-\eta)N(\eta)}\gamma_1 t\right) \leq 0, \tag{34}$$

$S_h(t) - S_h^*(t) = 0$.
Thus, we get:

$$S_h(t) = S_h^*(t). \tag{35}$$

This shows, in turn, that the system has unique solutions. We can also figure out the right results for the other functions by using the same process.

3.5 Numerical technique for Caputo–Fabrizio derivative

We discussed a process that uses the Caputo–Fabrizio arbitrary-order derivative [11]. We can look at a case where the fractional derivative operator takes part:

$$_0^{CF}D_t^{\eta} Y(t) = f(t, Y(t)). \tag{36}$$

By applying the fundamental theorem of calculus:

$$Y(t) - Y(0) = \frac{1-\eta}{N(\eta)} f(t, Y(t)) + \frac{\eta}{N(\eta)}\int_0^t f(\alpha, Y(t))\, d\alpha, \tag{37}$$

$$Y(t) - Y(0) = \frac{1-\eta}{N(\eta)} f(t, Y(t)) + \frac{\eta}{N(\eta)}\int_0^t f(\alpha, Y(t))\, d\alpha, \tag{38}$$

By generalization:

$$Y(t_{n+1}) - Y(0) = \frac{1-\eta}{N(\eta)} f(t_n, Y(t_n)) + \frac{\eta}{N(\eta)}\int_0^{t+1} f(\alpha, Y(t))\, d\alpha, \tag{39}$$

$$Y(t_{n+1}) - Y(0) = \frac{1-\eta}{N(\eta)} f(t_n, Y(t_n)) + \frac{\eta}{N(\eta)}\int_0^{t+1} f(\alpha, Y(t))\, d\alpha, \tag{40}$$

and

$$Y(t_n) - Y(0) = \frac{1-\eta}{N(\eta)} f(t_{n-1}, Y(t_{n-1})) + \frac{\eta}{N(\eta)}\int_0^{t_n} f(\alpha, Y(t))\, d\alpha, \tag{41}$$

Taking in the way as $Y(t_{n-1}) - Y(t_n)$ we have:

$$Y(t_n) - Y(t_{n-1}) = \frac{1-\eta}{N(\eta)} f(t_n, Y(t_n)) - f(t_{n-1}, Y(t_{n-1})) + \frac{\eta}{N(\eta)}\int_{t_n}^{t_{n+1}} f(t, Y(t))\, dt, \tag{42}$$

where

$$\int_{t_n}^{t_{n+1}} f(t, Y(t)) dt = \int_{t_n}^{t_{n+1}} \left[\frac{f(t_n, Y_n)(t - t_{n-1})}{h} - \frac{f(t_{n-1}, Y_{n-1})(t - t_n)}{h}\right] dt \tag{43}$$

$\int_{t_n}^{t_{n+1}} f(t, Y(t))dt = \int_{t_n}^{t_{n+1}} \left[\frac{3h}{2} f(t_n, Y_n) - \frac{h}{2} f(t_{n-1}, Y_{n-1})\right]$

from Equation (47), we obtain the solution in the form as:

$$Y(t_{n+1}) - Y(t_n) = \frac{1-\eta}{N(\eta)} \left[f(t_n, Y(t_n)) - f(t_{n-1}, Y(t_{n-1}))\right] + \frac{\eta}{N(\eta)} \left[\frac{3h}{2} f(t_n, Y_n) - \frac{h}{2} f(t_{n-1}, Y_{n-1})\right],$$

$$Y(t_{n+1}) - Y(t_n) = \left(\frac{1-\eta}{N(\eta)} + \frac{3\eta h}{2N(\eta)}\right) f(t_n, Y(t_n)) - \left(\frac{1-\eta}{N(\eta)} + \frac{\eta h}{2N(\eta)}\right) f(t_{n-1}, Y(t_{n-1})), \tag{44}$$

Solution in the form as:

$$Y(t_{n+1}) - Y(t_n) = \left(\frac{1-\eta}{N(\eta)} + \frac{3\eta h}{2N(\eta)}\right) f(t_n, Y(t_n)) - \left(\frac{1-\eta}{N(\eta)} + \frac{\eta h}{2N(\eta)}\right) f(t_{n-1}, Y(t_{n-1})), \tag{45}$$

This is the two-step ABM method designed for computing the CF arbitrary-order derivative.

3.6 Numerical technique on CF frame Dengue fever model

To obtain an approximate result for the Dengue fever disease system using the Caputo–Fabrizio derivative, we first implement the numerical ABM scheme (mentioned earlier) on the original system and then use it again on the modified system. By applying the suggested scheme, the equations of the system (13) can be written as:

$$\left.\begin{aligned}
S_{h(n+1)} &= S_{hn} + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_1(t, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}) \\
&\quad + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_1\left(t, S_{h(n-1)}, E_{D(n-1)}, I_{D(n-1)}, T_{D(n-1)}, V_{(n-1)}, R_{D(n-1)}, S_{V(n-1)}, I_{V(n-1)}\right), \\
E_{D(n+1)} &= E_{Dn} + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_2(t, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}) \\
&\quad + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_2\left(t, S_{h(n-1)}, E_{D(n-1)}, I_{D(n-1)}, T_{D(n-1)}, V_{(n-1)}, R_{D(n-1)}, S_{V(n-1)}, I_{V(n-1)}\right), \\
I_{D(n+1)} &= I_{Dn} + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_3(t, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}) \\
&\quad + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_3\left(t, S_{h(n-1)}, E_{D(n-1)}, I_{D(n-1)}, T_{D(n-1)}, V_{(n-1)}, R_{D(n-1)}, S_{V(n-1)}, I_{V(n-1)}\right), \\
T_{D(n+1)} &= T_{Dn} + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_4(t, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}) \\
&\quad + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_4\left(t, S_{h(n-1)}, E_{D(n-1)}, I_{D(n-1)}, T_{D(n-1)}, V_{(n-1)}, R_{D(n-1)}, S_{V(n-1)}, I_{V(n-1)}\right), \\
V_{(n+1)} &= V_n + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_5(t, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}) \\
&\quad + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_5\left(t, S_{h(n-1)}, E_{D(n-1)}, I_{D(n-1)}, T_{D(n-1)}, V_{(n-1)}, R_{D(n-1)}, S_{V(n-1)}, I_{V(n-1)}\right), \\
R_{D(n+1)} &= R_{Dn} + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_6(t, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}) \\
&\quad + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_6\left(t, S_{h(n-1)}, E_{D(n-1)}, I_{D(n-1)}, T_{D(n-1)}, V_{(n-1)}, R_{D(n-1)}, S_{V(n-1)}, I_{V(n-1)}\right), \\
S_{V(n+1)} &= S_{Vn} + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_7(t, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}) \\
&\quad + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_7\left(t, S_{h(n-1)}, E_{D(n-1)}, I_{D(n-1)}, T_{D(n-1)}, V_{(n-1)}, R_{D(n-1)}, S_{V(n-1)}, I_{V(n-1)}\right), \\
I_{V(n+1)} &= I_{Vn} + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_7(t, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}) \\
&\quad + \left(\tfrac{1-\eta}{N(\eta)} + \tfrac{3\eta h}{2N(\eta)}\right) F_7\left(t, S_{h(n-1)}, E_{D(n-1)}, I_{D(n-1)}, T_{D(n-1)}, V_{(n-1)}, R_{D(n-1)}, S_{V(n-1)}, I_{V(n-1)}\right).
\end{aligned}\right\} \tag{46}$$

3.7 Dengue fever Disease system with AB derivative

Following is an alternative way of describing the Dengue fever disease system using AB time derivatives of arbitrary orders. In what follows, we show a predictor–corrector algorithm with Adams type implementation [3] approaches the system's approximate solution using the AB arbitrary-order operator. A system for Dengue fever disease studied with AB variable-order derivative of a generalized Mittag–Leffler function is as follows:

$$\left.\begin{aligned}
{}_0^{ABC}D_t^{\eta} S_h(t) &= \pi_h + \alpha R_D + \tau_1 V - \lambda_D S_h - (\tau_2 + \mu_h) S_h,\\
{}_0^{ABC}D_t^{\eta} E_D(t) &= \lambda_D S_h - (\omega + \mu_h) E_D,\\
{}_0^{ABC}D_t^{\eta} I_D(t) &= \omega E_D - (\rho + \delta_1 + \mu_h) I_D,\\
{}_0^{ABC}D_t^{\eta} T_D(t) &= \rho I_D - (\sigma + \delta_2 + \mu_h) T_D,\\
{}_0^{ABC}D_t^{\eta} V(t) &= \tau_2 S_h - (\tau_1 + \mu_h) V,\\
{}_0^{ABC}D_t^{\eta} R_D(t) &= \sigma T_D - (\alpha + \mu_h) R_D,\\
{}_0^{ABC}D_t^{\eta} S_V(t) &= \pi_V - \lambda_V S_V - \mu_V S_V,\\
{}_0^{ABC}D_t^{\eta} I_V(t) &= \lambda_V S_V - (\delta_V + \mu_h) I_V.
\end{aligned}\right\} \tag{47}$$

For simplicity, we let

$$\left.\begin{aligned}
{}_0^{ABC}D_t^{\eta} S_h(t) &= F_1(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V),\\
{}_0^{ABC}D_t^{\eta} E_D(t) &= F_2(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)\\
{}_0^{ABC}D_t^{\eta} I_D(t) &= F_3(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V),\\
{}_0^{ABC}D_t^{\eta} T_D(t) &= F_4(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V),\\
{}_0^{ABC}D_t^{\eta} V(t) &= F_5(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V),\\
{}_0^{ABC}D_t^{\eta} R_D(t) &= F_6(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V),\\
{}_0^{ABC}D_t^{\eta} S_V(t) &= F_7(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V),\\
{}_0^{ABC}D_t^{\eta} I_V(t) &= F_8(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V).
\end{aligned}\right\} \tag{48}$$

After, applying the AB arbitrary-order integral on both sides of above system (48), we obtain:

$$\left.\begin{aligned}
S_h(t) &= S_{h0} + \tfrac{1-\eta}{AB(\eta)} F_1(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)\\
&\quad + \tfrac{\eta}{AB(\eta)} \int_0^t F_1(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)(t-\alpha)^{\eta-1} d\alpha,\\
E_D(t) &= E_{D0} + \tfrac{1-\eta}{AB(\eta)} F_2(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)\\
&\quad + \tfrac{\eta}{AB(\eta)} \int_0^t F_2(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)(t-\alpha)^{\eta-1} d\alpha,\\
I_D(t) &= I_{D0} + \tfrac{1-\eta}{AB(\eta)} F_3(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)\\
&\quad + \tfrac{\eta}{AB(\eta)} \int_0^t F_3(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)(t-\alpha)^{\eta-1} d\alpha,\\
T_D(t) &= T_{D0} + \tfrac{1-\eta}{AB(\eta)} F_4(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)\\
&\quad + \tfrac{\eta}{AB(\eta)} \int_0^t F_4(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)(t-\alpha)^{\eta-1} d\alpha,\\
V(t) &= V_0 + \tfrac{1-\eta}{AB(\eta)} F_5(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)\\
&\quad + \tfrac{\eta}{AB(\eta)} \int_0^t F_5(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)(t-\alpha)^{\eta-1} d\alpha,\\
R_D(t) &= R_{D0} + \tfrac{1-\eta}{AB(\eta)} F_6(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)\\
&\quad + \tfrac{\eta}{AB(\eta)} \int_0^t F_6(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)(t-\alpha)^{\eta-1} d\alpha,\\
S_V(t) &= S_V(t)_0 + \tfrac{1-\eta}{AB(\eta)} F_7(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)\\
&\quad + \tfrac{\eta}{AB(\eta)} \int_0^t F_7(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)(t-\alpha)^{\eta-1} d\alpha,\\
I_V(t) &= I_V(t)_0 + \tfrac{1-\eta}{AB(\eta)} F_8(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)\\
&\quad + \tfrac{\eta}{AB(\eta)} \int_0^t F_8(t, S_h, E_D, I_D, T_D, V, R_D, S_V, I_V)(t-\alpha)^{\eta-1} d\alpha.
\end{aligned}\right\} \tag{49}$$

The calculation for the above arbitrary-order integral is handled by the numerical technique applied to the AB arbitrary-order integral. The Adams-type predictor–corrector scheme (developed by Adams) is

applied here to solve the AB fractional integral equation. An AB integral called arbitrary-order integral exists.

$$ {}_0^{AB}I_t^{\eta}\left[Y\left(t\right)\right]=\frac{1-\eta}{AB\left(\eta\right)}Y\left(t\right)+\frac{\eta}{AB\left(\eta\right)\Gamma\left(\eta\right)}\int_0^t Y\left(\alpha\right)(t-\alpha)^{\eta-1}d\alpha. \tag{50} $$

Let $h=\frac{T}{N}, t_k=hk, \ \ (k=0,1,2,...,N)$

the solution is being considered in the interval $[0,T]$. Consequently, the corrector formula of integral version of AB derivative is suggested as below:

$$ Y_h\left(t_{n+1}\right)=Y_0\left(t_{n+1}\right)+\frac{\left(1-\eta\right)h^{\eta}}{AB\left(\eta\right)\Gamma\left(\eta+2\right)}f\left(t_{n+1},Y_h^p\left(t_{n+1}\right)\right)+\frac{\eta h^{\eta}}{AB\left(\eta\right)\Gamma\left(\eta+2\right)}\sum_{j=0}^{n}\theta_{j,n+1}f\left(t_j,Y_h\left(t_j\right)\right). \tag{51} $$

Where $\begin{cases} n^{\eta+1}-(n-\eta)(n+\eta)^{\eta}, if\ j=0 \\ (n-j+2)^{\eta+1}+(n-j)^{\eta+1}-2(n-j+1)^{\eta+1}, if\ 1\leq j\leq k \\ 1 \ \ if\ j=n+1 \end{cases}$

Also the predictor $Y_h^p\left(t_{n+1}\right)$ is suggested as follows:

$$ Y_h^p\left(t_{n+1}\right)=Y_0+\frac{\left(1-\eta\right)}{AB\left(\eta\right)}f\left(t_n,Y_h\left(t_n\right)\right)+\frac{\eta}{AB\left(\eta\right)\Gamma^2\left(\eta\right)}\sum_{j=0}^{n}\delta_{j,n+1}f\left(t_j,Y_h\left(t_j\right)\right), \tag{52} $$

where

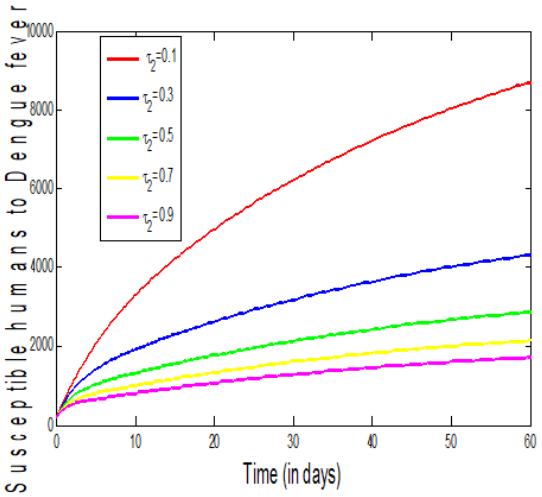
$\delta_{j,n+1}=\frac{h^{\eta}}{\eta}(n+j-1)^{\eta+1}-(n-j)^{\eta}, \ 0\leq j\leq n$. The above schemes transform the Dengue fever disease system (48) into the simple iterative rule given below:

$$
\left.\begin{aligned}
S_h(t) &= S_h(0) + \tfrac{(1-\eta)h^\eta}{AB(\eta)\Gamma(\eta+2)}\left[F_1\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)} \\ , R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma(\eta+2)}\sum_{j=0}^{n}\theta_{j,n+1}\left[F_1\left(t, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
E_D(t) &= E_D(0) + \tfrac{(1-\eta)h^\eta}{AB(\eta)\Gamma(\eta+2)}\left[F_2\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)}, \\ R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma(\eta+2)}\sum_{j=0}^{n}\theta_{j,n+1}\left[F_2\left(t, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
I_D(t) &= I_D(0) + \tfrac{(1-\eta)h^\eta}{AB(\eta)\Gamma(\eta+2)}\left[F_3\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)} \\ , R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma(\eta+2)}\sum_{j=0}^{n}\theta_{j,n+1}\left[F_3\left(t, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
T_D(t) &= T_D(0) + \tfrac{(1-\eta)h^\eta}{AB(\eta)\Gamma(\eta+2)}\left[F_4\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)}, \\ R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma(\eta+2)}\sum_{j=0}^{n}\theta_{j,n+1}\left[F_4\left(t, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
V(t) &= V(0) + \tfrac{(1-\eta)h^\eta}{AB(\eta)\Gamma(\eta+2)}\left[F_5\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)}, \\ R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma(\eta+2)}\sum_{j=0}^{n}\theta_{j,n+1}\left[F_5\left(t, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
R_D(t) &= R_D(0) + \tfrac{(1-\eta)h^\eta}{AB(\eta)\Gamma(\eta+2)}\left[F_6\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)}, \\ R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma(\eta+2)}\sum_{j=0}^{n}\theta_{j,n+1}\left[F_6\left(t, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
S_V(t) &= S_V(0) + \tfrac{(1-\eta)h^\eta}{AB(\eta)\Gamma(\eta+2)}\left[F_7\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)}, \\ R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma(\eta+2)}\sum_{j=0}^{n}\theta_{j,n+1}\left[F_7\left(t, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
I_V(t) &= I_V(0) + \tfrac{(1-\eta)h^\eta}{AB(\eta)\Gamma(\eta+2)}\left[F_8\left(\begin{array}{l} t_{n+1}, S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)}, \\ R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)} \end{array}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma(\eta+2)}\sum_{j=0}^{n}\theta_{j,n+1}\left[F_8\left(t, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right].
\end{aligned}\right\} \tag{53}
$$

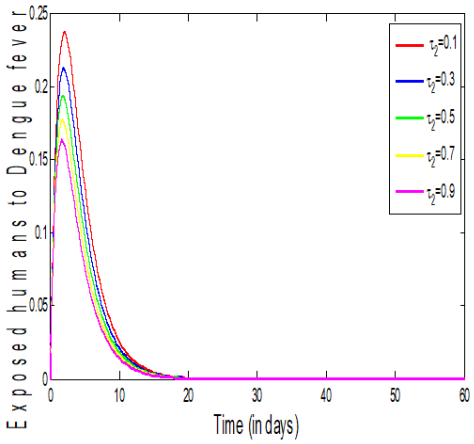
The predictor term $S^p_{h(n+1)}, E^p_{D(n+1)}, I^p_{D(n+1)}, T^p_{D(n+1)}, V^p_{(n+1)}, R^p_{D(n+1)}, S^p_{V(n+1)}, I^p_{V(n+1)}$ are given as:

$$
\left.
\begin{aligned}
S^{p}_{h(n+1)} &= S_h\left(0\right) + \tfrac{(1-\eta)}{\Gamma(\eta)}\left[F_1\left(t_n, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma^2(\eta)}\sum_{j=0}^{n}\delta_{j,n+1}\left[F_1\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
E^{p}_{D(n+1)} &= E_D\left(0\right) + \tfrac{(1-\eta)}{\Gamma(\eta)}\left[F_2\left(t_n, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma^2(\eta)}\sum_{j=0}^{n}\delta_{j,n+1}\left[F_2\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
I^{p}_{D(n+1)} &= I_D\left(0\right) + \tfrac{(1-\eta)}{\Gamma(\eta)}\left[F_3\left(t_n, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma^2(\eta)}\sum_{j=0}^{n}\delta_{j,n+1}\left[F_3\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
T^{p}_{D(n+1)} &= T_D\left(0\right) + \tfrac{(1-\eta)}{\Gamma(\eta)}\left[F_4\left(t_n, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma^2(\eta)}\sum_{j=0}^{n}\delta_{j,n+1}\left[F_4\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
V^{p}_{(n+1)} &= V\left(0\right) + \tfrac{(1-\eta)}{\Gamma(\eta)}\left[F_5\left(t_n, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma^2(\eta)}\sum_{j=0}^{n}\delta_{j,n+1}\left[F_5\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
R^{p}_{D(n+1)} &= R_D\left(0\right) + \tfrac{(1-\eta)}{\Gamma(\eta)}\left[F_6\left(t_n, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma^2(\eta)}\sum_{j=0}^{n}\delta_{j,n+1}\left[F_6\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
S^{p}_{V(n+1)} &= S_V\left(0\right) + \tfrac{(1-\eta)}{\Gamma(\eta)}\left[F_7\left(t_n, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma^2(\eta)}\sum_{j=0}^{n}\delta_{j,n+1}\left[F_7\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right], \\
I^{p}_{V(n+1)} &= I_V\left(0\right) + \tfrac{(1-\eta)}{\Gamma(\eta)}\left[F_7\left(t_n, S_{hn}, E_{Dn}, I_{Dn}, T_{Dn}, V_n, R_{Dn}, S_{Vn}, I_{Vn}\right)\right] \\
&\quad + \tfrac{\eta}{AB(\eta)\Gamma^2(\eta)}\sum_{j=0}^{n}\delta_{j,n+1}\left[F_7\left(t_j, S_{hj}, E_{Dj}, I_{Dj}, T_{Dj}, V_j, R_{Dj}, S_{Vj}, I_{Vj}\right)\right].
\end{aligned}
\right\} \tag{54}
$$

4 Numerical simulation

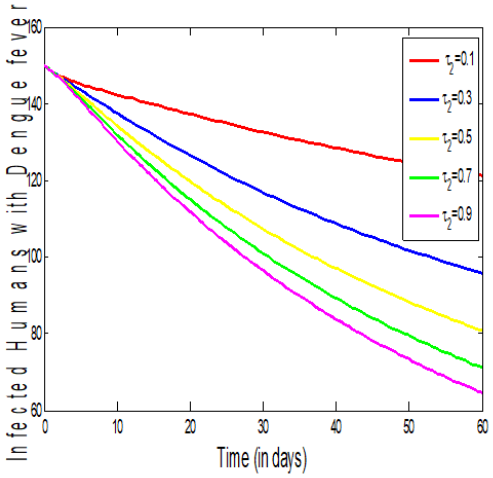


(a) Simulation of susceptible humans to Dengue fever

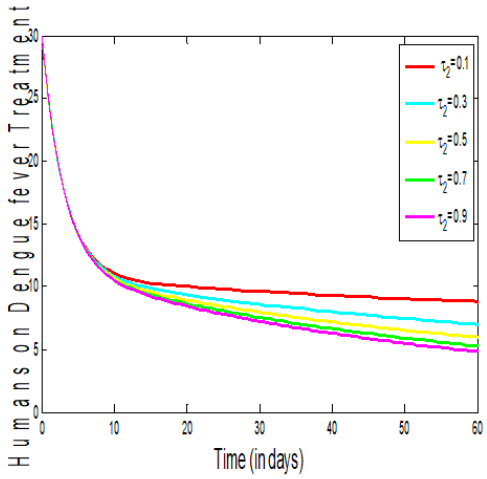


(b) Simulation of Exposed humans to Dengue fever

Figure 2: Simulation of the effect τ_2 on Susceptible and Exposed humans Dengue fever

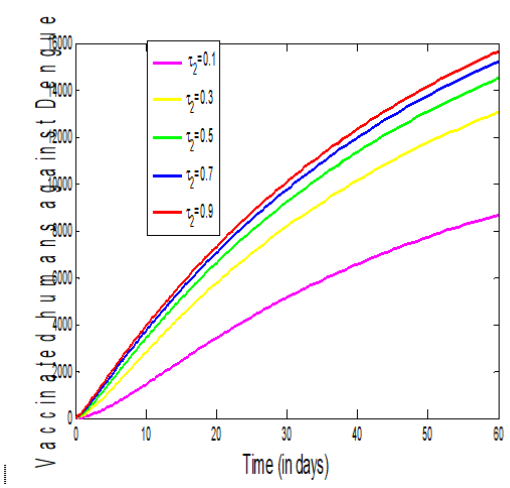


(a) Simulation of infected humans with Dengue fever

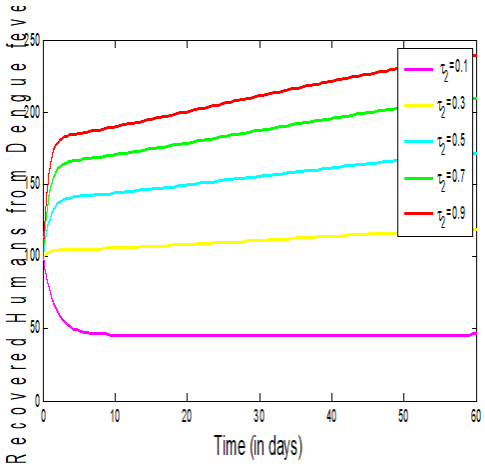


(b) Simulation of humans on Dengue fever treatment

Figure 3: Simulation of the effect τ_2 on infected humans with Dengue fever and humans on Dengue fever treatment

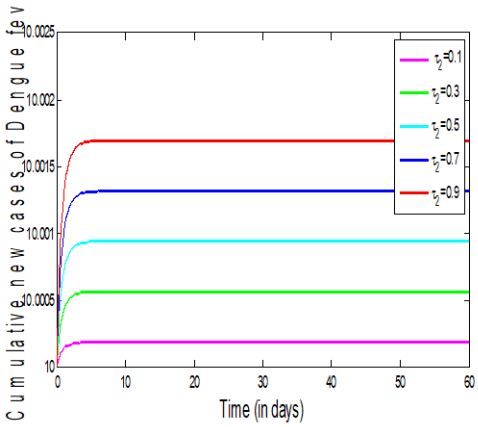


(a) Simulation of vaccinated humans against Dengue fever

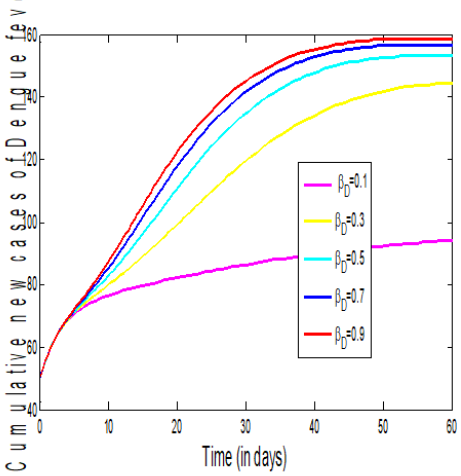


(b) Simulation of Recovered humans from Dengue fever

Figure 4: Simulation of the effect τ_2 on vaccinated humans against Dengue fever and Recovered humans from Dengue fever

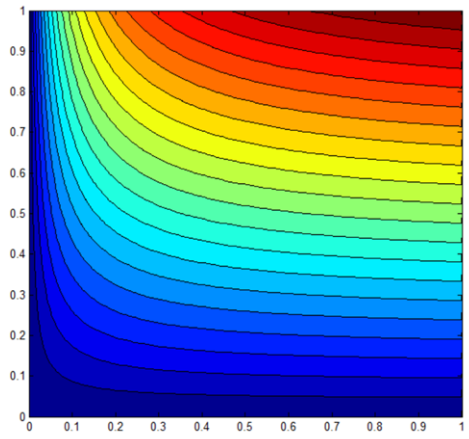


(a) Simulation of cumulative new cases of Dengue fever

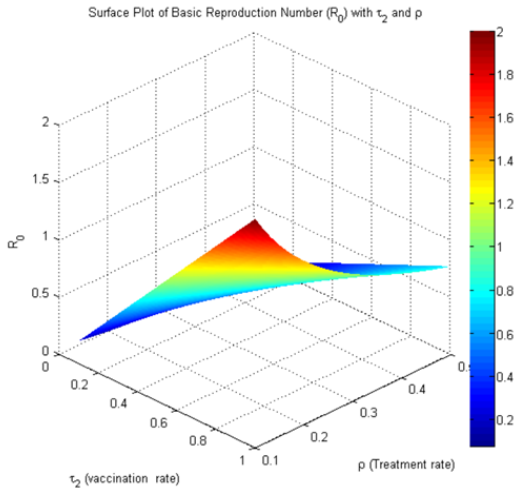


(b) Simulation of cumulative new cases of Dengue fever

Figure 5: Simulation of the effect τ_2 and β_D on cumulative new cases of Dengue fever



(a) **Contour plot showing the Impact of ρ and τ_2 on R_0^D**



(b) **Surface plot showing the Impact of ρ and τ_2 on R_D^0**

Figure 6: **Contour and Surface plot of the basic reproduction number**

(2a) demonstrates how vaccination rate (τ_2) affects those who are susceptible to Dengue fever. The number of people at risk of infection often decreases when the vaccination rate (τ_2) increases. (2b) shows how Dengue fever affects people in a population after vaccination rates (τ_2) change. It is noticed that when more people get vaccinated, fewer people are exposed to the virus. (3a) shows how Dengue fever affects people in a population after vaccination rates (τ_2) change. It is noticed that when more people get vaccinated, fewer people will be infected to the virus. (3b) shows how Dengue fever affects people in a population after vaccination rates (τ_2) change. It is noticed that when more people get vaccinated, fewer people receive Dengue fever treatment. (4a) shows how Dengue fever affects people in a population after vaccination rates (τ_2) change. It is noticed that when more people get vaccinated the number of people vaccinated against the Dengue fever increases. (4b) shows how Dengue fever affects people in a population after vaccination rates (τ_2) change. It is noticed that when more people get vaccinated the number of recovered humans from the Dengue fever increases. (5a) shows how Dengue fever affects people in a population after vaccination rates (τ_2) change. It is noticed that when more people get vaccinated, the cumulative new cases of Dengue fever decreases. (5b) shows how Dengue fever affects people in a population after contact rate (β_D)changes. It is noticed that, as the contact rate increases, the cumulative new cases of Dengue fever increases. (6a) demonstrates the shape of the relationship R_0^D. A look at the graph shows that the maximum reached by the parameters is 0.8 and the value of R_0^D is still smaller than unity (1). So, enhancing these factors will not likely result in a major increase in Dengue infections. (6b) shows that if the value of rho and tau_2 increases, the basic reproduction number drops below one (1) much earlier. It was found that increasing rho and tau_2 will eventually help reduce the impact of Dengue fever on human population.

Parameters	Values/day	Sources
π_h	500	[51]
π_V	10000000	[51]
μ_h	0.02041	[1]
μ_V	0.5	[1]
β_D	0.5	[51]
β_V	0.4	[51]
ρ	0.1	[51]
ω	0.3254	[51]
δ_1	0.365	[1]
δ_2	0.213	Estimated
δ_V	0.21	Estimated
α	0.3	Estimated
σ	0.5	Estimated
τ_1	0.03	Estimated
τ_2	0.5	Estimated

Table 1: **Parameter values and sources**

5 Conclusion

Studying a changing-order fractional outbreak model of Dengue fever allows us to see how the disease is transmitted in any general population. The suggested model was generalized further by including Caputo, CF and AB kinds of fractional derivatives. Three kinds of kernels were used to build the calculation tools which made the results more accurate. To solve in different systems, the choices were Caputo's fractional derivative with Adams–Bashforth–Moulton method, Adams–Bashforth–Moulton scheme for any-order CF derivative and the Adams predictor–corrector AB–Caputo fractional derivative. For the purpose of explaining CF derivative, the study also explored fixed-point theory. How dependable and fast the procedures are is shown in our visuals. Many pictures have been made to clarify the different outcomes possible under randomly chosen rules. In summary, using this operator clearer explains how the model works.

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Credit authorship contribution statement

Enejoh Jalija: Writing–original draft, Formal analysis. Jeremiah Amos: Writing – original draft, Formal analysis, William Atokolo Writing – original draft, Formal analysis., David Omale: Writing – original draft, Formal analysis., Emmanuel Abah: Writing – original draft., Ugbede Alih: Writing – original draft., Bolarinwa Bolaji: Writing – original draft, Formal analysis.

Declaration of competing interest

No financial conflicts or personal relationships affecting the research findings of this paper exist according to the authors.

Data availability

Values of parameters were fitted, and also parameters used are adequately cited and referenced.

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