International Journal of Mathematical Analysis and Modelling

(Formerly Journal of the Nigerian Society for Mathematical Biology)

Volume 7 Issue 2, 2024

ISSN (Print): 2682 - 5694
ISSN (Online): 2682 – 5708
www.tnsmb.org

Estimation of different types of entropies for the Gumbel Type-2 distribution

D.O. Omosigho*†, A.D. Adefolarin*, and N.A. Adeleye‡

Abstract

The estimation of the entropy of a random system is relevant in many scientific applications. This article is aimed at the analysis of the entropy of the Gumbel type-2 distribution, an aspect of the statistical analysis which has not been employed. On this basis, six different entropy measures are considered and expressed analytically. A numerical study is performed to discuss the behavior of these measures. In addition, we considered the method of maximum likelihood estimation approach to obtain the parameter estimates of the Gumbel type-2 distribution. Maximum likelihood estimates for the considered entropy measures are thus derived. The convergence properties of these estimates are proved through a simulated data, showing their numerical efficiency. Concrete applications to two real data sets are provided.

Keywords and phrases: entropy; maximum likelihood estimation; gumbel type-2 distribution; random system.

1 Introduction

Entropy, which is one of the important terms in statistical mechanics, was originally defined in physics especially in the second law of thermodynamics. In statistical theory, Entropy is defined as a measure of uncertainty associated with a distribution. In every probability distribution there is some element of uncertainty associated with it and Entropy is used to provide a reasonable explanation for the level of uncertainty present. Entropy can also be considered as a measure of randomness of a probabilistic system. The concept of Entropy was redefined by [12] as a measure of information, which provides a quantitative measure for the level of uncertainty of a random system. It is a measure of average uncertainty in a random variable. When a system is in a number of states randomly with equal probability of occurrences entropy attains its maximum value and reach minimum value i.e., 0 when the system is in one particular state, which indicates that there is no uncertainty in its description.

*Department of Mathematics and Statistics, Federal Polytechnic Ado-Ekiti, Nigeria
†Corresponding author
‡Department of Statistics, Ogun State Institute of Technology, Igbesa, Ogun State, Nigeria

Let X be a random variable with a continuous distribution function (CDF) $F(x)$ and probability density function (PDF) $f(x)$. The differential entropy $J(X)$ of the random variable is defined by [7] as:

$$J((X) = J(f) = -\int_{-\infty}^{\infty} f(x)F(x)dx$$

It could be observed that a very sharply peaked distribution has very low entropy, whereas if the probability is spread out, the entropy is higher. In this sense, $J(X)$ is a measure of uncertainty associated with f. Furthermore, as $J(f)$ increases, $f(x)$ approaches uniformity. Thus, entropy can be taking as a tool for measuring the uniformity of a distribution,[8]. Thus, entropy can be viewed as measuring the uniformity of a distribution, [8] Many authors have studied different techniques for estimating entropy for different life distributions. [4] developed and studied the entropy of upper record values and provided several upper and lower bounds for this entropy by using the hazard rate function. [5] examined the entropy in the Weibull distribution for progressive censoring. [6] developed and studied the estimators for the entropy function of a Rayleigh distribution based on doubly-generalized Type II hybrid censored samples by making use of maximum likelihood estimator (MLE), approximate MLEs and Bayes estimators.

The units of entropy majorly depend on the base of logarithm. If the base of the logarithm is e, then entropy is measured in nats and if the base of the logarithm is 2, then the entropy is measured in bits. Entropy is used to study a wide range of physical, biological and chemical phenomenon. We observe that the movement of molecule in biological sciences is random so, to study the different properties of the molecules we have to find the entropy of probability distribution of the molecule. In the present study, we have derived the expressions for seven different types of entropy for Gumbel type-2 distribution and also carried out a numerical study on this entropy formation.

2 Methodology/Model formulation

In probability theory, the Gumbel type-2 probability density function is given as

$$f(x;\alpha,\varphi) = \alpha\varphi x^{-\varphi-1}e^{-\alpha x^{-\varphi}}, \qquad x > 0; \alpha, \varphi > 0. \tag{1}$$

The graph of probability density function of Gumbel type-2 distribution for various values of the parameters is given below in figure 1

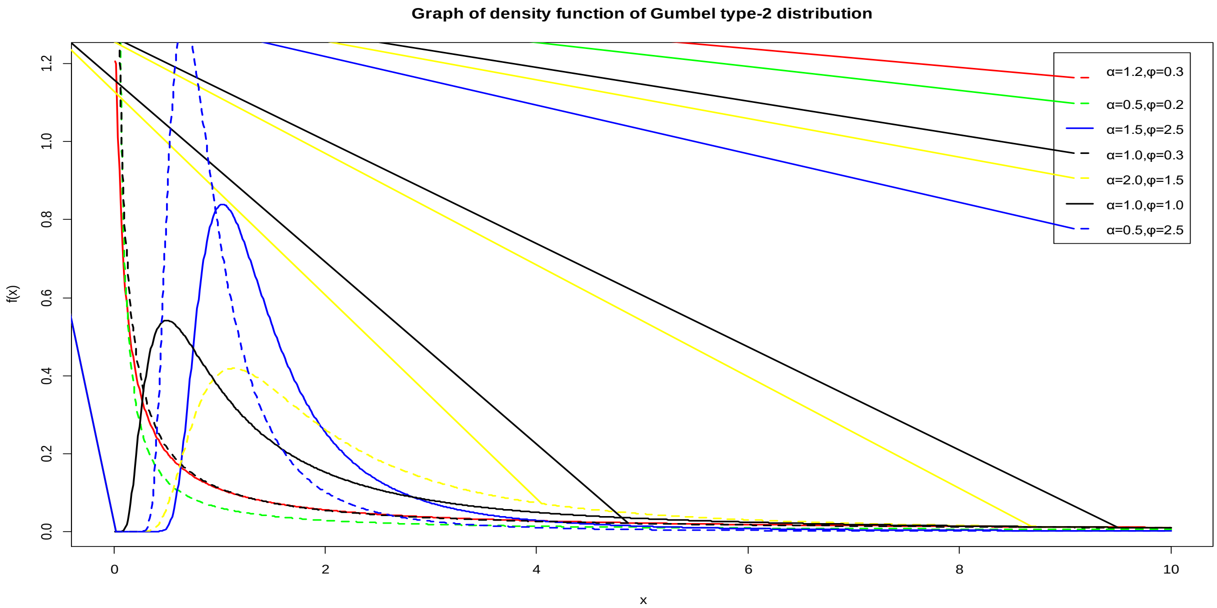

Figure 1. The graph of probability density function of Gumbel type-2 distribution.

2.1 An integral result

The following result given in Lemma 1 shows that a certain integral involving the PDF of the Gumbel type-2 distribution is pertinent in obtaining some properties of different measures of entropy that will be considered in this work.

Lemma 1: Let $\lambda > 0$, $f(x;\alpha,\varphi)$ be specified by equation (1) and

$$I_\lambda(\alpha,\varphi) = \int_{-\infty}^{+\infty} f(x;\alpha,\varphi)^\lambda dx, \tag{2}$$

Then, $I_\lambda(\alpha,\varphi)$ exist if and only if $\min(\alpha,\varphi) > \max\left(1 - {}^{1}/_{\lambda}, 0\right)$, and is given as

$$I_\lambda(\alpha,\varphi) = \alpha^\lambda \varphi^{\lambda-1} [\alpha\varphi]^{1-{}^{\lambda(\varphi+1)}/_{\lambda}} \Gamma[(\lambda-1)(\varphi+1)+1] \tag{3}$$

Equation (3) is a very important concept that is used in deriving different measures of entropy of Gumbel type-2 considered in this work.

2.2 Various entropy measures

Various approaches can be used to estimate the entropy of Gumbel type-2. The important entropy measures which is mostly applicable in literature is presented in Table 1 for a general distribution with pdf denoted by $f(x;\alpha,\varphi)$. Also, we adhere to basic assumptions that allow for the use of different entropy measures which is $\lambda > 0$ and $\lambda \neq 1$.

From Table 1, which summarizes different type of entropy considered, we can emphatically say that the integral $\int_{-\infty}^{+\infty} f(x;\alpha,\varphi)^{\lambda}dx$ is very important in determining the considered entropy measures. Now, we consider different entropy measures of Gumbel type-2 distribution. Based on Lemma 1, it is pertinent that α, φ and λ satisfy $min\,(\alpha,\varphi) > max\left(1-{}^{1}/_{\lambda},0\right)$.

Table 1. Important entropy measures of a distribution with pdf $\boldsymbol{f(x;\alpha,\varphi)}$ at $\boldsymbol{\lambda}$

$\boldsymbol{Name\ of\ entropy}$	$\boldsymbol{Notation}$	Reference	$\boldsymbol{Expression}$
$Renyi$	$R_{\lambda}(\alpha,\varphi)$	[10]	$\frac{1}{1-\lambda} log\left[\int_{-\infty}^{+\infty} f(x;\rho)^{\lambda}dx\right]$
$Tsallis$	$T_{\lambda}(\alpha,\varphi)$	[13]	$\frac{1}{\lambda-1}\left[1-\int_{-\infty}^{+\infty} f(x;\rho)^{\lambda}dx\right]$
$Arimoto$	$A_{\lambda}(\alpha,\varphi)$	[2]	$\frac{\lambda}{1-\lambda}\left(\left[\int_{-\infty}^{+\infty} f(x;\rho)^{\lambda}dx\right]^{\frac{1}{\lambda}}-1\right)$
$Havrda\ and\ Charvat$	$HC_{\lambda}(\alpha,\varphi)$	[9]	$\frac{1}{2^{1-\lambda}-1}\left[\int_{-\infty}^{+\infty} f(x;\rho)^{\lambda}dx-1\right]$
$Award\ and\ Alawneh\ 1$	$AA1_{\lambda}(\alpha,\eta)$	Awad and Alawneh (1987)	$\frac{1}{1-\lambda} log\left[\left(\operatorname{Sup}_{x\in R} f(x,\rho)\right)^{1-\lambda}\int_{-\infty}^{+\infty} f(x;\rho)^{\lambda}dx\right]$
$Award\ and\ Alawneh\ 2$	$AA2_{\lambda}(\alpha,\eta)$	Awad and Alawneh (1987)	$\frac{1}{2^{1-\lambda}-1}\left[\left(\operatorname{Sup}_{x\in R} f(x,\rho)\right)^{1-\lambda}\int_{-\infty}^{+\infty} f(x;\rho)^{\lambda}dx-1\right]$

2.2.1 Renyi entropy: Based on Table 1, equation (1) and Lemma 1, the Renyi entropy of the Gumbel type-2 distribution can be expressed as

$$
\begin{aligned}
R_{\lambda}(\alpha,\eta) &= \frac{1}{1-\lambda} log[I_{\lambda}(\alpha,\varphi)] \\
&= \frac{1}{1-\lambda} log\left\{\alpha^{\lambda}\varphi^{\lambda-1}[\alpha\varphi]^{1-{}^{\lambda(\varphi+1)}/_{\lambda}}\Gamma[(\lambda-1)(\varphi+1)+1]\right\} \qquad (4)
\end{aligned}
$$

2.2.2 Havrada and Charvat entropy: From Table 1, equation (1) and proposition 1, Havrda and Charvat entropy of the Gumbel type-2 distribution can be expressed as

$$HC_\lambda(\alpha,\eta) = \frac{1}{2^{1-\lambda}-1}\left(I_\lambda(\alpha,\eta)-1\right)$$
$$= \frac{1}{2^{1-\lambda}-1}\left\{\alpha^\lambda\varphi^{\lambda-1}[\alpha\varphi]^{1-\lambda(\varphi+1)/\lambda}\Gamma[(\lambda-1)(\varphi+1)+1]-1\right\} \quad (5)$$

2.2.3 Arimoto entropy: from Table 1, equation (1) and proposition 1, Arimoto entropy of the Gumbel type-2 distribution can be obtained as

$$A_\lambda(\alpha,\eta) = \frac{\lambda}{1-\lambda}\left\{[I_\lambda(\alpha,\varphi)]^{\frac{1}{\lambda}}-1\right\}$$
$$A_v(\alpha,\eta) = \frac{\lambda}{1-\lambda}\left(\left[\alpha^\lambda\varphi^{\lambda-1}[\alpha\varphi]^{1-\lambda(\varphi+1)/\lambda}\Gamma[(\lambda-1)(\varphi+1)+1]\right]^{\frac{1}{\lambda}}-1\right) \quad (6)$$

2.2.4 Tsallis entropy: From Table 1, equation (1) and proposition 1, Tsallis entropy of the Gumbel type-2 distribution can be represented by

$$T_\lambda(\alpha,\eta) = \frac{1}{\lambda-1}\left\{1-[I_\lambda(\alpha,\varphi)]^{\frac{1}{\lambda}}\right\}$$
$$= \frac{1}{\lambda-1}\left(1-\alpha^\lambda\varphi^{\lambda-1}[\alpha\varphi]^{1-\lambda(\varphi+1)/\lambda}\Gamma[(\lambda-1)(\varphi+1)+1]\right) \quad (7)$$

2.2.5 Award and Alawneh 1: From Table 1, equation (1), and proposition 1, Award and Alawneh 1 entropy of Gumbel type-2 distribution can be obtained as

$$AA1_\lambda(\alpha,\eta) = \frac{1}{1-\lambda}\log\left[\left(\operatorname*{Sup}_{x\in R} f(x,\rho)\right)^{1-\lambda}\int_{-\infty}^{+\infty} f(x;\rho)^\lambda dx\right] \quad (8)$$

Before going further, there is need to determine $sup_{0<x<\infty} f(x;\alpha,\varphi)$, which is provided ion the following lemma.

Lemma 2. Let $\boldsymbol{f}(x;\alpha,\varphi)$ be given as in (1). Then, $sup_{0<x<\infty} f(x;\alpha,\varphi)$ is finite if and only if $\alpha > 0$ and $\varphi > 0$, based on this we have

$$sup_{0<x<\infty} f(x;\alpha,\varphi) = \alpha\varphi\left(\frac{\alpha\varphi}{\varphi+1}\right)^{-\varphi-1/\varphi} e^{-\alpha\left(\frac{\alpha\varphi}{\varphi+1}\right)^{-1}}$$

Proof. We have

$$f'(x;\alpha,\varphi) = \alpha\varphi\left[(-\varphi-1)x^{-\varphi-2}e^{-\alpha x^{-\varphi}} + x^{-\varphi-1}(\alpha\varphi x^{-\varphi-1})e^{-\alpha x^{-\varphi}}\right]$$

Therefore, taking $f'(x;\alpha,\varphi) = 0$ implies that

$$x_* = \left(\frac{\alpha\varphi}{\varphi+1}\right)^{1/\varphi}$$

Since $f'(x;\alpha,\varphi) > 0$ for $x < x_*$ and $f'(x;\alpha,\varphi) < 0$ for $x > x_*$, x_* is considered as a maximum point for $f(x;\alpha,\varphi)$. Hence

$$\lim_{0<x<\infty} f(x;\alpha,\varphi) = f(x_*;\alpha,\varphi) = \alpha\varphi {x_*}^{-\varphi-1} e^{-\alpha {x_*}^{-\varphi}}$$
$$= \alpha\varphi\left(\frac{\alpha\varphi}{\varphi+1}\right)^{-\varphi-1/\varphi} e^{-\alpha\left(\frac{\alpha\varphi}{\varphi+1}\right)^{-1}}$$

It should be noted that, taking $\varphi = 1$, we have $f(x;\alpha,\varphi) = {4e^{-2}}/{\alpha}$ and for $\alpha = 1$, we have $f(x;\alpha,\varphi) = \varphi^{-1/\varphi}(\varphi+1)^{1+1/\varphi} e^{-\left(1+1/\varphi\right)}$.

2.2.6 Award and Alawneh 2: From Table 1, equation (1), and proposition 1, Award and Alawneh 2 entropy of Gumbel type-2 distribution can be obtained as

$$AA2_\lambda(\alpha,\eta) = \frac{1}{2^{1-\lambda}-1}\left[\left(\operatorname*{Sup}_{x\in R} f(x,\rho)\right)^{1-\lambda} \int_{-\infty}^{+\infty} f(x;\rho)^\lambda dx - 1\right] \tag{9}$$

Theoretically, it is complicated to study the characteristics behaviour of these entropy measures. Based on this reason, a numerical study will be used to obtain the values of the forms of entropy under study in the next section.

3 Estimation of the entropy measures

The estimation of the parameters of the Gumbel type-2 model using the maximum likelihood is well known in statistical literature and details can be found. Suppose $x_1, \dots, x_n$ is observed from a random variable X having a Gumbel type-2 distribution with parameters α and φ. The maximum likelihood estimation (MLEs) of α and φ, say, $\hat{\alpha}$ and $\hat{\varphi}$, are defined by

$$(\hat{\alpha}\,,\hat{\varphi}) = argmax_{(\alpha,\varphi)\in(0,+\infty)^2}\, l(\alpha,\varphi),$$

where $l(\alpha,\varphi)$ represent the likelihood function specified by

$$l(\alpha,\varphi) = nlog\alpha + nlog\varphi - (\varphi+1)\sum_{i=1}^{n} log(x_i) - \alpha\sum_{i=1}^{n} {x_i}^{-\varphi}$$

The MLEs are the solutions of the two equations given as follows:

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} {x_i}^{-\varphi} = 0, \quad \frac{\partial l}{\partial\varphi} = \frac{n}{\varphi} - \sum_{i=1}^{n} log(x_i) - \alpha\sum_{i=1}^{n} {x_i}^{-\varphi}\, log(x_i) = 0$$

and this implies that $\hat{\alpha}$ and $\hat{\varphi}$ satisfies the following simple relation

$$\hat{\alpha} = \frac{n}{\sum_{i=1}^{n} {x_i}^{-\varphi}}$$

Then, it follows that the properties of these MLEs follow the usual maximum likelihood theory. Based on this one can easily obtained the MLEs of the entropy measures.

3.1 Numerical values

We now investigate the numerical values for the six entropy measures presented in subsection 2.2 under the following configuration of the parameters: configuration 1, $\alpha = 0.5$ with $\varphi = \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0\}$ and $\lambda = 1.5$, configuration 2, $\alpha = 0.5$ with

$\varphi = \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0\}$ and $\lambda = 3.5$, configuration 3, $\varphi = 0.5$ with $\alpha = \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0\}$ and $\lambda = 1.5$ configuration 4, $\varphi = 0.5$ with $\alpha = \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0\}$ and $\lambda = 3.5$. The findings of all the six entropy measures are presented for this configurations in Table $2-5$, respectively.

Table 2-5 presents the numerical values for all the entropy measures considered for different values of the parameters of the Gumbel type-2 model.

Table 2. Numerical values of the considered entropy of the Gumbel type-2 distribution at $\boldsymbol{\alpha = 0.5}$ and $\boldsymbol{\lambda = 1.5}$

$\boldsymbol{\varphi}$	$R_\lambda(\alpha,\varphi)$	$T_\lambda(\alpha,\varphi)$	$A_\lambda(\alpha,\varphi)$	$HC_\lambda(\alpha,\varphi)$	$AA1_\lambda(\alpha,\eta)$	$AA2_\lambda(\alpha,\eta)$
1.5	0.9651	0.7656	0.8253	−0.9246	2.6130	2.4898
2.0	0.5256	0.4622	0.4821	−0.1586	2.1902	2.2721
2.5	0.4697	0.3371	0.4094	−0.0957	2.1887	2.0506
3.0	0.1225	0.1188	0.1200	−0.0246	1.8357	1.4889
3.5	−0.2420	−0.2572	−0.2520	0.0533	1.5320	1.8270
4.0	−0.5720	−0.6622	−0.6301	0.1371	1.2664	1.6017
4.5	−0.8720	−1.0930	−1.0194	0.2264	1.0314	1.3756
5.0	−1.1464	−1.5478	−1.3962	0.3206	0.8205	1.1489
5.5	−1.3989	−2.0254	−1.7823	0.4195	0.6292	0.9216
6.0	−1.6327	−2.5244	−2.1698	0.5228	0.4545	0.6939

From Table 2, it can be observed that the values of Renyi, Tsallis, Arimoto, Award and Alawneh 1 and Award and Alawneh 2 entropy increases as the value of $\boldsymbol{\varphi}$ increases and decrease for Havrada and Charvat entropy.

Table 3. Numerical values of the considered entropy of the Gumbel type-2 distribution at $\boldsymbol{\alpha = 0.5}$ and $\boldsymbol{\lambda = 3.5}$

$\boldsymbol{\varphi}$	$R_\lambda(\alpha,\varphi)$	$T_\lambda(\alpha,\varphi)$	$A_\lambda(\alpha,\varphi)$	$HC_\lambda(\alpha,\varphi)$	$AA1_\lambda(\alpha,\eta)$	$AA2_\lambda(\alpha,\eta)$
1.5	0.349	0.233	0.309	−2.709	2.068	1.208
2.0	0.217	0.168	0.201	−1.950	1.865	1.203
2.5	0.080	0.072	0.078	−0.843	1.745	1.199
3.0	−0.169	−0.206	−0.176	2.394	1.665	1.189
3.5	−0.049	−0.052	−0.049	0.600	1.572	1.196
4.0	−0.267	−0.379	−0.294	4.417	1.547	1.191
4.5	−0.371	−0.610	−0.423	7.106	1.533	1.188
5.0	−0.460	−0.864	−0.545	10.064	1.347	1.163
5.5	−0.541	−1.152	−0.645	13.447	1.259	1.151
6.0	−0.620	−1.483	−0.780	17.276	1.147	1.143

From Table 3, it can be observed that the values of Renyi, Tsallis, Arimoto, Award and Alawneh 1 and Award and Alawneh 2 entropy increases as the value of $\boldsymbol{\varphi}$ increases and decrease for Havrada and Charvat entropy.

Table 4. Numerical values of the considered entropy of the Gumbel type-2 distribution at $\boldsymbol{\varphi} = \mathbf{0.5}$ and $\boldsymbol{\lambda} = \mathbf{3.5}$

$\boldsymbol{\alpha}$	$R_\lambda(\alpha,\varphi)$	$T_\lambda(\alpha,\varphi)$	$A_\lambda(\alpha,\varphi)$	$HC_\lambda(\alpha,\varphi)$	$AA1_\lambda(\alpha,\eta)$	$AA2_\lambda(\alpha,\eta)$
$\mathbf{1.5}$	2.962	1.545	1.882	−0.320	4.754	3.097
$\mathbf{2.0}$	3.537	1.659	2.077	−0.344	4.753	3.097
$\mathbf{2.5}$	3.984	1.727	2.205	−0.358	4.755	3.097
$\mathbf{3.0}$	4.348	1.773	2.296	−0.368	4.753	3.097
$\mathbf{3.5}$	4.656	1.805	2.365	−0.394	4.753	3.097
$\mathbf{4.0}$	4.923	1.829	2.419	−0.379	4.754	3.097
$\mathbf{4.5}$	5.159	1.848	2.463	−0.383	4.754	3.097
$\mathbf{5.0}$	5.371	1.863	2.499	−0.386	4.754	3.097
$\mathbf{5.5}$	5.561	1.866	2.530	−0.389	4.754	3.097
$\mathbf{6.0}$	5.733	1.876	2.556	−0.391	4.752	3.097

From Table 4, it can be observed that the values of Renyi, Tsallis, Arimoto entropy decreases Award and Alawneh 1 and Award and Alawneh 2 entropy remain constant while Havrada and Charvat entropy increases as the value of $\boldsymbol{\alpha}$ increases.

Table 5. Numerical values of the considered entropy of the Gumbel type-2 distribution at $\boldsymbol{\varphi} = \mathbf{0.5}$ and $\boldsymbol{\lambda} = \mathbf{3.5}$

$\boldsymbol{\alpha}$	$R_\lambda(\alpha,\varphi)$	$T_\lambda(\alpha,\varphi)$	$A_\lambda(\alpha,\varphi)$	$HC_\lambda(\alpha,\varphi)$	$AA1_\lambda(\alpha,\eta)$	$AA2_\lambda(\alpha,\eta)$
$\mathbf{1.5}$	1.942	0.397	1.050	−4.621	3.733	1.215
$\mathbf{2.0}$	2.528	0.399	1.170	−4.649	3.798	1.215
$\mathbf{2.5}$	2.967	0.400	1.234	−4.654	3.738	1.215
$\mathbf{3.0}$	3.407	0.400	1.277	−4.655	3.812	1.215
$\mathbf{3.5}$	3.684	0.400	1.299	−4.656	3.781	1.215
$\mathbf{4.0}$	3.905	0.400	1.314	−4.657	3.736	1.215
$\mathbf{4.5}$	4.141	0.400	1.327	−4.657	3.736	1.215
$\mathbf{5.0}$	4.352	0.400	1.338	−4.657	3.736	1.215
$\mathbf{5.5}$	4.542	0.400	1.345	−4.657	3.736	1.215
$\mathbf{6.0}$	4.716	0.400	1.352	−4.657	3.736	1.215

From Table 5, it can be observed that the values of Renyi entropy decreases, Tsallis entropy increases and remain constant for $\boldsymbol{\alpha} = \mathbf{2.5} - \mathbf{6.0}$, Arimoto entropy decreases, Havrada and Charvat, Award and Alawneh 1 and Award and Alawneh 2 entropy remain constant for all values of $\boldsymbol{\alpha}$.

4 Illustrative example

For illustrative purposes, we present here a real data analysis using the proposed methods. . The data set refers to [10] which consists of thirty successive values of March precipitation (in inches) in Minneapolis/St Paul. To evaluate whether the data fit the Gumbel type-2 distribution or not also, to check for the goodness of fit, we compute the Anderson–Darling statistic and Cramer-von Misses statistic which values is respectively given as 0.7723 and 0.1261 with associated p-value of 0.4924. Since the p-value is quite high that is p-value > 0.05, we cannot reject the null hypothesis that the data are coming from the Gumbel type-2 distribution. The ordered data are as follows: 0.32, 0.47, 0.52 , 0.59, 0.77, 0.81, 0.90, 0.96, , 1.18, 1.20 , 1.20, , 1.35, 1.43, 1.74, 1.95, 1.51, 1.62, 1.31, 0.81, 1.87, 1.89, 2.05, 2.10, , 2.20, 2.48, 2.81, 3.00, 3.09, 3.37, 4.75.

5 Conclusion

This article studied the entropy of the Gumbel type-2 distribution which covered both the theoretical and practical aspects. In particular, six different entropy measures were examined. After determining the closed-form expressions of these measures, an estimation strategy was developed to evaluate them in a practical setting. A real-life data set is used to show how the different entropies can be estimated. The finding of this study aims to be applied by the statistician to assess the entropy of diverse data with values on such as modern rate, unit interval, percentage and proportion type data.

References

[1] Abo-Eleneen, Z. A. (2011). The entropy of progressively censored samples. Entropy, 13, 437–449.

[2] Arimoto, S. (1971). Information-theoretical considerations on estimation problems. Information and Control, Vol. 19, 3, pg. 181–194.

[3] Awad, A. M., and Alawneh, A. J. (1987). Application of entropy to a life-time model. IMA Journal of Mathematical Control and Information, 4, 2, 143–148.

[4] Baratpour, S. Ahmadi, J. and Arghami, N.R. (2007). Entropy properties of record statistics. Stat. Pap. 2007, 48, 197–213.

[5] Cramer, E., and Bagh, C. (2011). Minimum and maximum information censoring plans in progressive censoring. Communication in Statistics Theory and Methods, 40, 2511–2527.

[6] Cho, Y., Sun, H. and Lee, K. (2014). An estimation of the entropy for a Rayleigh distribution based on doubly-generalized Type-II hybrid censored samples. Entropy, 16, 3655–3669.

[7] Cover, T. M. and Thomas, J. A. (2005). Elements of Information Theory; Wiley: Hoboken, NJ, USA, 2005.

[8] Jiheel, A. K. and Shanubhoque, A. (2014). Shrinkage estimation of the entropy function for the exponential distribution under different loss functions using progressive Type II censored sample. International Journal of Mathematics and Computer Research 2014, 2, 394–402

[9] Havrda, J., and Charvat, F. (1967). Quantification method of classification processes, concept of structural entropy. Kybernetika, Vol. 3, pg. 30–35.

[10] Hinkley, D. (1977).On Quick Choice of Power Transformations. Journal of the Royal Statistical Society, Series(c), Applied Statistics, 26(1), 67–69.

[11] Renyi A. (1960). "On measures of entropy and information". Proc. 4th Berkeley Symposium on Mathematical Statistics and Probability. Vol. 1, pg. 47-561.

[12] Shannon, C. E. (1948). "A mathematical theory of communication". The Bell System Technical, Vol. 1, pg. 379–423.

[13] Tsallis, C. (1988). "Possible generalization of Boltzmann-Gibbs statistics". Journal of Statistical Physics, Vol., 52, Pg. 479–487.