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Stabilizers of Lojid algebras

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Abstract

In this paper, the notion of stabilizers are introduced and studied in lojid algebras. It is shown that to every lojid stabilizer, there is a corresponding lojid adjoint. The properties of stabilizers and their corresponding adjoints are investigated in some classes of lojid algebras.

Keywords and phrases: Lojid algebras; congruences; translations; stabilizers; adjoints

1 Introduction

An algebra of type (2, 0) is a non-empty set, having a constant element, on which is defined a binary operation such that certain axioms are satisfied. BCI algebras and BCK algebras, introduced in [1] and [2], are common varieties of such algebras. There are several other varieties of algebras of type (2, 0). There are also several generalizations of BCI algebras. In [3], BCH algebras were studied. In [4], d algebras were studied. In [5], the notion of BE algebras was introduced. Ideals and upper sets in BE algebras were investigated in [6] and [7]. Pre- commutative algebras were studied in [8]. Fenyves algebras were studied in [9], [10] and [11]. In [12], Q algebras were introduced. Homomorphisms of Q algebras were studied in [13].

Recently, it has been shown in [14] that algebras of type (2,0) have diverse applications in coding theory. Motivated by this, more research interest has been given to the study of algebras of type (2,0). Obic algebras were introduced in [15]. In [16], torian algebras were studied. It was shown that the class of torian algebras is a wider class than the class of obic algebras. Ideals of torian algebras were investigated in [17]. The dual and nuclei of ideals as well as congruences developed on ideals of torian algebras were studied. In [18], right distributive torian algebras were studied. Isomorphism Theorems of torian algebras were studied in [19]. In [20], lojid algebras were introduced. Properties of classes of lojid algebras were investigated Associates, commutants and regular maps were introduced and studied. Furthermore, congruences were introduced as a build up to the establishment of quotient lojid algebras.

In this paper, the notion of stabilizers are introduced and studied in lojid algebras. It is shown that to every lojid stabilizer, there is a corresponding lojid adjoint. The properties of stabilizers and their corresponding adjoints are investigated in some classes of lojid algebras.

2 Preliminaries

In this section, some basic concepts necessary for a proper understanding of this paper are discussed.

Definition 2.1. [20] An algebra $(X; *, 0)$ of type (2, 0) is called a semi-lojid algebra if $x * 0 = x$ for all $x \in X$.

Example 2.2. [20]. Let $X = \{0, 1, 2, 3\}$. Define a binary operation $*$ on X by the following table:

Then $(X; *, 0)$ is a semi-lojid algebra.

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$*$	0	1	2	3
0	0	2	3	1
1	1	0	2	2
2	2	3	0	3
3	3	1	1	0

Definition 2.3. [20]. Let a be an element of a semi-lojid algebra $(X;*,0)$. The mapping $L_a : X \to X$ given by $L_a(x) = a * x$ for all $x \in X$, is called a left translation on $(X;*,0)$.
Similarly, the mapping $R_a : X \to X$ given by $R_a(x) = x * a$ for all $x \in X$, is called a right translation on $(X;*,0)$.

We shall write xL_a instead of $L_a(x)$ and xR_a instead of $R_a(x)$.

Definition 2.4. [20]. A semi-lojid algebra $(X;*,0)$ is called a lojid algebra if all its left and right translations are bijections.

Example 2.5. [20]. Let $X = \{0,1,2,3,4\}$. Define a binary operation $*$ on X by the following table:

$*$	0	1	2	3	4
0	0	1	3	4	2
1	1	0	4	2	3
2	2	4	1	3	0
3	3	2	0	1	4
4	4	3	2	0	1

Then $(X;*,0)$ is a lojid algebra.

Example 2.6. [20]. Let $X = \{0,1,2,3\}$. Define a binary operation $*$ on X by the following table:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then $(X;*,0)$ is a lojid algebra.

We shall write X for a lojid algebra $(X;*,0)$ unless there is the need to emphasize the binary operation and the constant element of $(X;*,0)$.

Definition 2.7. [20]. A lojid algebra X is said to be completely lojid if $xR_x = 0$ for all $x \in X$.

Example 2.8. [20]. The algebra in Example 2.3. is a completely lojid algebra.

Definition 2.9. [20]. Let $(X;*,0)$ be a lojid algebra. A non-empty subset S of X is called a sub-lojid algebra (or simply a sub-algebra) of $(X;*,0)$ if $(S;*,0)$ is a lojid algebra.

3 Stabilizers and Adjoints

In this section, we present the notions of stabilizers in lojid algebras. By introducing a new binary operation in lojid algebras, we study adjoints of stabilizers.

Definition 3.1. A self map f on a lojid algebra is said to be regular if $f(0) = 0$.

Definition 3.2. A self map f on a lojid algebra $(X; *, 0)$ is called a stabilizer if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$.

Proposition 3.3. *Every stabilizer on a completely lojid algebra is regular.*

Proof. The proof is straightforward.

Definition 3.4. Let $(X; *, 0)$ be a lojid algebra. Define a binary operation $\odot$ on X by $x \odot y = y * (y * x)$ for all $x, y \in X$.

Definition 3.5. Let f be a stabilizer on a lojid algebra $(X; *, 0)$. A map $g : X \to X$ is called a right adjoint of f if $g(x * y) = (g(x) * f(y)) \odot (f(x) * g(y))$ for all $x, y \in X$.
If $g(x * y) = (f(x) * g(y)) \odot (g(x) * f(y))$ for all $x, y \in X$, then g is called a left adjoint of f
If g is both a right adjoint and a left adjoint of f, then g is called an adjoint of f.

Example 3.6. Let N^* be the set of non-negative integers. Define a binary operation $*$ on N^* by

$$x * y = \begin{cases} 0, & x \leq y \\ x - y, & x > y \end{cases}$$

Then $(N^*; *, 0)$ is a lojid algebra. Define the self map f on N^* by $f(x) = 7x$ for all $x \in N^*$. Then f is a stabilizer on N^*. The self map g on N^* defined by $g(x) = 0$ for all $x \in N^*$ is an adjoint of f.

Definition 3.7. A lojid algebra X is said to be prime if $0R_x = 0$ for all $x \in X$.

Definition 3.8. A lojid algebra X is said to be composite if $0R_x = x$ for all $x \in X$.

Definition 3.9. A self map f on a lojid algebra X is called a module if $f(x) = 0$ for all $x \in X$.

Proposition 3.10. *Let f be a stabilizer on a completely lojid algebra $(X; *, 0)$, and let g be a regular right adjoint of f. Then $(f(x) * g(x)) * ((f(x) * g(x)) * (g(x) * f(x))) = (f(y) * g(y)) * ((f(y) * g(y)) * (g(y) * f(y)))$ for all $x, y \in X$.*

Proof. Notice that $0 = g(0) = g(x * x) = (g(x) * f(x)) \odot (f(x) * g(x)) = (f(x) * g(x)) * ((f(x) * g(x)) * (g(x) * f(x)))$. So, $(f(x) * g(x)) * ((f(x) * g(x)) * (g(x) * f(x))) = 0$
Now, replacing x with y in the last expression, we have $(f(y) * g(y)) * ((f(y) * g(y)) * (g(y) * f(y))) = 0$. Hence, the conclusion follows.

Corollary 3.11. *Let f be a stabilizer on a completely lojid algebra $(X; *, 0)$, and let g be a regular right adjoint of f.*

1. *If f is a module, then $(0 * g(x)) * ((0 * g(x)) * g(x)) = (0 * g(y)) * ((0 * g(y)) * g(y))$ for all $x, y \in X$.*

2. *If g is a module, then $f(x) * (f(x) * (0 * f(x) = f(y) * (f(y) * (0 * f(y)$ for all $x, y \in X$.*

Proof.

1. Since f is a module, then expression $(f(x) * g(x)) * ((f(x) * g(x)) * (g(x) * f(x))) = (f(y) * g(y)) * ((f(y) * g(y)) * (g(y) * f(y)))$ in Proposition 3.10 becomes $(0 * g(x)) * ((0 * g(x)) * g(x)) = (0 * g(y)) * ((0 * g(y)) * g(y))$ for all $x, y \in X$ as required.

2. Since g is a module, then expression $(f(x) * g(x)) * ((f(x) * g(x)) * (g(x) * f(x))) = (f(y) * g(y)) * ((f(y) * g(y)) * (g(y) * f(y)))$ in Proposition 3.10 becomes $f(x) * (f(x) * (0 * f(x) = f(y) * (f(y) * (0 * f(y)$ for all $x, y \in X$ as required.

Proposition 3.12. *Let f be a stabilizer on a lojid algebra X, and let g be a right adjoint of f. Then the following hold for all $x, y \in X$:*

1. $(f(0) * g(x)) * ((f(0) * g(x)) * (g(0) * f(x))) = g(0 * x)$
2. *If X is prime, then* $(f(0) * g(x)) * ((f(0) * g(x)) * (g(0) * f(x))) = g(0)$
3. *If f is regular, then* $(0 * g(x)) * ((0 * g(x)) * g(0) * f(x))) = g(0 * x)$
4. *If X is prime and f is regular, then g is regular*
5. *If g is regular, then* $((f(0) * g(x)) * ((f(0) * g(x)) * (0 * f(x))) = g(0 * x)$
6. *If X is composite, then* $((f(0) * g(x)) * ((f(0) * g(x)) * (g(0) * f(x))) = g(x)$
7. *If X is composite and f is regular, then* $g(x) * (g(x) * (g(0) * f(x))) = g(x)$
8. *If f is composite and g is regular, then* $((f(0) * g(x)) * ((f(0) * g(x)) * f(x)) = g(x)$
9. *If f is a module, then* $(0 * g(x)) * ((0 * g(x)) * g(0) = g(0 * x)$
10. *If g is a module, then* $f(0) * (f(0) * (0 * f(x))) = 0$
11. *If g is a module, then* $f(0) * (f(0) * (0 * f(x))) = f(0) * (f(0) * (0 * f(y)))$

Proof.

1. Notice that $g(0*x) = (g(0)*f(x)) \odot (f(0)*g(x)) = (f(0)*g(x))*((f(0)*g(x))*(g(0)*f(x)))$ as required.
2. Apply 'primeness' to (1)
3. Apply regularity of f in (2)
4. Combine (2) and (3)
5. Apply regularity of g to (1)
6. Apply 'compositeness' of X to (1)
7. Apply regularity of f to (6)
8. Apply regularity of g to (6)
9. Apply 'moduleness' to f in (1)
10. Apply 'moduleness' to g in (1)
11. Follows from (10) after replacing x with y

Corollary 3.13. *Let f be a stabilizer on a completely lojid algebra X, and let g be a right adjoint of f. Then the following hold for all $x, y \in X$:*

1. $(0 * g(x)) * ((0 * g(x)) * (g(0) * f(x))) = g(0 * x)$
2. *If X is prime, then g is regular*
3. *If X is composite, then* $g(x) * (g(x) * (g(0) * f(x))) = g(x)$

4. *If g is a module, then* $0 * (0 * (0 * f(x))) = 0$
5. *if g is a module, then* $0 * (0 * (0 * f(x))) = 0 * (0 * (0 * f(y)))$

Proof.

1. Follows from Proposition 3.3 and Proposition 3.12(1).
2. Follows from Proposition 3.3 and Proposition 3.12(4).
3. Follows from Proposition 3.3 and Proposition 3.12(7).
4. Follows from Proposition 3.3 and Proposition 3.12(10).
5. Follows from (4) after replacing x with y.

Proposition 3.14. *Let f be a stabilizer on a completely lojid algebra $(X; *, 0)$, and let g be a regular right adjoint on f. If f is a module, then the following hold for all $x, y \in X$:*

1. $((0 * g(x)) * ((0 * g(x)) * g(x)) = 0$
2. $((0 * g(x)) * ((0 * g(x)) * g(x)) = ((0 * g(y)) * ((0 * g(y)) * g(y))$
3. $g(0 * x) = 0$
4. $g(0 * x) = g(0 * y)$
5. $((0 * g(x)) * ((0 * g(x)) * g(x)) = g(0 * x)$

Proof.

1. Notice that $0 = g(0) = g(x*x) = (g(x)*f(x)) \odot (f(x)*g(x)) = ((0*g(x))*((0*g(x))*g(x))$ as required.
2. This follows from (1) after replacing x with y
3. Notice that $g(0 * x) = (g(0) * f(x)) \odot (f(0) * g(x)) = 0$
4. This follows from (3) after replacing x with y
5. This follows from (1) and (3)

Proposition 3.15. *Let f be a stabilizer on a completely lojid algebra X, and let g be a regular left adjoint of f. Then* $(g(x) * f(x)) * ((g(x) * f(x)) * (f(x) * g(x))) = (g(y) * f(y)) * ((g(y) * f(y)) * (f(y) * g(y)))$ *for all $x, y \in X$.*

Proof. Notice that $0 = g(0) = g(x * x) = (f(x) * g(x)) \odot (g(x) * f(x)) = (g(x) * f(x)) * ((g(x) * f(x)) * (f(x) * g(x)))$. By replacing x with y, the conclusion follows.

Corollary 3.16. *Let f be a stabilizer on a completely lojid algebra X, and let g be a regular left adjoint of f. Then the following hold for all $x, y \in X$:*

1. *If f is a module, then* $g(x) * (g(x) * (0 * g(x))) = g(y) * (g(y) * (0 * g(y)))$
2. *If g is a module, then* $(0 * f(x)) * ((0 * f(x)) * f(x)) = (0 * f(y)) * ((0 * f(y)) * f(y))$

Proposition 3.17. *Let f be a stabilizer on a completely lojid algebra X, and let g be a left adjoint of f. Then the following hold for all $x, y \in X$:*

1. $(g(0) * f(x)) * ((g(0) * f(x)) * (f(0) * g(x))) = g(0 * x)$
2. *If X is prime, then* $(g(0) * f(x)) * ((g(0) * f(x)) * (f(0) * g(x))) = g(0)$

3. *If f is regular, then* $(g(0) * f(x)) * ((g(0) * f(x)) * (0 * g(x))) = g(0 * x)$
4. *If X is prime and f is regular, then g is regular*
5. *If g is regular, then* $(0 * f(x)) * ((0 * f(x)) * (f(0) * g(x))) = g(0 * x)$
6. *If X is composite, then* $(g(0) * f(x)) * ((g(0) * f(x)) * (f(0) * g(x))) = g(x)$
7. *If X is composite and f is regular, then* $(g(0) * f(x)) * ((g(0) * f(x)) * g(x)) = g(x)$
8. *If X is composite and g is regular, then* $f(x) * (f(x) * (f(0) * g(x))) = g(x)$
9. *If f is a module, then* $g(0) * (g(0) * (0 * g(x))) = g(0 * x)$
10. *if g is a module, then* $(0 * f(x)) * ((0 * f(x)) * f(0)) = 0$
11. *If g is a module, then* $(0 * f(y)) * ((0 * f(y)) * f(0))$

Proof.

1. Notice that $g(0*x) = (f(0)*g(x)) \odot (g(0)*f(x)) = (g(0)*f(x))*((g(0)*f(x))*(f(0)*g(x)))$ as required.
2. Apply 'primeness' of X to (1)
3. Apply regularity to f in (1)
4. Follows from (ii) and (3)
5. Apply regularity of g to (1)
6. Apply 'compositeness' of X to (1)
7. Apply regularity of f to (6)
8. Apply regularity of g to (6)
9. Apply 'moduleness' of f to (1)
10. Apply 'moduleness' of g to (1)
11. Follows from (10) after replacing x with y

Corollary 3.18. *Let f be a stabilizer on a completely lojid algebra X, and let g be a left adjoint of f. Then the following hold for all $x, y \in X$:*

1. $(g(0) * f(x)) * ((g(0) * f(x)) * (0 * g(x))) = g(0 * x)$
2. *If X is prime, then g is regular*
3. *If X is composite, then* $(g(0) * f(x)) * ((g(0) * f(x)) * g(x)) = g(x)$

Proof.

1. Follows from Proposition 3.3 and Proposition 3.17(1).
2. Follows from Proposition 3.3 and Proposition 3.17(4).
3. Follows from Proposition 3.3 and Proposition 3.17(6).

Proposition 3.19. *Let f be a stabilizer on a completely lojid algebra X, and let g be a regular left adjoint of f. If f is a module, then the following hold for all $x, y \in X$:*

1. $g(x) * (g(x) * (0 * g(x))) = 0$
2. $g(x) * (g(x) * (0 * g(x))) = g(y) * (g(y) * (0 * g(y)))$
3. $g(0 * x) = 0 * (0 * (0 * g(x)))$

Proof.

1. Notice that $0 = g(0) = g(x * x) = g(x) * (g(x) * (0 * g(x)))$ as required.
2. Follows from (1) after replacing x with y
3. Notice that $g(0 * x) = (f(0) * g(x)) \odot (g(0) * f(x)) = 0 * (0 * (0 * g(x)))$ as required.

By Proposition 3.10 and Proposition 3.15, we have have the following Theorem:

Theorem 3.20. *Let f be a stabilizer on a completely lojid algebra $(X; *, 0)$, and let g be a regular adjoint of f. Then the following hold for all $x, y \in X$:*

1. $(f(x) * g(x)) * ((f(x) * g(x)) * (g(x) * f(x))) = (f(y) * g(y)) * ((f(y) * g(y)) * (g(y) * f(y)))$
2. $(g(x) * f(x)) * ((g(x) * f(x)) * (f(x) * g(x))) = (g(y) * f(y)) * ((g(y) * f(y)) * (f(y) * g(y)))$

Proof.

1. Notice that $0 = g(0) = g(x * x) = (g(x) * f(x)) \odot (f(x) * g(x)) = (f(x) * g(x)) * ((f(x) * g(x)) * (g(x) * f(x)))$. So, $(f(x) * g(x)) * ((f(x) * g(x)) * (g(x) * f(x))) = 0$
   Now, replacing x with y in the last expression, we have $(f(y) * g(y)) * ((f(y) * g(y)) * (g(y) * f(y))) = 0$. Hence, the conclusion follows.
2. Notice that $0 = g(0) = g(x * x) = (f(x) * g(x)) \odot (g(x) * f(x)) = (g(x) * f(x)) * ((g(x) * f(x)) * (f(x) * g(x)))$. By replacing x with y, the conclusion follows.

By Corollary 3.11 and Corollary 3.16, we have the following Corollary:

Corollary 3.21. *Let f be a stabilizer on a completely lojid algebra $(X; *, 0)$, and let g be a regular adjoint of f. Then the following hold for all $x; y \in X$:*

1. *If f is a module, then* $(0 * g(x)) * ((0 * g(x) * g(x)) = (0 * g(y)) * ((0 * g(y)) * g(y)))$ *and* $g(x) * (g(x) * (0 * g(x))) = g(y) * (g(y) * (0 * g(y)))$
2. *If g is a module, then* $f(x) * (f(x) * (0 * f(x))) = f(y) * (f(y) * (0 * f(y)))$ *and* $(0 * f(x)) * ((0 * f(x)) * f(x)) = (0 * f(y)) * ((0 * f(y)) * f(y))$

By Proposition 3.12 and Proposition 3.17, we have the following Theorem:

Theorem 3.22. *Let f be a stabilizer on a lojid algebra $(X; *, 0)$, and let g be an adjoint of f. Then the following hold for all $x, y \in X$:*

1. $(f(0) * g(x)) * ((f(0) * g(x)) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * (f(0) * g(x)))$
2. *If X is prime, then* $(f(0) * g(x)) * ((f(0) * g(x)) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * (f(0) * g(x)))$
3. *If f is regular, then* $(0 * g(x)) * ((0 * g(x)) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * (0 * g(x)))$
4. *If X is prime and f is regular, then g is regular.*
5. *If g is regular, then* $((f(0) * g(x)) * ((f(0) * g(x)) * (0 * f(x))) = (0 * f(x)) * ((0 * f(x)) * (f(0) * g(x)))$

6. *If X is composite, then* $((f(0) * g(x)) * ((f(0) * g(x)) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * ((f(0) * g(x))))$
7. *If X is composite and f is regular, then* $g(x) * (g(x) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * g(x))$
8. *If X is composite and g is regular, then* $((f(0) * g(x)) * ((f(0) * g(x)) * f(x)) = f(x) * (f(x) * (f(0) * g(x)))$
9. *If f is a module, then* $(0 * g(x)) * ((0 * g(x)) * g(0)) = g(0) * (g(0) * (0 * g(x)))$
10. *If g is a module, then* $f(0) * (f(0) * (0 * f(x))) = (0 * f(x)) * ((0 * f(x)) * f(0))$
11. *If g is a module, then* $f(0) * (f(0) * (0 * f(x))) = f(0) * (f(0) * (0 * f(y)))$ *and* $(0 * f(x)) * ((0 * f(x)) * f(0) = (0 * f(y)) * ((0 * f(y)) * f(0))$

Proof.

1. Since g is an adjoint of f, then by Proposition 3.12 and Proposition 3.17, we have $(f(0)*g(x))*((f(0) * g(x)) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * (f(0) * g(x)))$ as required.
2. Since X is prime, then by Proposition 3.12 and Proposition 3.17, we have $(f(0)*g(x))*((f(0)*g(x)) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * (f(0) * g(x)))$ as required.
3. Since f is regular, then by Proposition 3.12 and Proposition 3.17, we have $(0 * g(x)) * ((0 * g(x)) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * (0 * g(x)))$ as required.
4. Since X is prime and f is regular, then by Proposition 3.12 and Proposition 3.17, g is regular as required.
5. Since g is regular, then by Proposition 3.12 and Proposition 3.17, we have $((f(0) * g(x)) * ((f(0) * g(x)) * (0 * f(x))) = (0 * f(x)) * ((0 * f(x)) * (f(0) * g(x)))$ as required.
6. Since X is composite, then by Proposition 3.12 and Proposition 3.17, we have $((f(0) * g(x)) * ((f(0) * g(x)) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * ((f(0) * g(x))))$ as required.
7. Since X is composite and f is regular, then by Proposition 3.12 and Proposition 3.17, we have $g(x) * (g(x) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * g(x))$ as required.
8. Since X is composite and g is regular, then by Proposition 3.12 and Proposition 3.17, we have $((f(0) * g(x)) * ((f(0) * g(x)) * f(x)) = f(x) * (f(x) * (f(0) * g(x)))$ as required.
9. Since f is a module, then by Proposition 3.12 and Proposition 3.17, we have $(0 * g(x)) * ((0 * g(x)) * g(0)) = g(0) * (g(0) * (0 * g(x)))$ as required.
10. Since g is a module, then by Proposition 3.12 and Proposition 3.17, we have $f(0) * (f(0) * (0 * f(x))) = (0 * f(x)) * ((0 * f(x)) * f(0))$ as required.
11. Since g is a module, then by Proposition 3.12 and Proposition 3.17, we have $f(0) * (f(0) * (0 * f(x))) = f(0)*(f(0)*(0*f(y)))$ and $(0*f(x))*((0*f(x))*f(0) = (0*f(y))*((0*f(y))*f(0))$ as required.

By Corollary 3.13 and Corollary 3.18, we have the following Corollary:

Corollary 3.23. *Let f be a stabilizer on a completely lojid algebra $(X; *, 0)$, and let g be an adjoint of f. Then the following hold for all $x, y \in X$:*

1. $(0 * g(x)) * ((0 * g(x)) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * (0 * g(x)))$
2. *If X is prime, then g is regular.*
3. *If X is composite, then* $g(x) * (g(x) * (g(0) * f(x))) = (g(0) * f(x)) * ((g(0) * f(x)) * g(x))$

By Proposition 3.14 and Proposition 3.19, we have the following Theorem:

Theorem 3.24. *Let f be a stabilizer on a completely lojid algebra $(X; *, 0)$, and let g be an adjoint of f. Then the following hold for all $x, y \in X$:*

1. $((0 * g(x)) * ((0 * g(x)) * g(x)) = g(x) * (g(x) * (0 * g(x)))$
2. $(0 * g(x)) * ((0 * g(x)) * g(x)) = (0 * g(y)) * ((0 * g(y)) * g(y))$ *and* $g(x) * (g(x) * (0 * g(x))) = g(y) * (g(y) * (0 * g(y)))$
3. $0 * (0 * (0 * g(x))) = 0$

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