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On Lojid Algebras

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Abstract

In this paper, lojid algebras are introduced. Properties of classes of lojid algebras are investigated. Associates, commutants and regular maps are introduced and studied. Furthermore, congruences are introduced as a build up to the establishment of quotient lojid algebras.

Keywords and phrases: Lojid algebras, commutants, congruences, translations.

1 Introduction

An algebra of type (2, 0) is a non-empty set, having a constant element, on which is defined a binary operation such that certain axioms are satisfied. BCI algebras and BCK algebras, introduced in [1] and [2], are common varieties of such algebras. There are several other varieties of algebras of type (2, 0). There are also several generalizations of BCI algebras. In [3], BCH algebras were studied. In [4], d algebras were studied. In [5], the notion of BE algebras was introduced. Ideals and upper sets in BE algebras were investigated in [6] and [7]. Pre- commutative algebras were studied in [8]. Fenyves algebras were studied in [9], [10] and [11]. In [12], Q algebras were introduced. Homomorphisms of Q algebras were studied in [13].
Recently, it has been shown in [14], that algebras of type (2,0) have diverse applications in coding theory. Motivated by this, more research interest has been given to the study of algebras of type (2,0). Obic algebras were introduced in [15]. In [16], torian algebras were studied. It was shown that the class of torian algebras is a wider class than the class of obic algebras. Ideals of torian algebras were investigated in [17]. The dual and nuclei of ideals as well as congruences developed on ideals of torian algebras were studied. In [18], right distributive torian algebras were studied. Isomorphism Theorems of torian algebras were studied in [19].
In this paper, lojid algebras are introduced. Properties of classes of lojid algebras are investigated. Associates, commutants and regular maps are introduced and studied. Furthermore, congruences are introduced as a build up to the establishment of quotient lojid algebras.

2 Lojid algebras

In this section, we introduce semi-lojid algebras as a prelude to the introduction of lojid algebras. Thereafter, lojid algebras are introduced. We also establish some properties of lojid algebras. Furthermore, we show that given a lojid algebra, there exist two other lojid algebras which are derived from it by obtaining two binary operations which are induced by the binary operation of the classical lojid algebra.

Definition 2.1. *An algebra* $(X;*,0)$ *of type (2, 0) is called a semi-lojid algebra if* $x*0=x$ *for all* $x\in X$.

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Example 2.2. Let $X = \{0, 1, 2, 3\}$. Define a binary operation $*$ on X by the following table:

$*$	0	1	2	3
0	0	2	3	1
1	1	0	2	2
2	2	3	0	3
3	3	1	1	0

Then $(X; *, 0)$ is a semi-lojid algebra.

Definition 2.3. *Let a be an element of a semi-lojid algebra $(X; *, 0)$. The mapping $L_a : X \to X$ given by $L_a(x) = a * x$ for all $x \in X$, is called a left translation on $(X; *, 0)$.*
Similarly, the mapping $R_a : X \to X$ given by $R_a(x) = x * a$ for all $x \in X$, is called a right translation on $(X; *, 0)$.

We shall write xL_a instead of $L_a(x)$ and xR_a instead of $R_a(x)$.

Remark 2.4. We are interested in semi-lojid algebras in which all the left and right translations are bijections.

Notice that not all the left and right translations in the semi-lojid algebra in Example 2.1. are bijections. We therefore have the following definition:

Definition 2.5. *A semi-lojid algebra $(X; *, 0)$ is called a lojid algebra if all its left and right translations are bijections.*

Example 2.6. Let $X = \{0, 1, 2, 3, 4\}$. Define a binary operation $*$ on X by the following table:

$*$	0	1	2	3	4
0	0	1	3	4	2
1	1	0	4	2	3
2	2	4	1	3	0
3	3	2	0	1	4
4	4	3	2	0	1

Then $(X; *, 0)$ is a lojid algebra.

Example 2.7. Let $X = \{0, 1, 2, 3\}$. Define a binary operation $*$ on X by the following table:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then $(X; *, 0)$ is a lojid algebra.

We shall write X for a lojid algebra $(X; *, 0)$ unless there is the need to emphasize the binary operation and the constant element of $(X; *, 0)$.

Definition 2.8. *A lojid algebra X is said to be completely lojid if $xR_x = 0$ for all $x \in X$.*

Example 2.9. The algebra in Example 2.3. is a completely lojid algebra.

Definition 2.10. *Let $(X; *, 0)$ be a lojid algebra. A non-empty subset S of X is called a sub-lojid algebra (or simply a sub-algebra) of $(X; *, 0)$ if $(S; *, 0)$ is a lojid algebra.*

Proposition 2.11. *Let X be a lojid algebra. A subset S (with $0 \in S$) of X is a sub algebra of X if and only if the following hold:*

1. *for any $a, b \in S$, there exists a unique $c \in S$ such that $a * c = b$.*
2. *for any $a, b \in S$, there exists a unique $d \in S$ such that $d * a = b$.*

Proof. Suppose S is a sub-algebra of X. Then S is a lojid algebra, and clearly, items (1) and (2) of the proposition hold.
Conversely, suppose items (1) and (2) hold in S. This implies that the left and right translations for each $a \in S$ are bijections. Hence, S is a lojid algebra.

Corollary 2.12. *A non-empty subset S of a completely lojid algebra X is a sub-algebra of X if and only if the following hold:*

1. *for any $a, b \in S$, there exists a unique $c \in S$ such that $a * c = b$.*
2. *for any $a, b \in S$, there exists a unique $d \in S$ such that $d * a = b$.*

Proof. The proof follows from Proposition 2.1 since if $x \in S$, then $x * x = 0 \in S$.

Definition 2.13. *Let $(X;*,0)$ be a lojid algebra. Define two new binary operations $\vee$ and $\wedge$ on X as follows:*
$x \vee y = yL_x^{-1}$ *and* $x \wedge y = xR_y^{-1}$ *for all* $x, y \in X$.

Notice that $x \vee y = z \Leftrightarrow x * z = y$ and $x \wedge y = z \Leftrightarrow z * y = x$ for all $x, y, z \in X$.
Hence, we have the following Lemma:

Lemma 2.14. *Let $(X;*,0)$ be a lojid algebra. Then $(X;\vee,0)$ and $(X;\wedge,0)$ are lojid algebras.*

Proposition 2.15. *Let $(X;*,0)$ be a lojid algebra. Then the following hold for all $x, y \in X$:*

1. $x * (x \vee y) = y$
2. $(y \wedge x) * x = y$
3. $x \vee (x * y) = y$
4. $(y * x) \wedge x = y$
5. $x * (x \vee y) = (y \wedge x) * x$
6. $x * (x \vee y) = x \vee (x * y)$
7. $x * (x \vee y) = (y * x) \wedge x$
8. $(y \wedge x) * x = x \vee (x * y)$
9. $(y \wedge x) * x = (y * x) \wedge x$
10. $x \vee (x * y) = (y * x) \wedge x$

Proof.

1. Notice that $x * (x \vee y) = x(yL_x^{-1}) = y$
2. Notice that $(y \wedge x) * x = (yR_x^{-1})x = y$
3. notice that $x \vee (x * y) = (x * y)L_x^{-1} = y$
4. Notice that $(y * x) \wedge x = (y * x)R_x^{-1} = y$
5. Follows from (1) and (2)
6. Follows from (1) and (3)
7. Follows from (1) and (4)
8. Follows from (2) and (3)
9. Follows from (2) and (4)
10. Follows from (3) and (4)

Theorem 2.16. *A subset S (with $0 \in S$) of a lojid algebra X is a sub-algebra of X if and only if $x * y,\ x \vee y,\ x \wedge S$ for all $x, y \in S$.*

Proof. Suppose $x * y,\ x \vee y,\ x \wedge S$ for all $x, y \in S$. Let $x, y \in S$. Then there exists a unique $z \in X$ such that $x * z = y$. This implies that $z = x \vee y$. So, $z \in S$. Since X is a lojid algebra, it follows that z is unique in S. Similar argument shows that there exists a unique $p \in S$ such that $p * x = y$. Hence, S is a lojid algebra.
The converse follows from Lemma 2.1. and Proposition 2.2.

Corollary 2.17. *A non-empty subset S of a completely lojid algebra X is a sub-algebra of X if and only if $x * y,\ x \vee y,\ x \wedge S$ for all $x, y \in S$.*

Proof. Clearly, $0 \in S$ since if $x \in X$, then $x * x = 0 \in S$. The result follows therefore from Proposition 2.2. and Theorem 2.1.

3 Associates and Commutants

Definition 3.1. *Let X be a lojid algebra. An element $a \in X$ is said to be left-associative (respectively right-associative, centre-associative) if $xL_aR_y = xR_yL_a$ (respectively $xR_aL_y = xL_yR_a$, $aL_xR_y = aR_yL_x$) for all $x, y \in X$.*

Let X be a lojid algebra. The collection of all left-associative (respectively right-associative, centre-associative) elements of X is denoted by $Ass_L(X)$ (respectively $Ass_R(X), Ass_C(X)$).
Notice that $0 \in Ass_R(X)$ for every lojid algebra X.

Definition 3.2. *A lojid algebra X is said to be bounded if $xL_0 = x$ for all $x \in X$.*

The following Lemma is straightforward from definition.

Lemma 3.3. *Let X be a bounded lojid algebra. Then $0 \in Ass_L(X) \bigcap Ass_C(X)$.*

Theorem 3.4. *Let $(X; *, 0)$ be a lojid algebra. Then $(Ass_R(X), *)$ is a groupoid.*

Proof. Clearly, $0 \in Ass_R(X)$. We show that $Ass_R(X)$ is closed with respect to $*$. Now, let $a, b \in Ass_R(X)$ and let $x, y \in X$. Then $xR_{(a*b)}L_y = y * (x * (a * b)) = y * ((x * a) * b) = (y * (x * a)) * b = ((y * x) * a) * b = (y * x) * (a * b) = xL_yR_{(a*b)}$. So, $Ass_R(X)$ is closed with respect to $*$ as required.

Theorem 3.5. *Let $(X; *, 0)$ be a bounded lojid algebra. Then $(Ass_L(X), *)$ and $(Ass_C(X), *)$ are groupoids.*

Proof. By Lemma 3.1, $0 \in Ass_L(X)$. Now, let $a, b \in Ass_L(X)$ and let $x, y \in X$. Then $xL_{(a*b)}R_y = ((a*b)*x)*y = (a*(b*x))*y = a*((b*x)*y) = a*(b*(x*y)) = (a*b)*(x*y) = xR_yL_{(a*b)}$ as required.
Similar argument shows that $(Ass_C(X), *)$ is a groupoid.

Definition 3.6. *Let X be a lojid algebra. An element $a \in X$ is called an associative element if $a \in Ass_R(X) \bigcap Ass_L(X) \bigcap (Ass_C(X)$.*

The collection of all associative elements of X is called the associate of X. It is denoted by $Ass(X)$.
Combining Theorem 3.1. and Theorem 3.2., we have the following Theorem:

Theorem 3.7. *Let X be a bounded lojid algebra. Then $Ass(X)$ is a groupoid.*

Definition 3.8. *Let X be a lojid algebra. An element $a \in Ass(X)$ is called a commutative element if $aR_x = aL_x$ for all $x \in X$.*

The collection of all commutative elements of X is called the commutant of X. It is denoted by $Cmm(X)$.

Let a be an element of a lojid algebra X. Since L_a and R_a are bijections, they belong to the symmetric group $S(X)$ of all permutations of the set X. Now, consider the following sets:
$L(X) = \{L_a, L_a^{-1} : a \in X\}$,
$R(X) = \{R_a, R_a^{-1} : a \in X\}$ and
$T(X) = \{L_a, R_a, L_a^{-1}, R_a^{-1} : a \in X\}$.
The collection of permutations of X which are members of $L(X)$ (respectively $R(X), T(X)$) forms a subgroup of $S(X)$ called the left (respectively right, centre) group of $S(X)$. They are denoted by $L_S(X), R_S(X)$ and $T_S(X)$ respectively.

Theorem 3.9. *Let X be a bounded lojid algebra. Then $Cmm(X)$ is isomorphic to $C(T_S(X))$; where $C(T_S(X))$ is the centre of $T_S(X)$*

Proof. Let $z \in Cmm(X)$. Then we have $L_zR_y = R_yL_z$, $R_zL_y = L_yR_z$ and $L_z = R_z$ for all $y, z \in X$. So, $R_z \in C(T_S(X))$. We show that the mapping $\phi : Cmm(X) \to C(T_S(X))$ defined by $\phi(z) = R_z$ for all $z \in Cmm(X)$, is a homomorphism. Let $z_1, z_2 \in Cmm(X)$. Then $\phi(z_1 * z_2) = R_{(z_1 * z_2)}$ since $xR_{z_1}R_{z_2} = (x * z_1) * z_2 = x * (z_1 * z_2) = xR_{(z_1 * z_2)}$ for all $x \in X$. Clearly, ϕ is bijective and hence an isomorphism.
Similar argument shows that the mapping $\eta : Cmm(X) \to C(T_S(X))$ defined by $\eta(z) = L_z$ for all $z \in Cmm(X)$, is an isomorphism.

Definition 3.10. *Let X be a lojid algebra. A mapping $\alpha \in T_S(X)$ is said to be regular if $\alpha(0) = 0$.*

The collection of all regular mappings on a lojid algebra X is denoted by $D(X)$.

Proposition 3.11. *Let X be bounded lojid algebra. Then the set $P = \{R_{(x,y)}, L_{(x,y)}, T_{(x)}\}$ is contained in $D(X)$; where $R_{(x,y)} = R_xR_yR^{-1}_{(x*y)}$, $L_{(x,y)} = L_xL_yL^{-1}_{(y*x)}$ and $T_{(x)} = R_xL^{-1}_x$ for all $x, y \in X$.*

Proof. Notice that $0R_{(x,y)} = 0R_{(x*y)}R^{-1}_{(x*y)} = 0$.
Similar argument shows that $L_{(x*y)}$ and $T_{(x)}$ are regular.

We now pose an Open Problem:
Does the set P in Proposition 3.1 account for all the regular mappings on a bounded lojid algebra?

4 Variants of Invertibility

It is known that the notion of invertibility of elements in an algebraic structure is only meaningful if the structure has an identity element. Although lojid algebras do not have identity elements, in this section, we discuss some variants of invertibility of elements in lojid algebras.
Henceforth, we shall write xy for $x * y$ whenever x, y are elements of a lojid algebra $(X; *, 0)$.

Definition 4.1. *Let X be a lojid algebra. An element $x \in X$ is said to be $\lambda-$ invertible if there exists a bijection λ on X defined by $\lambda(a) = a^\lambda$, $(a \in X)$, such that $a^\lambda(ax) = x$*

If every element of a lojid algebra X is $\lambda-$ invertible, then X is said to be $\lambda-$ invertible.

Definition 4.2. *Let X be a lojid algebra. An element $x \in X$ is said to be $\rho-$ invertible if there exists a bijection ρ on X defined by $\rho(a) = a^\rho$, $(a \in X)$ such that $(xa)a^\rho = x$.*

If every element of a lojid algebra is $\rho-$ invertible, then X is said to be $\rho-$ invertible.

Definition 4.3. *A lojid algebra X which is both $\lambda-$ invertible and $\rho-$ invertible is called an invertible lojid algebra.*

Proposition 4.4. *Let X be an invertible lojid algebra. Then the following hold for all $a, b, x, y \in X$:*

1. *The expression $x = a^\lambda b$ is the solution of $ax = b$*
2. *The expression $y = ba^\rho$ is the solution of $ya = b$*
3. *$(ab)^\lambda = b^\rho a^\rho$*
4. *$(ab)^\rho = b^\lambda a^\lambda$*

Proof.

1. Consider the expression $ax = b$. "Multiplying" both sides on the left by a^λ and simplifying gives $x = a^\lambda b$ as required.
2. Consider the expression $ya = b$. "Multiplying" both sides on the right by a^ρ and simplifying gives $y = ba^\rho$ as required.

3. Let $ab = c$. Then $a = cb^\rho,\ b^\rho = c^\lambda,\ c^\lambda = b^\rho a^\rho$. Thus the conclusion follows.

4. Let $ab = c$. Then $b = a^\lambda c,\ a^\lambda = bc^\rho,\ c^\rho = b^\lambda a^\lambda$. Thus the conclusion follows.

Definition 4.5. *A lojid algebra X is said to be partially absorbing if any two elements $x, y \in X$ satisfy $(xy)x^\rho = y$.*

Example 4.6. Let $X = \{0, 1, 2, 3, 4\}$. Define a binary operation $*$ on X by the following table:

*	0	1	2	3	4
0	0	1	2	3	4
1	1	0	3	4	2
2	2	4	0	1	3
3	3	2	4	0	1
4	4	3	1	2	0

Then $(X; *, 0)$ is a partially absorbing lojid algebra.

The following Lemma follows from definition.

Lemma 4.7. *Let X be a partially absorbing lojid algebra. Then $x(yx^\rho) = y$ and $(xy)^\rho = x^\rho y^\rho$ for all $x, y \in X$.*

Definition 4.8. *A lojid algebra X is said to be absorbing if any two elements $x, y \in X$ satisfy $y(xy)^\rho = x^\rho$.*

Example 4.9. Let $X = \{0, 1, 2, 3, 4, 5\}$. Define a binary operation $*$ on X by the following table:

$*$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	0	4	5	3
2	2	0	1	5	3	4
3	3	5	4	0	2	1
4	4	3	5	1	0	2
5	5	4	3	2	1	0

Then $(X; *, 0)$ is an absorbing lojid algebra.

Proposition 4.10. *Every partially absorbing lojid algebra is an absorbing lojid algebra.*

Proof. Let X be a partially absorbing lojid algebra. By Lemma 4.1., we have $y(xy)^\rho = y(x^\rho y^\rho) = x^\rho$ as required.

Remark 4.11. The converse of Proposition 4.2 does not hold. Notice that the absorbing lojid algebra in Example 4.2 is not partially absorbing.

5 Congruences and Quotient Algebras

In this section, we discuss the notion of congruences in lojid algebras and use in the establishment of quotient lojid algebras.

Definition 5.1. *Let X be a lojid algebra. A relation $\sim$ on X is called a congruence if the following hold for all $a, b, c, d \in X$:*

1. $ca \sim cb \Rightarrow a \sim b$.
2. $ac \sim bc \Rightarrow a \sim b$.
3. $a \sim b$ *and* $c \sim d \Rightarrow ac \sim bd$.

Example 5.2. Let $X = \{0, 1, 2, 3, 4\}$. Define a binary operation $*$ on X by the following table:

$*$	0	1	2	3	4
0	0	1	3	4	2
1	1	0	4	2	3
2	2	4	1	3	0
3	3	2	0	1	4
4	4	3	2	0	1

Then $(X; *, 0)$ is a lojid algebra. Now, define a relation $\sim$ on X by $x \sim y \Rightarrow x^2 = y^2$. Then $\sim$ is a congruence on X.

Definition 5.3. *Let $(X; *, 0)$ and $(Y; *', 0')$ be lojid algebras. A function $f : X \to Y$ is called a homomorphism if $f(x * y) = f(x) *' f(y)$ for all $x, y \in X$.*

Let X and Y be lojid algebras and let $f : X \to Y$ be a homomorphism. Then f induces a relation $\sim$ on X defined by $a \sim b \Leftrightarrow f(a) = f(b)$. Clearly, $\sim$ is a congruence on X.
Conversely, if $\sim$ is a congruence on a lojid X, then $\sim$ determines a homomorphism $f : X \to Y$. To show this, let $K_a, K_b, ...$ be the congruence classes with respect to $\sim$. Then $bK_a = \{ba_i : a_i \in K_a\} = K_{ba}$. So, we can define $K_a *' K_b = K_{ab}$. Let Y be the collection $\{K_a, K_b, ...\}$ of all congruence classes. Then $(Y; *', K_0)$ is a lojid algebra. The equations $K_x *' K_a = K_b$ and $K_a *' K_y = K_b$ have unique solutions determined by the unique solutions of $x * a = b$ and $a * y = b$ in the lojid algebra $(X; *, 0)$. Let $f : X \to Y$ be such that $f(a) = K_a$ for all $a \in X$. Then $f(a * b) = f(a) *' f(b)$. Hence f is a homomorphism.

We therefore have the following Theorems:

Theorem 5.4. *Let X and Y be lojid algebras. If $f : X \to Y$ is a homomorphism, then f induces a congruence on X.*

Theorem 5.5. *Let X and Y be lojid algebras. If $\sim$ is a congruence on X, then $\sim$ induces a homomorphism $f : X \to Y$.*

Let $(X; *, 0)$ be a lojid algebra, and let $\sim$ be a congruence on X. Let $[x]$ denote the class of $x \in X$. That is, $[x] = \{y \in X : x \sim y\}$. Let $X(\sim)$ be the collection of all congruence classes in X. Define a binary operation $\circ$ on $X(\sim)$ by $[x] \circ [y] = [x * y]$ for all $[x], [y] \in X(\sim)$.

Theorem 5.6. *Let $\sim$ be a congruence on a lojid algebra $(X; *, 0)$. Then $(X(\sim); \circ, [0])$ is a lojid algebra.*

Proof. Clearly $(X(\sim); \circ, [0])$ is a semi-lojid algebra. Also, the maps $L_{[a]} : X(\sim) \to X(\sim)$ and $R_{[a]}$ defined by $L_{[a]}([x]) = [a] \circ [x]$ and $R_{[a]}([x]) = [x] \circ [a]$ respectively for all $[a] \in X(\sim)$ are bijections.

Definition 5.7. *Let $\sim$ be a congruence on a lojid algebra $(X; *, 0)$. The lojid algebra $(X(\sim); \circ, [0])$ is called the quotient lojid algebra induced by $\sim$.*

Theorem 5.8. *Let $\sim$ be a congruence on a lojid algebra X. Then a congruence class K_h has unique solvability property if and only if $h \sim h^2$.*

Proof. Let $K_h = H$. Suppose H has the unique solvability property. Let $a, b \in H$. Then we have $ab \sim h^2$. So, $ab \in H$ or $ab \sim h$, and we have $h \sim h^2$ as required.
Conversely, let H be such that $h \sim h^2$. Let $a, b \in H$. Then $a \sim h$, $b \sim h \Rightarrow ab \sim h^2$ and $ab \sim h \Rightarrow ab \in H$. So, H is closed. We show the existence of solutions of the equation $ax = b$ in H. Let $a, b \in H$. Then $b = ax \sim hx \Rightarrow b \sim h \sim h^2 \Rightarrow hx \sim h^2 \Rightarrow x \sim h \Rightarrow x \in H$. Similar argument shows that for the equation $ya = b$, we have $y \in H$. The uniqueness of the solutions is obvious.

6 Conclusion

In this paper, we have initiated the study of lojid algebras. We hope that this paper would activate further research on the theory of lojid algebras. In particular, we hope that the open problem posed in this paper would be solved in the nearest future. Several notions and concepts which we have studied in lojid algebras could also be extended to other algebras of type (2, 0).

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