International Journal of Mathematical Analysis and Modelling

(Formerly Journal of the Nigerian Society for Mathematical Biology)

Volume 7 Issue 2, 2024

ISSN (Print): 2682 - 5694
ISSN (Online): 2682 – 5708
www.tnsmb.org

Mathematical model showing preventive measure as a key to reducing Coronavirus disease (COVID-19) spread

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Abstract

A set of non-linear Mathematical equations of COVID-19 model, presented in a flow diagram was proposed to study and ascertain the impact of preventive measures (self isolation and social distancing) to curtailing the spread of the disease. To this effect, we computed the threshold quantity R_{eff} (Effective Reproduction Number), carried out the stability analysis on the equilibrium points obtained using Jacobian technique (Eigenvalue approach), Castilo-Chavez method and a suitable Lyapunov function. We discovered that the Disease Free equilibrium point failed the long term test whereas the Endemic Equilibrium was globally asymptotically stable, thus implying that the COVID-19 spread could be curtailed or curbed in the presence of preventive measures (self isolation and social distancing) as advocated.

Keywords and phrases: COVID-19; preventive measures; effective reproduction number; equilibrium point; stability

1 Introduction

At the moment, the world is seized, hanged on a serious lockdown, with capitulating economy and heralded in panic. Coronavirus disease-19 often referred to as "COVID-19" is a viral infectious disease. The outbreak started in late 2019 and became a global pandemic by March 2020. The World health organization labelled it a "pandemic" due to its global network of spread, infact the first pandemic caused by a Coronavirus [1].

As at the time of this writing, the number of confirmed cases, deaths and recovery stood at 553, 244; 25, 034; and 126,630 respectively spanning about 176 countries of the world [2]. Other viral infections such as Ebola and Lassa Fever have models already proposed for them with possible treatment but this could not suffice for COVID-19 as a result of its peculiar mode of spread. The virus spreads through person to person, between people who are in close contact with one another (within about 5 feet or 3 metres) and through respiratory droplets produced when an infected person coughs or sneezes. These droplets can land on mouth, nose or eyes which are openings into the human system. The virus can stay on stainless steel, plastic and cardboard close to $48, 72$ and 24 hours respectively [3] . The symptoms include; sore throat, dry cough, fever, fatigue, diarrhea (mild/moderate cases), difficulty in breathing (severe case) [4]. There is an already developed screening test for identifying and confirming cases. The more diagnosis are made, the more control on the level of exposure [5].

Most pathetic, there are two exposure periods. The first case, an individual shows no sign until after 5 days and the second level is the full blown COVID 19 which shows up $6-7$ days later from the first

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exposure. If the infection is not COVID-19, the individual recovers with treatment after a few days of incubation, otherwise he/she becomes infectious and needs to be quarantined to avoid further spread . The quarantined stays under serious surveillance and likely recovers afterwards if a mild infection.

COVID-19 if not confirmed early or managed as soon as confirmed could lead to breakdown in respiration, chronic pneumonia, failure of vital organs and eventual death. The global death rate is 4.4% though Nationality death rate varies [6].

It has been proposed that self isolation and social distancing as preventive measures could cut down on the spread of COVID-19 outbreak [7]. Other necessary measures include; washing of hands with sanitizer, wearing a face mask and keeping the immune system strong with proper diet [8].

In this paper, we employed the mathematical modelling tool in order to ascertain the effectiveness of the preventive measures to curtailing the pandemic.

The threshold quantity known as the effective reproduction number (R_{eff}) was computed [9] and the equilibrium points obtained. The impact of the preventive measures were studied on the short and long run (local and global stability) using the approaches by Castilo- Chavez [10] and Lyapunov method [11].

2 Methodology/Model formulation

We present a flow diagram to illustrate the pattern of dynamism associated with COVID-19. The total population is denoted by $N(t)$ since population is time dependent. Hence

$$N(t) = S_p(t) + S(t) + E(t) + E_T(t) + I(t) + Q(t) + R(t) \tag{1}$$

S is the Susceptible population; S_p the population guarded via self isolation and social distancing; E is the exposed individuals; E_T the exposed but treated individuals; I the infective/infectious individuals; Q the Quarantined individuals and R the Recovered individuals.
$(I - \phi)\Lambda$ is the recruitment rate into the population susceptible to COVID-19; $\phi\Lambda$ is the recruitment rate of individuals prevented /protected /guarded against COVID-19 ; μ is the Natural death/mortality rate; β is the exposure rate of susceptible individuals to COVID-19; α is the rate of treatment of the exposed; $\lambda = 1 - \alpha$ is the rate at which the untreated exposed (those that are actual carriers-under incubation period) becomes infectious (tests positive/confirmed cases); η is the rate at which the infectious individuals become quarantined; τ_1 and τ_2 are the rates at which the quarantined and exposed treated recovers respectively; δ_1 is the COVID-19 induced death rate.
All variables and parameters are positive for all time $t > 0$ in the feasible region Ω where $S_p(t)$, $S(t)$, $E(t)$, $E_T(t)$, $I(t)$, $Q(t)$, $R(t) \in \Omega \subset \Re^7_+$
The solution of each equation are bounded in Ω , $\forall\, t > 0$ such that $0 \leq N \leq \frac{(1-\phi)\Lambda}{\mu}$.
It follows that the Model is Epidemiologically and Biologically well posed in the region. Model flow diagram is seen in Figure 1:

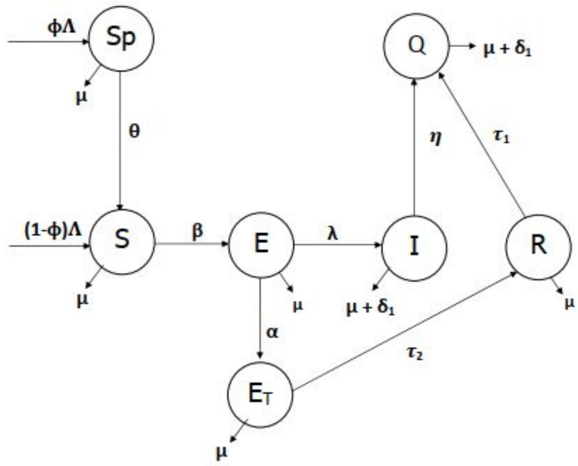

Figure 1

3 Analysis of the model

The Governing equation

$$\begin{aligned}
\frac{dS_p}{dt} &= \phi\Lambda - (\theta+\mu)S_p \\
\frac{dS}{dt} &= (1-\phi)\Lambda + \theta S_p - \beta SE - \mu S \\
\frac{dE}{dt} &= \beta SE - \lambda E - \alpha E - \mu E \\
\frac{dE_T}{dt} &= \alpha E - (\tau_2+\mu)E_T \\
\frac{dI}{dt} &= \lambda E - (\mu+\delta_1+\eta)I \\
\frac{dQ}{dt} &= \eta I - (\tau_1+\mu+\delta_1)Q \\
\frac{dR}{dt} &= \tau_1 Q + \tau_2 E_T - \mu R
\end{aligned} \tag{2}$$

The force of infection is given by

$$\beta E = \frac{\gamma\varepsilon(1-\xi)E}{N} \tag{3}$$

where γ is the probability rate of contracting COVID−19; ε is the infection contact rate; and ξ denotes the chance of success of preventive measures (self isolating and social distancing) against COVID−19; and hence the effective reproductive number (R_{eff})is given as

$$R_{eff} = \frac{\beta S^\circ}{(\lambda+\alpha+\mu)} = \frac{\gamma\varepsilon(1-\xi)}{(\lambda+\alpha+\mu)}. \tag{4}$$

Note:

- S° is the COVID−19 free equilibrium point obtained as $\frac{(1-\phi)\Lambda}{\mu} \geq N$.
- R_{eff} is the average number of secondary infection per infectious case in a population made up of both susceptible and non-susceptible (protected/prevented) population [9].

The disease free equilibrium point DFE is given by solving (2).

$$E_\circ = (S_p^\circ, S, E^\circ, E_T^\circ, I^\circ, Q^\circ, R^\circ) = (0, \frac{(1-\phi)\Lambda}{\mu}, 0, 0, 0, 0, 0)$$

where

$$S_p^\circ = E^\circ = E_T^\circ = I^\circ = Q^\circ = R^\circ = 0 \ \text{ and } S^\circ > 0$$

and the endemic equilibrium point

$$E_* = (S_p^*, S^*, E^*, E_T^*, I^*, Q^*, R^*)$$

where

$$S_p^* \neq 0,\ S^* \neq 0,\ E^* \neq 0,\ E_T^* \neq 0,\ I^* \neq 0,\ Q^* \neq 0,\ R^* \neq 0.$$

$$S_p^* = \frac{\phi\Lambda}{\theta+\mu}\ ;\ S^* = \frac{\lambda+\alpha+\mu}{\beta} \ = \ \frac{1}{R_{eff}} > 0 \ \text{ iff } \ R_{eff} > 1.$$

Thus less susceptible individuals get exposed.

$$\begin{aligned} E^* &= \frac{(1-\phi)\Lambda + \theta S_p^* - \mu S^*}{\beta S^*} \\ &= \frac{(1-\phi)\Lambda}{(\lambda+\alpha+\mu)} + \frac{\phi\theta\Lambda}{(\lambda+\alpha+\mu)(\theta+\mu)} - \frac{\mu}{\beta} \\ &= \frac{\mu}{\beta}\Big[\frac{(1-\phi)\Lambda(\theta+\mu)+\phi\theta\Lambda}{(\theta+\mu)(\lambda+\alpha+\mu)}.\frac{\beta}{\mu} - 1\Big] \\ &= \frac{\mu}{\beta}\Big[\frac{(1-\phi)\Lambda(\theta+\mu)+\phi\theta\Lambda}{\mu(\theta+\mu)}.R_{eff} - 1\Big] \end{aligned}$$

$$E^* > 0 \ \text{ provided that } R_{eff} > 1 \ \text{ and } \frac{\phi\theta\Lambda}{\mu(\theta+\mu)} > \frac{(1-\phi)\Lambda}{\mu} \ \text{and} \ \frac{\mu}{\beta} > 0$$

(since all parameters are positive), this implies that exposure to COVID-19 reduces with an increase in preventive measures (self isolation and social distancing).

$$I^* = \frac{\lambda E^*}{\mu+\delta_1+\eta} = \frac{\mu\lambda}{\beta(\mu+\delta_1+\eta)}\Big[\frac{(1-\phi)\Lambda(\theta+\mu)+\phi\theta\Lambda}{\mu(\theta+\mu)}.R_{eff} - 1\Big]$$

$$I^* > 0 \ \text{ if and only if } \ R_{eff} > 1 \ \text{ and } \ \frac{\phi\theta\Lambda}{\mu(\theta+\mu)} > \frac{(1-\phi)\Lambda}{\mu} \ \text{ with } \ \frac{\mu\lambda}{\beta(\mu+\delta_1+\eta)} > 0$$

This suggests that infection from COVID-19 decreases due to a decrease in exposure level resulting from increased preventive measures.

$$E_T^* = \frac{\alpha E^*}{\tau_2+\mu}\ ;\ Q^* = \frac{\eta I^*}{\tau_1+\mu+\delta_1}\ ;\ R^* = \frac{\tau_1 Q^* + \tau_2 E_T^*}{\mu}$$

3.1 Local stability of DFE and Endemic equilibrium points

Theorem 3.1. *The COVID−19 free and endemic equilibrium points are locally asymptotically stable (LAS), if $R_{eff} < 1$ and $\det(J) > 0,\ Tr(J) < 0$.*

Proof:
We obtain Jacobian of (2) evaluated at DFE and EEP and check if $\det(J) > 0$, and $Tr(J) < 0$

$$|J - \lambda I| = \begin{bmatrix} A & 0 & 0 & 0 & 0 & 0 & 0 \\ \theta & B & -\beta S & 0 & 0 & 0 & 0 \\ 0 & \beta E & C & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & D & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & F & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & \tau_2 & 0 & \tau_1 & -\mu-\lambda \end{bmatrix}$$

where $A = -(\theta + \mu) - \lambda$, $B = -(\beta E + \mu) - \lambda$, $C\beta S - (\lambda + \alpha + \mu) - \lambda$ $D = -(\tau_2 + \mu) - \lambda$, $F = -(\mu + \delta_1 + \eta) - \lambda$, $G = \eta - (\tau_1 + \mu + \delta_1) - \lambda$.

$$\lambda_1 = -(\theta + \mu)\ ;\ \lambda_2 = -\mu\ ;\ \lambda_3 = -(\tau_1 + \mu + \delta_1)\ ;\ \lambda_4 = -(\mu + \delta_1 + \eta)\ ;\ \lambda_5 = -(\tau_2 + \mu)$$

The Jacobian reduces to

$$\begin{pmatrix} -(\beta E + \mu) - \lambda_6 & -\beta S \\ \beta E & \beta S - (\lambda + \alpha + \mu) - \lambda_7 \end{pmatrix}$$

Let $a = \beta E + \mu\ ;\ b = \beta S\ ;\ c = \beta E\ \ d = \lambda + \alpha + \mu = \frac{\beta}{R_{eff}}$

$$\begin{aligned} &\Rightarrow\ (-a - \lambda)(b - d - \lambda) + bc = 0 \\ &\Rightarrow\ \lambda^2 + \lambda(a + d - b) + ad + bc - ab = 0. \end{aligned}$$

If $a+d > b$ then $a+d-b > 0$; also if $ad+bc > ab$, then, $ad+bc-ab > 0$. Hence by Routh-Hurwitz Criterion $\lambda_6, \lambda_7 < 0$. Since $\lambda_i's$, $i = 1, 2, ...7$, are < 0 if $R_{eff} < 1$, and clearly $\det(J) > 0$ and $Tr(J) > 0$ then DFE and EEP are Locally asymptotically stable (LAS)

3.2 Global stability of Disease free equilibrium

Lemma 3.2. *Consider a Model System written in the form*

$$\begin{aligned} \frac{dz_1}{dt} &= F(z_1, z_2) \\ \frac{dz_2}{dt} &= G(z_1, z_2),\ \ G(z_1, 0) \end{aligned}$$

where $z_1 \in \Re^m$ and $z_2 \in \Re^n$ and $z_0 = (z_1, 0)$ is an equilibrium point of the system. Assume that
$(H1)$: for $\frac{dz_1}{dt} = F(z_1, 0)$, z_1^* is GAS
$(H2)$: $G(z_1, z_2) = Az_2 - \hat{G}(z_1, z_2),\ \ G(z_1, z_2) \geq 0$ For $z_1,\ z_2 \in \Omega$, where the Jacobian $A = \frac{\partial G}{\partial z_2}(z_1, 0)$ is an $M - matrix$ (the off diagonal elements of A are non-negative) the model makes biological sense. Hence z_0 is GAS provided $R_{eff} < 1$

We need to show that both $(H1)$ and $(H2)$ hold for $R_{eff} < 1$.
Let $z_1 = (S_p, S, E_T, Q, R)$, $z_2 = (E, I)$ and $z_1^* = (S_p^\circ, S^\circ, E_T^\circ, Q^\circ, R^\circ)$. $G(z_1, z_2)$ is given by

$$G(z_1, z_2) = \begin{pmatrix} \beta SE & -(\lambda + \alpha + \mu)E \\ \lambda E & -(\mu + \delta_1 + \eta)I \end{pmatrix}$$

This can be rewritten as $Az_2 - \hat{G}(z_1, z_2)$

$$A = \begin{pmatrix} \beta S^{\circ} - (\lambda + \alpha + \mu) & 0 \\ \lambda & -(\mu + \delta_1 + \eta) \end{pmatrix}$$

$\hat{G} = (z_1, z_2) = \beta E(S^{\circ} - S)$ then $\hat{G}(z_1, z_2) = \beta E S^{\circ} > 0$ it is obvious that $S^{\circ} > S$, $AZ = 0$, hence $G(z_1, z_2) = -\beta E S^{\circ} = -\gamma\varepsilon\frac{(1-\xi)}{N} E.S^{\circ}$.
Since $G(z_1, z_2) < 0$, it follows that condition $(H2)$ fails.
The DFE, E_0 is not globally asymptotically stable, this implies a continuous outbreak is imminent if condition that gives rise to it are not properly handled. This encourages compulsory enlightenment on self isolation and social distancing.

3.3 Global Stability of the Endermic Equilibrium

We seek this by means of Lyapunov direct method and Lasalle's invariant principle.
We assume the non-linear Lyapunov function

$$L : \{(S_p, S, E, E_T, I, Q, R) \in \Omega \subset \Re^7_+ : S_p(t), S(t), E(t), E_T(t), I(t), Q(t), R(t) > 0\}$$

Then

$$\begin{aligned} L : (S_p, S, E, E_T, I, Q, R) \quad = \quad & \beta(S - S^* - S^* \log\frac{S}{S^*}) + \beta(S_p - S_p^* - S_p^* \log\frac{S_p}{S_p^*}) \\ & +\beta(E - E^* - E^* \log\frac{E}{E^*}) \\ & +\beta(E_T - E_T^* - E_T^* \log\frac{E_T}{E_T^*}) \\ & +\beta(I - I^* - I^* \log\frac{I}{I^*}) \\ & +\beta(Q - Q^* - Q^* \log\frac{Q}{Q^*}) \\ & +\beta(R - R^* - R^* \log\frac{R}{R^*}), \end{aligned}$$

where L is C' in the region of Ω, L on Ω has a global minimum E_* and $L : (S_p, S, E, E_T, I, Q, R) = 0$. Thus

$$\begin{aligned} \frac{dL}{dt} = \dot{L} \quad &= \quad \beta(1 - \frac{S^*}{S})\frac{dS}{dt} + \beta(1 - \frac{S_p^*}{S_p})\frac{dS_p}{dt} + \beta(1 - \frac{E^*}{E})\frac{dE}{dt} + \beta(1 - \frac{E_T^*}{E_T})\frac{dE_T}{dt} \\ &+ \quad \beta(1 - \frac{I^*}{I})\frac{dI}{dt} + \beta(1 - \frac{Q^*}{Q})\frac{dQ}{dt} + \beta(1 - \frac{R^*}{R})\frac{dR}{dt} \\ &= \quad \beta(\frac{S - S^*}{S})\frac{dS}{dt} + \beta(\frac{S_p - S_p^*}{S_p})\frac{dS_p}{dt} + \beta(\frac{E - E^*}{E})\frac{dE}{dt} + \beta(\frac{E_T - E_T^*}{E_T})\frac{dE_T}{dt} \\ &+ \quad \beta(\frac{I - I^*}{I})\frac{dI}{dt} + \beta(\frac{Q - Q^*}{Q})\frac{dQ}{dt} + \beta(\frac{R - R^*}{R})\frac{dR}{dt}. \end{aligned}$$

Clearly, we see that $\dot{L} = 0$ provided $S = S^*,\ S_p = S_p^*,\ E = E^*,\ E_T = E_T^*, I = I^*,\ Q = Q^*$ and $R = R^*$.
Hence E_* is globally asymptotically stable within the region Ω. We have succeeded in showing Mathematically that in the long run that active and constant preventive approach yields expected results as regards outbreak intervention with special reference to COVID-19 being considered.

4 Discussions and conclusion

The Mathematical analysis shows that the DFE, E_0 is not globally asymptotically stable meaning the spread continues unless there be an enforcement of self isolation and social distancing across the infected regions and even uninfected, as these reduces the exacerbation of COVID-19 spread.

The spread of COVID-19 can be effectively and efficiently controlled over a given population with time, if the individuals take responsibility through preventive measures (self isolation and social distancing for the carriers and non-carriers) as encouraged.

Exposure to spread decreases with a suitable increase in preventive/protective measures resulting in subsequent lowering in number of new cases/infection.

Acknowledgment

This research is sponsored by TETFUND Nigeria under the IBR funding with
project number:TETFUND/DR&D/CE/UNI/NSUKKA/RP/VOL.I

References

[1] World Health Organization (WHO) https://www.who.int/europe/emergencies/situations/covid-19 last accessed 27/9/2024

[2] John Hopkins COVID-19 Resource Center, John Hopkins University of Medicine, https://github.com/CSSEGISandData/COVID-19/tree/master/csse_covid_19_data last accessed 27/9/2024.

[3] John Hopkins University (JHU) HUB, https://hub.jhu.edu/search/?q=coronavirus, last accessed 27/9/2024

[4] Adhikari, S. P., Meng, S., Wu, Y. J., Mao, Y. P., Ye, R. X., Wang, Q. Z., & Zhou, H. (2020). Epidemiology, causes, clinical manifestation and diagnosis, prevention and control of coronavirus disease (COVID-19) during the early outbreak period: a scoping review. Infectious diseases of poverty, 9, 1-12.

[5] http: // www.who.int>docs.

[6] https: // doi.org /10.1186/540249.020.00646.

[7] https : // Coronavirus.jhu.edu.

[8] http://www.fda.gov/

[9] Hethcote, H. W. (2000). The mathematics of infectious diseases. SIAM review, 42(4), 599-653.

[10] Castillo-Chavez, C., Feng, Z. and Huang, W. (2002) On the Computation of RO and Its Role on Global Stability. In: Castillo-Chavez, P.C., Blower, S., Driessche, P., Kirschner, D. and Yakubu, A.-A., Eds., Mathematical Approaches for Emerging and Reemerging Infectious Diseases: An Introduction, Springer, Berlin, 229. https://doi.org/10.1007/978-1-4757-3667-0_13

[11] Korobeinikov, A., & Wake, G. C. (2002). Lyapunov functions and global stability for SIR, SIRS, and SIS epidemiological models. Applied Mathematics Letters, 15(8), 955-960.