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Mathematical model for transmission dynamics of Malaria disease: impact of invasive alien plants

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Abstract

A mathematical model to study the impact of invasive alien plants on the dynamics of malaria transmission and its analyses is studied in this work. The resulted model equations are divided into homogeneous and non-homogeneous equations. The homogeneous equations are solved to determine its disease free equilibrium (DFE) and their stabilities. A basic reproduction number was determined from the DFE. It was found that when $R_0 < 1$, the disease will die out, when $R_0 = 1$, the model undergoes a backward bifurcation, $R_0 = 0$, the model undergoes forward bifurcation and whenever $R_0 > 1$, the disease will persist in the population. At $R_0 > 1$, the global analysis of the model was carried out and it was found to be globally and asymptotically stable (GAS). A sensitivity analysis of the model parameters was also investigated to determine the parameters that are sensitive for malaria transmission. A travelling wave equations and solutions were also provided for possible understanding of the behaviour of mosquito's mobility in human environment.

Keywords and phrases: malaria; invasive alien plant; mathematical model; stability, wave equations

1 Introduction

Malaria is a life-threatening disease caused by plasmodium parasites. It is transmitted when an infected female anopheles mosquito bite a susceptible human [11, 12]. The parasite inform of a sporozoite is injected into the blood stream and that is where infection started, and can also be transmitted through blood transfusion and contaminated needles. Infections by malaria parasites may result in a wide variety of symptoms, mild symptoms are fever, chills and headache. Severe symptoms include fatigue, confusion, seizures, and difficulty breathing. In general, malaria is a curable disease if diagnosed and treated promptly and correctly [2]. 608,000 malaria deaths occurred in 2022, with 95% of them occurring in the African Region.

The African region carries the highest share of the global malaria burden, with 94% of malaria cases and 95% of malaria deaths in 2022. The Sub-Saharan Africa, with the highest transmission found near the equator, particularly in Nigeria, the Democratic Republic of the Congo, Uganda, and Mozambique. Estimated 233 million malaria cases was reported in African Region in 2022 [12]. Children under 5 years old accounted for 80% of all malaria deaths in the African Region in 2022. Nigeria accounted for 27% of the global malaria burden and 31% of estimated deaths was due to malaria in 2022.

Malaria control strategy agenda using Dichlorodiphenyltrichloroethane (DDT) that became the main tool for the World Health Organisation's (WHO) malaria eradication programme from 1956 – 1967 fell out because of its effects on environmental management. Since the failure of this campaign to sustainably eliminate malaria from the tropics, the effectiveness of environmental management has remained tragically under-exploited. The authors in [8] promised a modern way of vector control

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strategies such as: invasive plant control, vaccination, treatments, control of vectors, insecticide treated net (ITN). It is based on this tragic background that we wish to develop a sound spatial nonlinear mathematical model to investigate the effects of invasive plants that allow for vector proliferation and spatial dynamics of vectors on the dynamics of malaria within a given habitat.

Invasive alien plants have been shown to have significant impacts on malaria transmission in various ways. These plants can create suitable habitats for mosquito breeding, leading to increased mosquito populations and greater transmission of the malaria parasite. In addition, invasive plants can alter local ecosystems and disrupt the natural balance of predator-prey relationships, further contributing to increased mosquito populations. Some invasive plants also have medicinal properties that can affect malaria treatment and drug resistance [8].

Overall, the invasion of alien plants poses a serious threat to efforts to control and prevent malaria transmission. It is crucial for researchers and policymakers to consider the impact of invasive plants on malaria transmission when developing strategies for disease control and management. Additionally, more research is needed to better understand the specific mechanisms through which invasive plants influence malaria transmission and to develop effective interventions to mitigate their negative effects. Mathematical models have been used to study the effects of invasive alien plants on malaria transmission. These models typically involve a combination of ecological and epidemiological factors to simulate how the presence of invasive plants can impact mosquito populations, parasite transmission, and disease dynamics [13].

One key aspect of these models is the influence of invasive plant habitats on mosquito breeding and population dynamics. By incorporating data on mosquito behaviour, habitat preferences, and reproductive rates, researchers can assess how invasive plants create environments conducive to mosquito proliferation, leading to increased malaria transmission.

Additionally, mathematical models can examine how invasive plants alter local ecosystems and disrupt predator-prey relationships that may regulate mosquito populations. These models can help identify mechanisms through which invasive plants indirectly affect malaria transmission by altering the abundance of mosquito predators or competitors. Furthermore, some mathematical models have explored the impact of invasive plants on the efficacy of malaria control strategies, such as insecticide spraying and bed nets. By considering how invasive plants affect mosquito behaviour and distribution, researchers can evaluate the effectiveness of traditional control.

One mathematical model that has been used to study the effects of invasive plants on malaria transmission is the "Larval Habitat Model." This model focuses on the relationship between the abundance of mosquito larval habitats, which can be influenced by the presence of invasive plants, and the density of mosquito populations. By quantifying the impact of invasive plant habitats on mosquito breeding sites, researchers can predict changes in malaria transmission dynamics.

Another mathematical model is the "Vector-Host Model," which considers the interactions between mosquito vectors, human hosts, and invasive plant habitats. This model can help assess how invasive plants create environments that favour mosquito breeding, leading to increased transmission of the malaria parasite to human hosts. By incorporating data on mosquito feeding habits, parasite development, and human movement patterns, researchers can better understand the complex dynamics of invasive plants on malaria transmission. Additionally, the "Ecosystem Disruption Model" is a mathematical model that examines how the invasion of alien plants disrupts local ecosystems and influences malaria transmission. This model can The Larval Habitat Model uses a system of differential equations to describe the dynamics of mosquito larvae in response to changes in habitat availability. The basic equation for the larval habitat model is:

$$\frac{dL}{dt} = cE - mL$$

Where $\frac{dL}{dt}$ represents the rate of change in the number of mosquito larvae over time, L represents the number of mosquito larvae in the habitat, c represents the rate at which mosquito eggs hatch and develop into larvae in the habitat, E represents the availability of suitable habitat for mosquito larvae to breed, and m represents the mortality rate of mosquito larvae in the habitat. This equation captures the balance between the hatching of mosquito eggs and the mortality of larvae in the habitat, which is influenced by the availability of suitable breeding sites (habitat quality) determined by the presence of invasive alien plants. By manipulating the variables in this equation, researchers can assess how changes in habitat availability due to invasive plants impact mosquito larval populations on malaria transmission dynamics.

2 Methodology/Model formulation

We shall begin our study of the mathematical dynamics of malaria motivated by invasive alien plants presence in this section.

2.1 Assumptions of the model equations

To begin the formulation, we briefly state some important assumptions of our proposed model as follows.

M_1 It is assumed that vegetation harbour mosquito population.

M_2 It is assumed that mosquitoes are in human environment without vegetation.

M_3 It is also assumed that mosquitoes migrate from vegetation in random motion.

M_4 we also assume that some recovered humans lost immunity and became susceptible.

M_5 It is assumed that mosquitoes rest in the Vegetation and not all of them leave the vegetation to human environment due to their reproduction cycle.

2.2 Model variables and parameters

Our proposed mathematical model is divided into eight (8) sub-population, and nineteen (19) parameters as shown below.

2.3 Model equations

$$
\begin{aligned}
\frac{\partial S_h}{\partial t} &= \Lambda_h - \lambda_h S_h + \alpha_h R_h - \mu_h S_h \\
\frac{\partial E_h}{\partial t} &= \lambda_h S_h - (q_h + \mu_h) E_h \\
\frac{\partial I_h}{\partial t} &= q_h E_h - (\delta_h + \mu_h + v_h) I_h \\
\frac{\partial R_h}{\partial t} &= v_h I_h - (\mu_h + \alpha_h) R_h \\
\frac{\partial P}{\partial t} &= g(1 - \frac{P}{K_P}) P - hP \\
\frac{\partial S_m}{\partial t} &= D\frac{\partial^2 S_m}{\partial x^2} - K\frac{\partial S_m}{\partial x} + \Lambda_m + \theta_m P - \lambda_m S_m - (\xi_m + \mu_m) S_m \\
\frac{\partial E_m}{\partial t} &= D\frac{\partial^2 E_m}{\partial x^2} - K\frac{\partial E_m}{\partial x} + \lambda_m S_m - (q_m + \xi_m + \mu_m) E_m \\
\frac{\partial I_m}{\partial t} &= D\frac{\partial^2 I_m}{\partial x^2} - K\frac{\partial I_m}{\partial x} + q_m E_m - (\xi_m + \mu_m) I_m
\end{aligned}
\tag{1}
$$

Where,

$$
\lambda_h = (1 - \epsilon\gamma)\beta_{hm}\frac{bI_m}{N_h}, \lambda_m = (1 - \epsilon\gamma)\beta_{mh}\frac{bI_h}{N_h}
$$

$$
0 < \epsilon < 1, 0 < \gamma < 1
$$

Table 1: Description of model variables and parameters

Variable/Parameter	Interpretation
$S_h(t)$	Number of susceptible human at time t
$E_h(t)$	Number of exposed humans at time t
$I_h(t)$	Number of infected humans at time t
$R_h(t)$	Number of recovered humans at time t
$P(t)$	Invasive Plants population
$S_m(t)$	Number of susceptible mosquitoes at time t
$E_m(t)$	Number of Exposed mosquitoes at time t
$I_m(t)$	Number of infected mosquitoes at time t
b	Number of bite per mosquito/vector
D	The coefficient of diffusion
K	The coefficient of advection
q_h	Rate at which exposed human population become infected
v_h	Recovered rate of infected individual
α_h	Recovery rate of human
h	Control rate of invasive plants
θ_m	recruitment rate of mosquitoes from the density of invasive plants into the general susceptible mosquito
μ_h	Natural death rate of humans
δ_h	Disease induced death rate
g	Growth rate of plant
K_p	Plant carrying capacity
μ_m	Natural death rate of mosquitoes
ξ_m	Death rate of mosquito due to insecticide spray
$\lambda_h(t)$	Force of infection for humans
$\lambda_m(t)$	Force of infection for vectors
β_{hv}	probability of malaria from infectious humans to susceptible mosquitoes
β_{mh}	probability of malaria from vectors to humans
ϵ	Availability of bed-nets
γ	Efficacy of bed-nets

The homogeneous part of system (1) is given by

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h - \lambda_h S_h + \alpha_h R_h - \mu_h S_h \\
\frac{dE_h}{dt} &= \lambda_h S_h - (q_h + \mu_h) E_h \\
\frac{dI_h}{dt} &= q_h E_h - (\delta_h + \mu_h + v_h) I_h \\
\frac{dR_h}{dt} &= v_h I_h - (\mu_h + \alpha_h) R_h \\
\frac{dP}{dt} &= g(1 - \frac{P}{K_P}) P - hP \\
\frac{dS_m}{dt} &= \Lambda_m + \theta_m P - \lambda_m S_m - (\xi_m + \mu_m) S_m \\
\frac{dE_m}{dt} &= \lambda_m S_m - (q_m + \xi_m + \mu_m) E_m \\
\frac{dI_m}{dt} &= q_m E_m - (\xi_m + \mu_m) I_m
\end{aligned} \tag{2}$$

from (1),we substituted $N = S_m + E_m + I_m$
or

$$\frac{\partial N}{\partial t} = \frac{\partial S_m}{\partial t} + \frac{\partial E_m}{\partial t} + \frac{\partial I_m}{\partial t}$$

Therefore, the non-homogeneous part of system (1) given as

$$D\frac{\partial^2 N}{\partial x^2} - K\frac{\partial N}{\partial x} \tag{3}$$

where D and K represent Difffusion and advection

Note: the constant K was introduced into the known parabolic partial differential equation to capture the behaviour of mosquito mobility. The resulted equation (3) can be solved with space in time and distance.We then omitted the solution since actual grid for its life application be determined explicitly.

3 Analysis of the model

We begin our analysis with the homogeneous model equations of system (2) as follows:
To find the disease free equilibrium (DFE), we equate the left hand sides of system (2) to zero, that is, $\frac{dS_h}{dt} = \frac{dE_h}{dt} = \frac{dI_h}{dt} = \frac{dR_h}{dt} = \frac{dP}{dt} = \frac{dS_m}{dt} = \frac{dE_m}{dt} = \frac{dI_m}{dt} = 0$. At the same time, the infected variables are equate to zero. We then solve for the rest of non-infected variables as follows. Hence, system (2) becomes

$$\begin{aligned}
0 &= \Lambda - \lambda_h S_h + \alpha_h R_h - \mu_h S_h \\
0 &= \lambda_h S_h - (q_h + \mu_h)E_h \\
0 &= q_h E_h - (\delta_h + \mu_h + v_h)I_h \\
0 &= v_h I_h - (\mu_h + \alpha_h)R_h \\
0 &= g(1 - \frac{P}{K_P})P - hP \\
0 &= \Lambda_m + \theta_m P - \lambda_m S_m - (\xi_m + \mu_m)S_m \\
0 &= \lambda_m S_m - (q_m + \xi_m + \mu_m)E_m \\
0 &= q_m E_m - (\xi_m + \mu_m)I_m
\end{aligned} \tag{4}$$

3.1 Case 1: Where there is plant

Also at DFE,we have $E_h = I_h = R_h = E_m = I_m = 0$, substituting these into (4), we have

$$\begin{aligned}
\Lambda_h - \mu_h S_h &= 0 \\
g(1 - \frac{P}{K_P})P - hP &= 0 \\
\Lambda_m + \theta_m p - (\xi_m + \mu_m)S_m &= 0
\end{aligned} \tag{5}$$

Solving equation (5) for S_h, P and S_m, we have

$$\begin{aligned}
S_h^* &= \frac{\Lambda_h}{\mu_h} \\
P^* &= \frac{K_p(g-h)}{g} \\
S_m^* &= \frac{V_h g + K_p\theta_m(g+h)}{g(\xi_m + \mu_m)}
\end{aligned} \tag{6}$$

Therefore, the Disease Free-equilibrium corresponding to the presence of invasive plants is given by

$$\begin{aligned} E_1 &= (S_h^*, E_h^*, I_h^*, R_h^*, P^*, S_m^*, E_m^*, I_m^*) \\ &= (S{*}_h, 0, 0, 0, P^*, S_m^*, 0, 0) \end{aligned}$$

where

$$\begin{aligned} S_h^* &= \frac{\Lambda}{\mu_h}, \\ P^* &= \frac{K_p(g-h)}{g}, g > h \\ S_m^* &= \frac{\Lambda_m g + K_p\theta_m(g-h)}{g(\xi_m + \mu_m)}, g > h \end{aligned}$$

3.1.1 Biological implication of E_1 equlibrium

The equilibrium, E_1 is the equilibrium state where there is plant in human environment with mosquito present.In this equilibrium there is a contact between human and mosquito population and breeding site for mosquito population to thrive.This may result to malaria in human population as time progresses.

3.2 Case II: When there is no invasive plant in human environment

In this case, we solve for the equilibrium where $P = 0$, that is, the plant population and obtain the second Disease Free equilibrium (DFE) as follows:

$$\begin{aligned} E_2 &= (S_h^+, E_h^+, I_h^+, R_h^+, P^+, S_m^+, E_m^+, I_m^+) \\ &= \Big(\frac{\Lambda}{\mu_h}, 0, 0, 0, 0, \frac{\Lambda_m}{\xi_m + \mu_m}, 0, 0\Big) \end{aligned} \tag{7}$$

3.3 Biological implication of E_2,equilibrium

The Equilibrium E_2 is a Disease Free Equilibrium at which there is no invasive plant in the host environment.The contact rate of mosquito and human is low as the mosquito do not have a resting place to reside around the human environment after bite.Malaria transmission is very low and it can easily be controlled by mere use of mosquito net.

3.4 Analysis of basic reproduction number (R_0)

To calculate the basic reproduction number, we divide system (2) into appearance of infection and transfer of infection as matrix $F_i(x)$ and $V_i(x)$ where $F_i(x)$ be the rate of appearance of new infections in compartment i and $V_i(x)$ be the difference between the transfer rate of individuals out of compartment i by all other means. Thus, the $F_i(x)$ and $V_i(x)$ of model (2) is shown below: Let x_0 be the DFE of model (2). Thus, we have the following partitioned derivatives,

$$DF(x_0) = \begin{pmatrix} F & 0 \\ 0 & 0 \end{pmatrix}$$

$$DV(x_o) = \begin{pmatrix} V & 0 \\ J_3 & J_4 \end{pmatrix}$$

Where F and V are MxN matrices defined by

$$F = \left(\frac{\partial F_i}{\partial x_j(x_o)}\right)$$

and

$$V = \left(\frac{\partial V_i}{\partial x_j(x_o)}\right)$$

$1 \leq 1, j \leq m$

Here,the partial derivatives of F_i and V_i are with respect to the infected classes only. The basic reproduction number is defined as

$$R_o = \rho(FV^{-1}) \tag{8}$$

Where ρ is the spectral radius of FV^{-1}(by spectral radius we mean the maximum eigenvalue of $FV-1$)

Theorem 1

Consider the disease transmission model (2) with $f(x)$ satisfying the stability conditions if x_0 is a DFE of the model,then x_o is locally asymptotically stable if $R_0 < 1$, but unstable if $R_0 > 1$, where R_0 is defined by (8).For the purpose of easy access and clarity, we shall rewrite model (2) as follows:

$$\begin{aligned}
\dot{S_h} &= \Lambda - \lambda_h S_h + \alpha_h R_h - \mu_h S_h \\
\dot{E_h} &= \lambda_h S_h - (q_h + \mu_h)E_h \\
\dot{I_h} &= q_h E_h - (\delta_h + \mu_h + v_h)I_h \\
\dot{R_h} &= v_h I_h - (\mu_h + \alpha_h)R_h \\
\dot{P} &= g(1 - \frac{P}{K_P})P - hP \\
\dot{S_m} &= \Lambda_m + \theta_m P - \lambda_m S_m - (\xi_m + \mu_m)S_m \\
\dot{E_m} &= \lambda_m S_m - (q_m + \xi_m + \mu_m)E_m \\
\dot{I_m} &= q_m E_m - (\xi_m + \mu_m)I_m
\end{aligned} \tag{9}$$

Where,

$$\lambda_h = (1 - \epsilon\gamma)\beta_{hv}\frac{bI_m}{N_h}, \lambda_m = (1 - \epsilon\gamma)\beta_{mh}\frac{bI_h}{N_h}$$

Thus, F and V are given as

$$F = \begin{pmatrix} (1 - \epsilon\gamma)\beta_h m\frac{bI_m}{N_h}S_h \\ 0 \\ (1 - \epsilon\gamma)\beta_m h\frac{bI_h}{N_h}S_m \\ 0 \end{pmatrix}$$

and

$$V = \begin{pmatrix} (q_h + \mu_h)E_h \\ (\delta_h + \mu_h + v_h)I_h - q_h E_h \\ (q_m + \xi_m + \mu_m)E_m \\ (\xi_m + \mu_m)I_m - q_m E_m \end{pmatrix} \tag{10}$$

Since we have only four infected classes, E_h, I_h, E_m and I_m, it follows that $m = 4$.
The DFE is $E_h^* = I_h^* = R_h^* = E_m^* = I_m^* = 0$ and

$$\begin{aligned}
S_h^* &= \frac{\Lambda}{\mu_h}, P^* = \frac{K_p(g - h)}{g} \\
S_m^* &= \frac{V_h g + K_p\theta_m(g - h)}{g(\xi_m + \mu_m)}
\end{aligned} \tag{11}$$

putting these into consideration, we have

$$F = \begin{pmatrix} 0 & 0 & 0 & (1-\epsilon\gamma)\beta_{hm}b \\ 0 & 0 & 0 & 0 \\ 0 & \frac{V_h g + K_p\theta_m(g-h)}{g(\xi_m+\mu_m)}(1-\epsilon\gamma)\beta_{hm}b\frac{\Lambda}{\mu_h} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{12}$$

$$V = \begin{pmatrix} q_h+\mu_h & 0 & 0 & 0 \\ -q_h & \delta_h+\mu_h+v_h & 0 & 0 \\ 0 & 0 & q_m+\xi_m+\mu_m & 0 \\ 0 & 0 & -q_m & \xi_m+\mu_m \end{pmatrix}$$

and

$$V^{-1} = \begin{pmatrix} \frac{1}{q_h+\mu_h} & 0 & 0 & 0 \\ \frac{q_h}{(q_h+\mu_h)(v_h+\delta_h+\mu_h)} & \frac{1}{\delta_h+\mu_h+v_h} & 0 & 0 \\ 0 & 0 & \frac{1}{q_m+\xi_m+\mu_m} & 0 \\ 0 & 0 & \frac{q_m}{(\mu_m+\xi_m)(q_m+\mu_m+\xi_m)} & \frac{1}{(\mu_m+\xi_m)} \end{pmatrix} \tag{13}$$

Therefore,

$$FV^{-1} = \begin{pmatrix} 0 & 0 & b(1-\epsilon\gamma)q_m\beta_{hm} & \frac{b(1-\epsilon\gamma)\beta_{hm}}{\mu_m+\xi_m} \\ 0 & 0 & 0 & 0 \\ \omega & \frac{b(1-\epsilon\gamma)q_m\beta_{hm}(gV_h+(g+h)K_p\theta_m)\Lambda_h}{g\mu_h(v_h+\delta_h+\mu_h)(\mu_m+\xi_m)} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{14}$$

where $\omega = \frac{b(1-\epsilon\gamma)q_m\beta_{hm}(gV_h+(g+h)K_p\theta_m)\Lambda_h}{g\mu_h(q_h+\mu_h)(v_h+\delta_h+\mu_h)(\mu_m+\xi_m)}$

Thus, calculating the eigenvalues of equation (14), we have,
$\lambda_1 = 0$, $\lambda_2 = 0$
$\lambda_3 = -\frac{b(-1+\epsilon\gamma)\sqrt{q_h}\sqrt{q_m}\sqrt{\beta_{hm}}\sqrt{\beta_{mh}}\sqrt{gv_h+gK_p\theta_m+hK_p\theta_m}\sqrt{\Lambda_h}}{\sqrt{g}\sqrt{\mu_h}\sqrt{q_h+\mu_h}\sqrt{v_h+\delta_h+\mu_h}(\mu_m+\xi_m)\sqrt{q_m+\mu_m+\xi_m}}$
$\lambda_4 = \frac{b(-1+\epsilon\gamma)\sqrt{q_h}\sqrt{q_m}\sqrt{\beta_{hm}}\sqrt{\beta_{mh}}\sqrt{gv_h+gK_p\theta_m+hK_p\theta_m}\sqrt{\Lambda_h}}{\sqrt{g}\sqrt{\mu_h}\sqrt{q_h+\mu_h}\sqrt{v_h+\delta_h+\mu_h}(\mu_m+\xi_m)\sqrt{q_m+\mu_m+\xi_m}}$
More so, the dominant eigenvalue is the basic reproduction number. Therefore,

$$R_{01} = \lambda_4 = \frac{b(1-\epsilon\gamma)}{(\mu_m+\xi_m)}\sqrt{\frac{q_h q_m\beta_{hm}\beta_{mh}(gv_h+gk_p\theta_m+hk_p\theta_m)\Lambda_h}{g\mu_h(q_h+\mu_h)(v_h+\delta_h+\mu_h)(q_m+\mu_m+\xi_m)}}$$

3.5 Basic reproduction analysis in the absence of invasive plants

Here, we are interesting in calculating basic reproduction number of E_2 of the Disease Free Equilibrium where there are no invasive plant. Substituting the equilibrium, E_2 in (10) after its partial derivatives, and taking the inverse of V_2, where,

$$\begin{aligned} E_2 &= (S_h^+, E_h^+, I_h^+, R_h^+, P^+, S_m^+, E_m^+ I_m^+) \\ &= \Big(\frac{\Lambda_h}{\mu_h}, 0, 0, 0, 0, \frac{\Lambda_m}{(\xi_m+\mu_m)}, 0, 0\Big) \end{aligned}$$

Thus, the basic reproduction number of E_2 is computed as:

$$\bar{F_2} = DF_2(E2) = \begin{pmatrix} 0 & 0 & 0 & (1-\epsilon\gamma)\beta_{hm}b \\ 0 & 0 & 0 & 0 \\ 0 & \frac{(1-\epsilon\gamma)b\beta_{mh}\Lambda_h\Lambda_m}{\mu_h(\xi_m+\mu_m)} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$\bar{V_2}^{-1} = \begin{pmatrix} \frac{1}{q_h+\mu_h} & 0 & 0 & 0 \\ \frac{q_h}{(q_h+\mu_h)(v_h+\delta_h+\mu_h)} & \frac{1}{V_h+\delta_h+\mu_h} & 0 & 0 \\ 0 & 0 & \frac{1}{q_m+\mu_h+\xi_m} & 0 \\ 0 & 0 & \frac{\xi_m}{(\mu_m+\xi_m)(q_m+\mu_m+\xi_m)} & \frac{1}{\mu_m+\xi_m} \end{pmatrix}$$

Therefore,

$$\bar{F_2}\bar{V_2}^{-1} = \begin{pmatrix} 0 & 0 & \frac{b(1-\epsilon\gamma)q_m\beta_h m}{(\mu_m+\xi_m)(q_m+\mu_m+\xi_m)} & \frac{b(1-\epsilon\gamma)\beta_{hm}}{\mu_m+\xi_m} \\ 0 & 0 & 0 & 0 \\ \frac{b(1-\epsilon\gamma)q_h\beta_m h\Lambda_h\Lambda_m}{\mu_h(q_h+\mu_h)(v_h+\delta_h+\mu_h)(\mu_m+\xi_m)} & \frac{b(1-\epsilon\gamma)\beta_m h\Lambda_h\Lambda_m}{\mu_h(v_h+\delta_h+\mu_h)(\mu_m+\xi_m)} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{15}$$

The eigenvalues corresponding to equation (15) are:

$$\lambda_1 = 0, \lambda_2 = 0$$

and

$$\lambda_3 = -\frac{b(1-\epsilon\gamma)\sqrt{q_h q_m \beta_{hm}\Lambda_h\Lambda_m}}{(\mu_m+\xi_m)\sqrt{\mu_h(q_h+\mu_h)(v_h+\delta_h+\mu_h)(q_m+\mu_m+\xi_m)}} g > h$$

$$\lambda_4 = \frac{b(1-\epsilon\gamma)\sqrt{q_h q_m \beta_{hm}\Lambda_h\Lambda_m}}{(\mu_m+\xi_m)\sqrt{\mu_h(q_h+\mu_h)(v_h+\delta_h+\mu_h)(q_m+\mu_m+\xi_m)}}$$

Therefore, we obtained the spectral radius $\rho(\bar{F_2}\bar{V_2}^{-1})$ as

$$R_{02} = \frac{b(1-\epsilon\gamma)}{(\mu_m+\xi_m)}\sqrt{\frac{q_h q_m \beta_{hm}\Lambda_h\Lambda_m}{\mu_h(q_h+\mu_h)(v_h+\delta_h+\mu_h)(q_m+\mu_m+\xi_m)}} \tag{16}$$

3.6 Biological implication $R_{01,02}$

As in the work of [9, 10], the epidemiological meaning of the square root value of the R_{01}, R_{02} are that, it takes two generation for infected host to produce new infected host. The biological implication of these R_{01}, R_{02} depends on its value: If $R_{01}, R_{02} < 1$ there will be no malaria in future but if R_{01}, $R_{02} > 1$ malaria will become endemic in the host population.

3.7 Local asymptotic stability (LAS) of E_1 equilibrium

Here, we investigate the local asymptotic stability of our model equations in the case where there are invasive plants in the human environment. To do this, we linearized the homogeneous part of our model equations with the corresponding equilibrium, E_1 and obtained the following:

$$J(E_1) = \begin{pmatrix} C_1 & 0 & 0 & \alpha_h & 0 & 0 & 0 & \frac{-b(1-\epsilon\gamma)\beta_{hm}}{S_h^*} \\ 0 & C_2 & 0 & 0 & 0 & 0 & 0 & \frac{b(1-\epsilon\gamma)\beta_{hm}}{S_h^*} \\ 0 & q_h & D_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_h & -\lambda-\xi_m+\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_2-\lambda & 0 & 0 & 0 \\ 0 & 0 & \frac{-b(1-\epsilon\gamma)\beta_{mh}}{S_h^*} & 0 & \theta_m & -\lambda+G_3 & 0 & 0 \\ 0 & 0 & D_2 & 0 & 0 & 0 & -\lambda-G_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_m & D_3 \end{pmatrix} \tag{17}$$

where $C_1 = -\lambda - \mu_h, C_2 = -\lambda - G_1, D_1 = -\lambda - (\delta_h + \mu_h + v_h), D_2 = \frac{b(1-\epsilon\gamma)\beta_{mh}}{S_h^*}, D_3 = -\lambda - \xi_m + \mu_m$
where

$$G_1 = (q_h + mu_h), G_2 = \alpha_h + \mu_h, G_3 = g - \frac{2gP^*}{K_p} - h, G_4 = q_m + \xi_m + \mu_m$$

The characteristic polynomial corresponding to the matrix is given by

$$T_1\lambda^4 + T_2\lambda^3 + T_3\lambda^2 + T_4\lambda + T_5 = 0 \tag{18}$$

$T_1 = (S^*)_h^2$

$$T_2 = (S^*)_h^2 q_m + 2(S^*)_h^2 \mu_m + 2(S^*)_h^2 \xi_m + q_h (S^*)_h^2 + \delta_h (S^*)_h^2 + 2\mu_h (S^*)_h^2 + (S^*)_h^2 v_h$$

$T_3 = \delta_h (S^*)_h^2 q_m + 2\mu_h (S^*)_h^2 q_m + 2q_h (S^*)_h^2 \mu_m + (S^*)_h^2 \mu_m q_m + 2q_h (S^*)_h^2 \xi_m + (S^*)_h^2 \xi_m q_m + (S^*)_h^2 v_h q_m + q_h (S^*)_h^2 q_m$

$T_4 = \delta_h\mu_h (S^*)_h^2 q_m + 2\delta_h q_h (S^*)_h^2 \mu_m + \delta_h (S^*)_h^2 \mu_m q_m + 2\delta_h q_h (S^*)_h^2 \xi_m + \delta_h (S^*)_h^2 \xi_m q_m + \delta_h q_h (S^*)_h^2 q_m + 2\mu_h q_h (S^*)_h^2 \xi_m + 2\mu_h (S^*)_h^2 \xi_m q_m + 2q_h (S^*)_h^2 \mu_m \xi_m + \mu_h^2 (S^*)_h^2 q_m + q_h (S^*)_h^2 \mu_m^2 + \mu_h q_h (S^*)_h^2 q_m + q_h (S^*)_h^2 \mu_m q_m + 2\mu_h q_h (S^*)_h^2 \mu_m + 2\mu_h (S^*)_h^2 \mu_m q_m + q_h (S^*)_h^2 \xi_m^2 + q_h (S^*)_h^2 \xi_m q_m + \mu_h (S^*)_h^2 v_h q_m + 2q_h (S^*)_h^2 v_h \mu_m + (S^*)_h^2 v_h \mu_m q_m + 2q_h (S^*)_h^2 v_h \xi_m + (S^*)_h^2 v_h \xi_m q_m + q_h (S^*)_h^2 v_h q_m + 2\delta_h\mu_h (S^*)_h^2 \xi_m + 2\delta_h (S^*)_h^2 \mu_m \xi_m + \delta_h (S^*)_h^2 \mu_m^2 + 2\delta_h\mu_h (S^*)_h^2 \mu_m + \delta_h (S^*)_h^2 \xi_m^2 + 2\mu_h (S^*)_h^2 \xi_m^2 + 2\mu_h^2 (S^*)_h^2 \xi_m + 4\mu_h (S^*)_h^2 \mu_m \xi_m + 2\mu_h (S^*)_h^2 \mu_m^2 + 2\mu_h^2 (S^*)_h^2 \mu_m + 2\mu_h (S^*)_h^2 v_h \xi_m + 2(S^*)_h^2 v_h \mu_m \xi_m + (S^*)_h^2 v_h \mu_m^2 + 2\mu_h (S^*)_h^2 v_h \mu_m + (S^*)_h^2 v_h \xi_m^2$
$T_5 = b^2 q_h \beta_{hm} q_m \beta_{mh} + 2b^2\gamma\epsilon q_h \beta_{hm} q_m \beta_{mh} + q_h (S^*)_h^2 v_h \mu_m q_m + \delta_h q_h (S^*)_h^2 \mu_m q_m + \mu_h q_h (S^*)_h^2 \mu_m q_m + \mu_h (S^*)_h^2 v_h \mu_m q_m + \delta_h\mu_h (S^*)_h^2 \mu_m q_m + \mu_h^2 (S^*)_h^2 \mu_m q_m + q_h (S^*)_h^2 v_h \mu_m^2 + \delta_h q_h (S^*)_h^2 \mu_m^2 + \mu_h q_h (S^*)_h^2 \mu_m^2 + \mu_h (S^*)_h^2 v_h \mu_m^2 + \delta_h\mu_h (S^*)_h^2 \mu_m^2 + \mu_h^2 (S^*)_h^2 \mu_m^2 + q_h (S^*)_h^2 v_h \xi_m q_m + \delta_h q_h (S^*)_h^2 \xi_m q_m + \mu_h q_h (S^*)_h^2 \xi_m q_m + \mu_h (S^*)_h^2 v_h \xi_m q_m + \delta_h\mu_h (S^*)_h^2 \xi_m q_m + \mu_h^2 (S^*)_h^2 \xi_m q_m + 2q_h (S^*)_h^2 v_h \mu_m \xi_m + 2\delta_h q_h (S^*)_h^2 \mu_m \xi_m + 2\mu_h q_h (S^*)_h^2 \mu_m \xi_m + 2\mu_h (S^*)_h^2 v_h \mu_m \xi_m + 2\delta_h\mu_h (S^*)_h^2 \mu_m \xi_m + 2\mu_h^2 (S^*)_h^2 \mu_m \xi_m + q_h (S^*)_h^2 v_h \xi_m^2 + \delta_h q_h (S^*)_h^2 \xi_m^2 + \mu_h q_h (S^*)_h^2 \xi_m^2 + \mu_h (S^*)_h^2 v_h \xi_m^2 + \delta_h\mu_h (S^*)_h^2 \xi_m^2 + \mu_h^2 (S^*)_h^2 \xi_m^2 (1 - R_{01}^2)$

To show the local asymptotic stability (LAS) of the disease free equilibrium (DFE) where there are invasive plants in human environment, we establish the conditions that will make the polynomial (18) have negative roots. To do this, we simply apply the Routh-Hurwitz Criterion. The following theorem suffices the claim:

3.8 Theorem 2

The polynomial (18) have negative roots only when $T_1, T_2, ..., T_5 > 0$, $T_1T_2 > T_3$ and $T_1T_2T_3 > T_3^2 + T_1^2T_4$.

3.8.1 Remark

Clearly, $T_1, T_2, ..., T_4 > 0$ and $T_5 > 0$ provided that $R_{01} < 0$, thus, the DFE corresponding to the invasive plants in human environment is locally asymptotically stable (LAS).

3.9 Epidemiological implication of E_1 stability

The stability of E_1 implies that the host population is malaria free for a long time period provided that $R_{01} < 1$.

3.10 Local asymptotic stability (LAS) of E_2 equilibrium

We shall investigate the local asymptotic stability of our model equations in the case where there are no invasive plants in the human environment. Thus, we linearized the homogeneous part of our model equations with the corresponding equilibrium, E_2 and obtained the following:

$$J(E_2) = \begin{pmatrix} \mu_h - \lambda & 0 & 0 & \alpha_h & 0 & 0 & 0 & -B_1 \\ 0 & -\lambda - A_1 & 0 & 0 & 0 & 0 & 0 & B_1 \\ 0 & q_h & -\lambda - A_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_h & -\lambda - A_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda - A_4 & 0 & 0 & 0 \\ 0 & 0 & -B_2 & 0 & \theta_m & -\lambda - A_5 & 0 & 0 \\ 0 & 0 & B_2 & 0 & 0 & 0 & -\lambda - A_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_m & -\lambda - A_5 \end{pmatrix} \tag{19}$$

where

$$A_1 = q_h + \mu_h, A_2 = \delta_h + \mu_h + v_h, A_3 = \alpha_h + \mu_h, A_4 = -h + g$$
$$A_5 = \xi_m + \mu_m, A_6 = q_m + \xi_m + \mu_m, B_1 = b\frac{1 - \epsilon\gamma\beta_{hm}}{S_h^{**}}, B_2 = b\frac{1 - \epsilon\gamma\beta_{mh}}{S_h^{**}}$$

The characteristic polynomial corresponding to the matrix is given by

$$x^4 + c_1x^3 + c_2x^2 + c_3x + c4 = 0 \tag{20}$$

where

$$\begin{aligned} c_1 &= A_1 + A_2 + A_5 + A_6 \\ c_2 &= A_1A_2 + A_1A_5 + A_2A_5 + A_1A_6 + A_2A_6 + A_5A_6 \\ c_3 &= A_1A_2A_5 + A_1A_2A_6 + A_2A_5A_6 \\ c_4 &= A_1A_2A_5A_6 - B_1B_2q_hq_m \end{aligned}$$

or

$$a_4 = A_1A_2A_5A_6(1 - R_{02}^2) \tag{21}$$

3.10.1 Remark

Clearly, $c_1, c_2, c_3 > 0$ and $c_4 > 0$ provided that $R_{02} < 0$, thus, the DFE corresponding to where there are no invasive plants in human environment is locally asymptotically stable (LAS).

3.11 Epidemiological implication of E_2 stability

The stability of E_2 equilibrium is conditioned on the value of R_{02} being less than a unit.The stability of E_2 equilibrium implies a host population free of malaria.

3.12 Disease endemic equilibrium

In this section, we are investigating the existence of an endemic equilibrium, however, we term this equilibrium as E_3. Thus, the equilibrium, E_3 is computed by solving for each state variable of the model (2), we then have,

$$E_3 = (S_h^{**}, E_h^{**}, I_h^{**}, R_h^{**}, P^{**}, S_m^{**}, E_m^{**}, I_m^{**})$$

where,

$$
\begin{aligned}
S_h^{**} &= \frac{\Lambda_h a_1 a_2 a_3}{(a_1a_2a_3 - V_h q_h \lambda_h \alpha_h)\lambda_h^{**} + \mu_h a_1 a_2 a_3}\\
E_h^{**} &= \frac{\Lambda_h a_1, a_3 \lambda_h^{**}}{(a_1a_2a_3 - v_h q_h \alpha_h)\lambda_h^{**} + \mu_h a_1 a_2 a_3}\\
I_h^{**} &= \frac{q_h \lambda_h^{**} \Lambda_h a_2}{(a_1a_2a_3 - v_h q_h \alpha_h)\lambda_h^{**} + \mu_h a_1 a_2 a_3}\\
R_h^{**} &= \frac{\Lambda_h v_h q_h \lambda_h^{**}}{(a_1a_2a_3 - v_h q_h \alpha_h)\lambda_h^{**} + \mu_h a_1 a_2 a_3}
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
P^{**} &= \frac{K_p(g-h)}{g}, g > h\\
S_m^{**} &= \frac{\Lambda_m g + \theta_m K_p(g-h)}{g\lambda_m^{**} + a_4}, g > h\\
E_m^{**} &= \frac{\lambda_m^{**}(\Lambda_m g + \theta_m K_p(g-h))}{(g\lambda_m^{**} + a_4)a_5}, g > h\\
I_m^{**} &= \frac{\lambda_m^{**}(\Lambda_m g + \theta_m K_p(g-h)}{(g\lambda_m^{**} + a_4)a_4 a_5}, g > h\\
N_h^{**} &= \frac{\Lambda_h(a_1a_2a_3 + \lambda_h^{**}(a_1a_3 + a_2q_h + q_hV_h))}{(a_1a_2a_3 - v_h q_h \alpha_h)\lambda_h^{**} + \mu_h a_1 a_2 a_3}
\end{aligned} \tag{23}
$$

where

$$
\begin{aligned}
a_1 = \delta_h + \mu_h + v_h, a_2 = q_h + \mu_h, a_3 = \mu_h + \alpha_h,\\
a_4 = \xi_m + \mu_m, a_5 = q_m + \xi_m + \mu_m
\end{aligned}
$$

and

$$
\lambda_h^{**} = (1-\epsilon\gamma)\beta_{hm}\frac{bI_m^{**}}{N_h^{**}}, \lambda_m^{**} = (1-\epsilon\gamma)\beta_{mh}\frac{bI_h^{**}}{N_h^{**}} \tag{24}
$$

substituting equations (24) in equations (22) show that the non-zero equilibrium of the model can be written in a compact form as:

$$
Z_1\lambda_h^{**} + Z_2\lambda_h^{**} + Z_3 = 0 \tag{25}
$$

where

$$
\begin{aligned}
Z_1 &= [(a_1a_3 + a_2q_h + q_hm_h)(1-\epsilon\gamma)\beta_{mh}q_ha_2\Lambda + (a_1a_3 + a_2q_h + q_hm_h)^2 g(\xi_m + \mu_m)^2\Lambda_h]\\
Z_2 &= [\Lambda_h(1-\epsilon\gamma)\beta_{mh}bq_ha_1a_2^2a_3 + a_1a_2a_3g(\xi_m+\mu_m)^2(a_1a_3 + a_2q_h + q_hm_h)\Lambda + g(\xi_m+\mu_m)^2(a_1a_3\\
&\quad + a_2q_h + q_hm_h)\Lambda_ha_1a_2a_3 - a_1a_2^2a_3(1-\epsilon)b)^2\beta_vh\beta_{hm}\theta_mK_p(g-h)q_h\\
&\quad + ((1-\epsilon\gamma)b)^2\beta_{mh}\beta_{hv}\theta_mK_p(g-h)q_h^2v_h\alpha_h]\\
Z_3 &= a_1^2a_2^2a_3^2\Lambda g(\xi_m+\mu_m)^2 - a_1a_2^2a_3((1-\epsilon\gamma)b)^2\beta_{mh}\beta_{hv}\theta_mK_p(g-h)q_h\mu_h\\
&= a_1^2a_2^2a_3^2\Lambda_hg(\xi_m+\mu_m)^2[1-R_{01}^2]
\end{aligned} \tag{26}
$$

The endemic equilibria of the homogeneous part of the model (2) can be obtained by solving for λ_h^{**} from (24), and substituting the positive values of λ_h^{**} in (25) can be analyzed for the possibility of endemic equilibria when $R_{01} < 1$. It is obvious that the coefficients, Z_1 and Z_2, of the equation (26) is always positive provided that $(a_1a_3 + a_2q_h + q_hm_h)(1-\epsilon\gamma)\beta_{mh}q_ha_2\Lambda > (a_1a_3 + a_2q_h + q_hm_h)^2g(\xi_m+\mu_m)^2\Lambda$ and $[\Lambda(1-\epsilon\gamma)\beta_{mh}bq_ha_1a_2^2a_3 + a_1a_2a_3g(\xi_m+\mu_m)^2(a_1a_3 + a_2q_h + q_hm_h)\Lambda > g(\xi_m+\mu_m)^2(a_1a_3 + a_2q_h + q_hm_h)\Lambda a_1a_2a_3 - a_1a_2^2a_3(1-\epsilon)b)^2\beta_mh\beta_{hm}\theta_mK_p(g-h)q_h + ((1-\epsilon\gamma)b)^2\beta_{mh}\beta_{hm}\theta_mK_p(g-h)q_h^2v_h\alpha_h]$ respectively. Also, Z_3 is positive (negative) if $R_{01} < 1 (R_{01} > 1)$. Hence, the following claim is established:

Theorem 3

The homogeneous part of the model (2), has

1. a single endemic equilibrium solution if any of the following three conditions is satisfied: R_{02} or $Z_2 < 0$ and $Z_1 = 0$ or $Z_2 < 0$ and $Z_2^2 - 4Z_1Z_3 = 0$
2. two endemic equilibria solutions if $Z_1 > 0$,$Z_2 < 0$ and $Z_2^2 - 4Z_1Z_3 > 0$.
3. no endemic equilibrium solution if both conditions above are not satisfied.

3.13 Biological implication

Different endemic equilibrium state will be observed in the host population.The first condition known as single endemic equilibrium implies malaria in all area of human population.The two endemic equilibrium states implies different level of malaria endemicity in the host population.

3.14 Backward bifurcation phenomenon

Backward bifurcation may occur under certain conditions for R_{01}. In such a case, we must have $R_{01} < 1$. From the expression of R_{01}, we know that if the transmission rate of malaria from mosquito to human is too low, there is no backward bifurcation. The presence of backward bifurcation indicates that the necessary requirement of $R_{01} < 1$, is not sufficient for disease elimination. The Centre Manifold theory will be used to improve the conditions on existence of backward bifurcation. To investigate the existence of the backward bifurcation of model (2), we shall rewrite (2) in the vector form as:

$$\frac{dY}{dt} = H(y) \tag{27}$$

where

$$Y = (y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8)^T$$

and

$$H = (h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8)^T$$

so that

$$S_h = y_1, E_h = y_2, I_h = y_3, R_h = y_4, P = y_5, S_m = y_6, E_m = y_7, I_m = y_8$$

.
Thus, model (2) becomes

$$\begin{aligned}
\frac{dy_1}{dt} &= \Lambda_h - \lambda_h y_1 + \alpha_h y_4 - \mu_h y_1 = h_1 \\
\frac{dy_2}{dt} &= \lambda_h y_1 - (q_h + \mu_h) y_2 := h_2 \\
\frac{dy_3}{dt} &= q_h y_2 - (\delta_h + \mu_h + v_h) y_3 := h_3 \\
\frac{dy_4}{dt} &= v_h y_3 - (\mu_h + \alpha_h) y_4 := h_4 \\
\frac{dy_5}{dt} &= g(1 - \frac{P}{K_P})P - h y_5 := h_5 \\
\frac{dy_6}{dt} &= \Lambda_m + \theta_m y_5 - \lambda_m y_6 - (\xi_m + \mu_m) y_6 := h_6 \\
\frac{dy_7}{dt} &= \lambda_m y_6 - (q_m + \xi_m + \mu_m) y_7 := h_7 \\
\frac{dy_8}{dt} &= q_m y_7 - (\xi_m + \mu_m) y_8 := h_8
\end{aligned} \tag{28}$$

with

$$\lambda_h = \phi\beta_{mh}\frac{bI_m}{N_h}, \lambda_m = \phi\beta_{mh}\frac{bI_h}{N_h}, \phi = 1-\epsilon\gamma$$

Choosing β_{mh} as the bifurcation parameter at $R_{01}=1$, We have

$$\beta_{mh} = \beta^*_{mh} = \frac{(\mu_m+\xi_m)g\mu_h(q_h+\mu_h)(v_h+\delta_h+\mu_h)(q_m+\mu_m+\xi_m)}{b(1-\epsilon\gamma)q_h q_m\beta_{mh}(gV_h+gK_p\theta_m+hK_p\theta_m)\Lambda_h} \tag{29}$$

so that the disease-free equilibrium,E_1,is locally stable when $\beta_{mh}<\beta^*_{mh}$,and is unstable when $\beta_{mh}>\beta^*_{mh}$. Thus, β^*_{mh} is a bifurcation parameter. Linearizing matrix of the system (2) around DFE, E_1 and evaluated at β^*_{mh}, we have the following.

$$J(E_1,\beta^*_{mh}) = \begin{pmatrix} -\mu_h & 0 & 0 & \alpha_h & 0 & 0 & 0 & K_1 \\ 0 & -(q_h+\mu_h) & 0 & 0 & 0 & 0 & 0 & K_2 \\ 0 & q_h & K_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_h & -(\alpha_h+\mu) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{g-2gp^*}{K_p} & 0 & 0 & 0 \\ 0 & 0 & \frac{-b\phi\beta_{mh}}{S^*_h} & 0 & K_3 & 0 & 0 & \\ 0 & 0 & \frac{-b\phi\beta_{mh}}{S^*_h} & 0 & 0 & 0 & K_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_m & K_6 \end{pmatrix} \tag{30}$$

where $K_1 = \frac{-b\phi\beta^*_{hm}}{S^*_h}, K_2 = \frac{b\phi\beta^*_{hm}}{S^*_h}, K_3 = \theta_m-(\xi_m+\mu_m), K_4 = -(q_m+\xi_m+\mu_m), K_5 = -(\delta_h+\mu_h+v_h), -\xi_m+\mu_m$

The eigenvalues of $J(E,\beta^*_{hm})$ given by (31) are the roots of the characteristic equation of the form:

$$P(\lambda) = (\lambda+\mu_h)(\lambda+\alpha_h+\mu_h)(\lambda+\mu_m+\xi_m)(\lambda+\frac{g(2p-1)}{K_p}), \tag{31}$$

where $P(\lambda)$ is a polynomial of degree four whose roots are real and negative except one zero eigenvalue. Thus, the right eigenvector, $w=(w_1,w_2,w_3,w_4,w_5,w_6,w_7,w_8)^T$, associated with the simple zero eigenvalue can be obtained from $J(E_1,\beta^*_{hm}).w=0$. Thus, we get

$$\begin{aligned} w_1 &= \frac{\alpha_h w_4 - \frac{b\phi\beta^*_{hm}}{s^*_h}}{\mu_h}, w_2 = \frac{b\phi\beta^*_{hm}}{q_h+\mu_h}s^*_h w_8, \\ w_3 &= \frac{q_h w_2}{\delta_h+\mu_h+v_h}, w_4 = \frac{v_h w_3}{\alpha_h+\mu_h}, \\ w_5 &= 0, w_6 = -\frac{-b\phi\beta_{mh}}{\xi_m+\mu_m}S^*_h w_3, w_7 = -\frac{-b\phi\beta_{mh}}{q_m+\xi_m+\mu_m}S^*_h w_3 \\ w_8 &= \frac{q_m w_7}{\xi_m+\mu_m} \end{aligned} \tag{32}$$

More so, the left eigenvector, $v=(v_1,v_2,...,v_8)$, corresponding to the simple zero eigenvalues of (30) is obtained from $V.J(E_1,\beta^*_{mh})=0$. Thus,

$$\begin{aligned} v_1 &= 0, v_2 = \frac{q_h v_3}{q_h+\mu_h}, v_3 = \frac{\frac{b\phi\beta_{mh}}{s^*_h}(v_7-v_6)+v_h v_4}{\delta_h+\mu_h+v_h}, \\ v_4 &= \frac{\alpha_h v_1}{\alpha_h+\mu_h} = 0, v_5 = 0, v_6 = 0, v_7 = \frac{q_m v_8}{q_m+\xi_m+\mu_m}, \\ v_8 &= \frac{\frac{b\phi\beta^*_{hm}}{s^*_h}(w_2-w_1}{\xi_m+\mu_m} \end{aligned} \tag{33}$$

Therefore, the second-order partial derivatives of h_i,$i = 1, 2, ..., 8$ from system (30) are zero at point (E_1, β^*_{mh}) except the following.

$$\begin{aligned}
\frac{\partial^2 h_2}{\partial y_1 \partial y_8} &= \frac{\phi \beta_{hm} \mu_h b}{\mu_h} - \frac{\phi \beta^*_{mh} \mu_h^2 b S^*_h}{\Lambda_h^2} \\
\frac{\partial^2 h_2}{\partial y_2 \partial \beta^*_{mh}} &= \frac{\phi b \mu_h S^*_h}{\Lambda_h} \\
\frac{\partial^2 h_2}{\partial y_2 \partial y_8} &= \frac{\partial^2 h_2}{\partial y_3 \partial y_8} = \frac{\partial^2 h_2}{\partial y_4 \partial y_8} = -\frac{\phi \beta^*_{mh} b \mu_h^2 S^*_h}{\Lambda_h^2} \\
\frac{\partial^2 h_7}{\partial y_3^2} &= \frac{-2\phi \beta_{hm} b \mu_h^2 S^*_m}{\Lambda_h^2} \\
\frac{\partial^2 h_7}{\partial y_3 \partial y_6} &= \frac{\phi \beta_{hm} b \mu_h}{\Lambda_h}
\end{aligned} \tag{34}$$

Theorem 4

The authors in [1] Consider a general system of ordinary differential equations with a parameter δ as follow:

$$\frac{dx}{dt} = f(x, \delta), f : R^n x R \implies R, \tag{35}$$

and

$$f \epsilon C^2(R^n x R)$$

without loss of generality, it is assumed that 0 is an equilibrium of the system for all values of the parameter δ, (that is $f(0, \delta) \equiv 0$).

Assume

1. A = $D_x h(0, 0)$ = $(\frac{\partial h_i}{\partial x_j}, 0, 0)$ is the linearized matrix of the system around the equilibrium 0 with σ evaluated at 0. Zero is a simple eigenvalue of **A** and all other eigenvalues of **A** have negative real parts;

2. Matrix A have a non-negative right eigenvector, w and a left eigenvector, v corresponding to the zero eigenvalue. Let h_k be the $k - th$ component of h and

$$\begin{aligned}
a &= \sum_{k,i,j=1}^{n} v_k w_i w_j \frac{\partial^2 h_k}{\partial y_i \partial y_j}(0, 0), \\
b &= \sum_{k,j=1}^{n} v_k w_i \frac{\partial^2 h_k}{\partial y_i \partial \sigma}(0, 0).
\end{aligned} \tag{36}$$

   The local dynamics of system (28) around *0* are totally determined by a and b

- $a > 0$, $b > 0$, when $\sigma < 0$ with $|\sigma| << 1$, 0 is locally asymptotically stable and there exists a positive unstable equilibrium; when $0 < \sigma << 1$, 0 is unstable and there exists a negative and locally asymptotically stable equilibrium;

- $a < 0$, $b < 0$, when $\sigma < 0$ with $|\sigma| << 1$,0 is unstable when $0 < \delta << 1$,0 is locally asymptotically stable,and there exists a positive unstable equilibrium;

- $a > 0, b < 0$, when $\sigma < 0$ with $|\sigma| << 1$,0 is unstable and there exists a locally asymptotically stable negative equilibrium; when $0 < \sigma << 1$,0 is stable and a positive unstable equilibrium appears;

- a < 0, $b > 0$. When σ changes from negative to positive, 0 changes its stability from stable to unstable. Correspondingly, a negative unstable equilibrium becomes positive and locally asymptotically stable.In particular, if $a > 0$ and $b > 0$, then a backward bifurcation occurs at $\sigma = 0$.

In order to assess the value of a and b, we substitute equations (32) - (34) in equations (36), we get

$$\begin{aligned}
a &= v_7 w_3^2 \frac{\delta^2 h_7}{\delta y_3^2} + 2v_7 w_3 w_6 \frac{\delta^2 h_7}{\delta y_3 \delta y_6} \\
&= v_7 w_3^2 (\frac{-2\phi\beta_{hm} b\mu_h^2 S_m^*}{\Lambda_h^2}) + 2v_7 w_3 w_6 \frac{\phi_h m b\mu_h}{\Lambda_h} \\
&= -2[(\frac{q_m}{q_m+\xi_m+\mu_m})(\frac{q_h}{\delta_h+\mu_h+v_h})^2 (\frac{\theta\beta_h m b\mu_h^2 S_m^*}{\Lambda_h^2}) v_8 w_2^2 \\
&+ (\frac{q_m}{q_m+\xi_m+\mu_m})(\frac{q_h}{\delta_h+\mu_h+v_h})(\frac{\theta\beta_{hm}^2 b^2\mu_h}{\Lambda_h(\xi_m+\mu_m)S_h^*}) v_8 w_2] \\
b &= v_2 w_2 \frac{q_h}{\delta_h+\mu_h+v_h} \frac{\phi b\mu_h S_h^*}{\Lambda_h} > 0
\end{aligned} \tag{37}$$

Remark: The coefficient b is always positive. Theorem 4 implies that system (2) will undergo backward bifurcation at $R_{01}(\beta_{mh}^*) = 1$ if $a > 0$. we recalled the fact that all the parameters of model (2) are positive and also the value of v_8 and w_2 are positive, it is easy to see that $a < 0$ and $b > 0$. Therefore, from Theorem 2 item (iv),the model (2) exhibit a forward (super-critical) bifurcation as R_{01} crosses the threshold, $R_{01} = 1$ (i.e; a locally asymptotically stable endemic equilibrium, $E_3 = (S_h^{**}, E_h^{**}, I_h^{**}, R_h^{**}, P^{**}, S_m^{**}, E_m^{**}, I_m^{**})$ representing the non-trivial positive steady-states of model (2) exists)

Biological implication

A host environment with free malaria can become malaria persistent environment when an individual previously affected by malaria transmit to other individuals with a given population.

3.15 Global stability of endemic equilibrium

The global stability of the Endemic Equilibrium E_3 of model (2) is very important in this study because both mosquito, vegetation and human co-exist. We will study this by considering the homogeneous spatial model of system (2) given as:

$$\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h - \lambda_h S_h + \alpha_h R_h - \mu_h S_h \\
\frac{dE_h}{dt} &= \lambda_h S_h - (q_h + \mu_h)E_h \\
\frac{dI_h}{dt} &= q_h E_h - (\delta_h + \mu_h + v_h)I_h \\
\frac{dR_h}{dt} &= v_h I_h - (\mu_h + \alpha_h)R_h \\
\frac{dP}{dt} &= g(1 - \frac{P}{K_P})P - hP \\
\frac{dE_m}{dt} &= \phi\tilde{\beta_m} hbI_h S_m - (q_m + \xi_m + \mu_m)E_m \\
\frac{dS_m}{dt} &= \Lambda_m + \theta_m P - \lambda_m S_m - (\xi_m + \mu_m)S_m \\
\frac{dI_m}{dt} &= q_m E_m - (\xi_m + \mu_m)I_m
\end{aligned} \tag{38}$$

where

$$\lambda_h = (1-\epsilon\gamma)\beta_{hm}\frac{bI_m}{N_h}, \lambda_m = (1-\epsilon\gamma)\beta_{mh}\frac{bI_h}{N_h}$$

with

$$N = \frac{\Lambda_h}{\mu}$$

and

$$\phi = 1 - \epsilon\gamma,$$

we have

$$\lambda_h = \phi\beta_{hm}\frac{\mu bI_m}{\Lambda} = \phi\tilde{\beta}_h mbI_m$$

$$\lambda_m = \phi\tilde{\beta}_m hbI_h$$

We shall employ the method of Goh-Volterra type Lyapunov function to investigate the GAS of the EEP, E_3. Let

$$\begin{aligned}
W = (S_h - S_h^{**} - S_h^{**}ln\frac{S_h}{S_h^{**}}) + (E_h - E_h^{**} - E_h^{**}ln\frac{E_h}{E_h^{**}}) + C(I_h - I_h^{**} - I_h^{**}ln\frac{I_h}{I_h^{**}}) \\
+(S_m - S_m^{**} - S_m^{**}ln\frac{S_m}{S_m^{**}}) + (E_m - E_m^{**} - E_m^{**}ln\frac{E_m}{E_m^{**}}) + D(I_m - I_m^{**} - I_m^{**}ln\frac{I_m}{I_m^{**}})
\end{aligned} \tag{39}$$

be the Lyaponov function of the Goh-Voltera type. Taking the time derivative of equation (39), we have

$$\begin{aligned}
\dot{W} = (\dot{S_h} - \frac{S_h^{**}}{S_h}\dot{S_h}) + (\dot{E_h} - \frac{E_h^{**}}{E_h}\dot{E_h}) + C(\dot{I_h} - \frac{I_h^{**}}{I_h}\dot{I_h}) \\
+(\dot{S_m} - \frac{S_m^{**}}{S_m}\dot{S_m}) + (\dot{E_m} - \frac{E_m^{**}}{E_m}\dot{E_m}) + D(\dot{I_m} - \frac{I_m^{*}*}{I_m}\dot{I_m})
\end{aligned} \tag{40}$$

Substituting the right hand side of the equations (38) in (40)
we have

$$
\begin{aligned}
\dot{W} &= \Lambda_h - \phi\tilde{\beta}_{hm}bI_mS_h + \alpha_hR_h - \mu_hS_h - \frac{S^{*}*}{S}[\Lambda_h - \phi\tilde{\beta}_hmbS_h + \alpha_hR_h - \mu_hS_h] \\
&+ \phi\tilde{\beta}_hmbI_mS_h - (q_h + \mu_h)E_h - \frac{E_h^{**}}{E_h}[(\phi\tilde{\beta}_{hm}bI_mS_h - (q_h + \mu_h)E_h] \\
&+ C[q_hE_h - (\delta_h + \mu_h + v_h)I_h] - \frac{I_h^{**}}{I_h}[(q_hE_h - (\delta_h + \mu_h + v_h)I_h)] \\
&+ \Lambda_m + \theta_mp - [\phi\tilde{\beta_m}hbI_hS_m - (\xi_m + \mu_m)S_m] - \frac{S_m^{**}}{S_m}[\Lambda_m + \theta_mP - [\phi\tilde{\beta_m}hbI_hS_m - (\xi_m + \mu_m)S_m] \\
&+ \phi\tilde{\beta_m}hbI_hS_m - (q_m + \xi_m + \mu_m)E_m - \frac{E_m^{**}}{E_m}(\phi\tilde{\beta_m}hbI_hS_m - (q_m + \xi_m + \mu_m)E_m) \\
&+ D(q_mE_m - (\xi_m + \mu_m)I_m - \frac{I_m^{**}}{I_m}(q_m + (\xi_m + \mu_m)I_m))
\end{aligned} \tag{41}
$$

At steady state, we observe that

$$
\begin{aligned}
\Lambda_h &= \phi\tilde{\beta}_{hm}bI_m^{**}S_h^{**} + \mu_hS_h^{**} \\
\Lambda_m &= \phi\tilde{\beta}_{mh}bI_h^{**}S{**}_m + (\xi_m + \mu_m)S_m^{**} - \theta_mP^{**}
\end{aligned} \tag{42}
$$

Now, setting $\delta_h = 0$ and substituting (41) in (40), we have

$$
\begin{aligned}
\dot{W} &= \phi\tilde{\beta}_{hm}bI_m^{**}S_h^{**} + \mu_hS_h^{**} - \phi\tilde{\beta}_{hm}bI_mS_h + \alpha_hR_h - \mu_hS_h \\
&- \phi\tilde{\beta}_{hm}bI_m^{**}\frac{S_h^{**2}}{S_h} - \frac{\mu S_h^{**2}}{S_h} + \phi\tilde{\beta}_{hm}bI_mS_h^{**} - \alpha_hR_h\frac{S_h^{**}}{S_h} \\
&+ \mu_hS_h^{**} + \phi\tilde{\beta}_{hm}bI_mS_h - (q_h + \mu_h)E_h - \phi\tilde{\beta}_{hm}bI_mS_h\frac{E_h^{*}*}{E_h} \\
&+ (q_h + \mu_h)E_h^{**} + Cq_hE_h - C(v_h + \mu_h)I_h - Cq_hE_h\frac{I_h^{*}*}{I_h} \\
&+ C(v_h + \mu_h)I_h^{*}* \\
&+ \phi\tilde{\beta}_{hm}bI_h^{**}S_m^{**} + (\xi_m + \mu_h)S_m^{**} - \phi\tilde{\beta}_{mh}bI_hS_m \\
&+ \phi\tilde{\beta}_{mh}bI_h^{**}S_m^{**} + (\xi_m + \mu_h)S_m^{**} - \theta_mP + \theta_mP - \phi\tilde{\beta}_{mh}bI_hS_m - (\xi_m + \mu_h)S_m
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{S_m^{**2}}{S_m}\phi\tilde{\beta}_{mh}bI_h^{**} - \frac{S_m^{**2}}{S_m}(\xi_m + \mu_m) + \frac{S_m^{**}}{S_m}\theta_mP - \theta_mP\frac{S_m^{**}}{S_m} + \phi\tilde{\beta}_{mh}bI_hS_m^{**} \\
&+ (\xi_m + \mu_m)S_m^{**} + \phi\tilde{\beta}_{mh}bI_hS_m - (q_m + \xi_m + \mu_m)E_m - \frac{E_m^{**}}{E_m}\phi\tilde{\beta}_{mh}bI_hS_m \\
&+ (q_m + \xi_m + \mu_m)E_m^{**} + Dq_mE_m - D(\xi_m + \mu_m)I_m - D\frac{I_m^{**}}{I_m}q_mE_m + D(\xi_m + \mu_m)I_m^{**}
\end{aligned}
$$

Separating the infected classes without double stars on them (1.e E_h, I_h, I_m), we have

$$
\begin{aligned}
\dot{W} &= \phi\tilde{\beta}_{hm}bI_m^{**}S_h^{**} + \mu_hS_h^{**} + \alpha_hR_h - \mu_hS_h \\
&- \phi\tilde{\beta}_{hm}bI_m^{**}\frac{S_h^{**2}}{S_h} - \frac{\mu S_h^{**2}}{S_h} + \phi\tilde{\beta}_{hm}bI_mS_h^{**} - \alpha_hR_h\frac{S_h^{**}}{S_h}
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
&+\mu_h S_h^{**}-(q_h+\mu_h)E_h-\phi\tilde{\beta}_{hm}bI_mS_h\frac{E_h^{**}}{E_h}\\
&+(q_h+\mu_h)E_h^* * +Cq_hE_h-C(\mu_h+v_h)I_h-C(q_hE_h\frac{I_h^{**}}{I_h})+C(\mu_h+v_h)I_h^{**}\\
&+\phi\tilde{\beta}_{hm}bI_h^{**}S_m^{**}+(\xi_m+\mu_h)S_m^{**}-\phi\tilde{\beta}_{mh}bI_hS_m\\
&-(\xi_m+\mu_h)S_m-\phi\tilde{\beta}_{mh}bI_h\frac{S_m^{**2}}{S_m}-(\xi_m+\mu_h)\frac{S_m^{**2}}{S_m}+\phi\tilde{\beta}_{mh}bI_hS_m^{**}\\
&+(\xi_m+\mu_m)S_m^{**}-(q_m+\xi_m+\mu_m)E_m-\frac{E_m^{**}}{E_m}\phi\tilde{\beta}_{mh}bI_hS_m\\
&+(q_m+\xi_m+\mu_m)E_m^{**}+Dq_mE_m-D(\xi_m+\mu_m)I_m-D\frac{I_m^{**}}{I_m}-D(\xi_m+\mu_m)I_m^{**}
\end{aligned}
\tag{44}
$$

We then collect the terms without double stars on them together as

$$
\begin{aligned}
&\phi\tilde{\beta}_{hm}bI_mS_h^{**}-(q_h+\mu_h)E_h+Cq_hE_h-C(\mu_h+v_h)I_h\\
&+\phi\tilde{\beta}_{mh}bI_hS_m^{**}-(q_m+\xi_m+\mu_m)E_m+Dq_mE_m-D(\xi_m+\mu_m)I_m=0
\end{aligned}
\tag{45}
$$

Solving for C and D from (45) in terms of I_h and I_m, we have

$$
\begin{aligned}
I_h(\phi\tilde{\beta}_{mh}bI_hS_m^{**}-(\mu_h+v_h))=0\\
C=\frac{\phi\tilde{\beta}_{mh}bS_m^{**}}{\mu_h+v_h}
\end{aligned}
$$

$$
\begin{aligned}
I_m(\phi\tilde{\beta}_{mh}bS_h^{**}-D(\xi_m+\mu_m))=0\\
D=\frac{\phi\tilde{\beta}_{mh}bS_h^{**}}{(\xi_m+\mu_m)}
\end{aligned}
$$

Also, we have from (45) at steady state

$$
(q_h+\mu_h)=\frac{\phi\tilde{\beta}_{hm}bI_m^{**}S_h^{**}}{E_h^{**}}
$$

$$
q_h=\frac{(v_h+\mu_h)I_h^{**}}{E_h^{**}}
$$

$$
=\frac{\phi\tilde{\beta}_{mh}bI_h\frac{S_m^{**2}}{S_m}+\phi\tilde{\beta}_{mh}bS_m^{**}}{E_h^* *}
$$

$$
(q_m+\xi_m+\mu_m)=\frac{\phi\tilde{\beta}_{mh}bI_h^{**}S_m^{**}}{E_m^{**}}
$$

$$
q_m=\frac{(\xi_m+\mu_m)I_m^{**}}{E_m}
$$

Rewritten (44) without including terms with double stars on them, we have

$$
\begin{aligned}
\dot{W} &= \phi\tilde{\beta}_{hm} b I_m^{**} S_h^{**} + \mu_h S_h^{**} - \mu_h S_h - \phi\tilde{\beta}_{hm} b I_m^{**} \frac{S_h^{**2}}{S_h} \\
&- \frac{\mu_h S_h^{**}}{S_h} + \mu_h S_h^{**} - \phi\tilde{\beta}_{hm} b I_m S_h \frac{E_h^* *}{E_h} \\
&+ (q_h + \mu_h) E_h^{**} - C(q_h E_h \frac{I_h^{**}}{I_h}) + C(\mu_h + v_h) I_h^{**} \\
&+ \phi\tilde{\beta}_{hm} b I_h^{**} S_m^{**} + (\xi_m + \mu_m) S_m^{**} - (\xi_m + \mu_m) S_m \\
&- \frac{S_m^{**2}}{S_m} \phi\tilde{\beta}_{mh} b I_h^{**} - (\xi_m + \mu_h) \frac{S_m^{**2}}{S_m} \\
&+ (\xi_m + \mu_m) S_m^{**} + (\xi_m + \mu_m) S_m^{**} - \frac{E_m^* *}{E_m} \phi\tilde{\beta}_{mh} b S_m I_h + (q_m + \xi_m + \mu_m) E_m^{**} \\
&- D \frac{I_m^{**}}{I_m} q_m E_m + D(\xi_m + \mu_m) I_m^{**}
\end{aligned} \tag{46}
$$

Substituting for C, D and q_h, $(q_h + \mu_h)$ and $(q_m + \xi_m + \mu_m)$ and q_m into (46) we have

$$
\begin{aligned}
\dot{W} &= \phi\tilde{\beta}_{hm} b I_m^{**} S_h^{**} + \mu_h S_h^{**} - \mu_h S_h \\
&- \phi\tilde{\beta}_{mh} b I_m^{**} \frac{S_h^{**2}}{S_h} - \mu_h \frac{S_h^{**2}}{S_h} + \mu_h S_h^{**} - \phi\tilde{\beta}_{hm} b I_m S_h \frac{E_h^{**}}{E_h} \\
&+ \phi\tilde{\beta}_{hm} b I_m^{**} S_h^{**} - \phi\tilde{\beta}_{mh} b S_m^{**} \frac{I_h^{**2}}{I_h} \frac{E_h}{E_h^{**}} + \phi\tilde{\beta}_{mh} b I_h^{**} S_m^{**} \\
&+ \phi\tilde{\beta}_{mh} b I_h^{**} S_m^{**} + \xi_m S_m^{**} + \mu_m S_m^{**} - \xi_m S_m - \mu_m S_m - \frac{S_m^{**2}}{S_m} \phi\tilde{\beta}_{hm} b I_h^{**} - \frac{S_m^{**2}}{S_m} \xi_m \\
&- \mu_m \frac{S_m^{**2}}{S_m} + \xi_m S_m^{**} + \mu_m S_m ** - \frac{E_m^{**}}{E_m} \phi\tilde{\beta}_{mh} I_h S_m \\
&+ \phi\tilde{\beta}_{mh} b I_h^{**} S_m^{**} - \phi\tilde{\beta}_{hm} S_h^{**} \frac{I_m^{**2}}{I_m} \frac{E_m}{E_m^{**}} + \phi\tilde{\beta}_{hm} b S_h^{**} I_m^{**}
\end{aligned} \tag{47}
$$

By gathering the terms with $\mu_h S_h, \mu_m S_m, \phi\tilde{\beta}_{hm}, \phi\tilde{\beta}_{mh}$ and factorizing (47), we have

$$
\begin{aligned}
\mu_h S_h^{**} (2 - \frac{S_h^{**}}{S_h} - \frac{S_h}{S_h^{**}}) \\
\mu_m S_m^{**} (2 - \frac{S_m}{S_m^{**}} - \frac{S_m^{**}}{S_m}) \\
\phi\tilde{\beta}_{hm} b I_m^{**} S_h^{**} (3 - \frac{S_h^{**}}{S_h} - \frac{S_h}{S\ h}^{**} \frac{E_h^{**}}{E_h} \frac{I_m}{I_m^{**}} - \frac{I_m^{**}}{I_m} \frac{E_m}{E_m^{**}}) \\
\phi\tilde{\beta}_{mh} b I_h^{**} S_m^{**} (3 - \frac{S_h^{**}}{I_h} \frac{E_h}{E_h^{**}} - \frac{S_m^{**}}{S_m} - \frac{I_h}{I_h^{**}} \frac{E_m^{**}}{E_m})
\end{aligned} \tag{48}
$$

Finally, since the arithmetic mean exceeds the geometric mean, the following inequalities hold.

$$\mu_h S_h^{**}(2 - \frac{S_h^{**}}{S_h} - \frac{S_h}{S_h^{**}}) \leq 0$$

$$\mu_m S_m^{**}(2 - \frac{S_m}{S_m^{**}} - \frac{S_m^{**}}{S_m}) \leq 0$$

$$\phi\tilde{\beta}_{hm} b I_m^{**} S_h^{**}(3 - \frac{S_h^{**}}{S_h} - \frac{S_h}{S_h}^{**} \frac{E_h^{**}}{E_h} \frac{I_m}{I_m}^{**} - \frac{I_m^{**}}{I_m} \frac{E_m}{E_m^{**}}) \leq 0$$

$$\phi\tilde{\beta}_{mh} b I_h^{**} S_m^{**}(3 - \frac{S_h^{**}}{I_h} \frac{E_h}{E_h^{**}} - \frac{S_m^{**}}{S_m} - \frac{I_h}{I_h^{**}} \frac{E_m^{**}}{E_m}) \leq 0$$

Thus, we have $\dot{W} \leq 0$ for $R_{01} > 1$. Hence, W is a Lyaponov function and the equilibrium E_3 is globally asymptotically stable.

Biological implication

The disease will persist in the host environment provided that those that are infected with the disease keep on transmitting the disease.The less than or equal ($\leq$) sign of the lyapunov function shows the decrease of the susceptible population.

3.16 Sensitivity analysis

In this section, we performed the sensitivity analysis in order to determine the relative importance of the model parameters on disease transmission. The analysis will enable us to find out parameters that have high impact on the basic reproduction number and which should be targeted for intervention strategies. We perform sensitivity analysis by calculating the sensitivity indices of the effective reproduction number R_{01} since our major emphasis is on the invasive plants in order to determine whether malaria fever can be controlled or not. These indices tell us how crucial each parameter is on the transmission of malaria fever. The sensitivity index of R_{01} to a parameter say τ, where τ is any of the parameters in Table 3 is given by

$$\Gamma_y^{R_{01}} = \frac{\partial R_{01}}{\partial y} \times \frac{y}{R_{01}} \tag{49}$$

Table (2) provides the indices of R_{01} to its parameters. From Table 2, it is obvious that the reproduction number, R_{01} is most sensitive to number of bite per mosquito (b), plants carrying capacity(K_p), recovered rate of humans(v_h), migration rate of mosquitoes from the vegetation (θ_m), disease induced death rate (δ_h), plants growth rate (g),deforestation rate (h) probability of transmission of malaria from infectious human to susceptible human (β_{hm}) and the probability of transferring malaria from vectors to human (β_{mh}) while the R_{02} is less sensitive to the recruitment rate of human population (Λ), rate at which exposed humans become infected (q_h), death rate of mosquitoes due to insecticides spray (ξ), availability of bed-nets and the efficacy of bed-nets (γ) [7].

The most sensitive parameter is number of bite per mosquito (b),followed by Λ_h recruitment rate of human population,Λ_m recruitment rate of mosquitoes δ_h disease induced death rate, natural death rate of humans (μ_h). Any increment in the values of the parameters with negative indices would reduce the value of the basic reproduction number, also any increment in the value of the parameters having positive indices will increase the endemicity of the disease. However Table 2 shows the relationship between the basic reproduction number and number of bite per mosquito, it shows that as number of bite per mosquito increases, the basic Reproduction number increases. Also as the treatment of exposed class increases, the basic reproduction number reduces, this means that, with intensity control measure against mosquitoes and treatment of exposed class, we could eliminate malaria from the host environment. The basic reproduction number increases in all cases as all these parameters increases, which means that all these parameters have to be kept at minimal level to have a complete eradication of malaria.

Biological implication

The parameters with positive values have impact on the malaria transmission. Reducing them or decreasing them lead to the decrease of the basic reproduction number.

3.17 Travelling wave solution

In this section, we shall assess the spatial spread of malaria disease when some infected mosquitoes flies from the invasive plant to human environment. With this regard, we shall find the travelling wave linking the equilibrium points and to determine the conditions for the existence of the propagation speed of such wave[3, 9]. In our model, we assumed that the vector population takes both advection movement (k) which increase the wave speed and dispersal coefficient D which depend on the wave propagation speed, v.
Travelling wave solution with respect to our model equation are of the form:
$E_h(x,t) = e_h(z), I_h(x,t) = i_h(z), E_m(x,t) = e_m(z), I_m(x,t) = i_m(z)$, where $z = x - vt$ and v is the constant wave speed, which we shall determine. Replacing system (1) in wave equation form, we obtain the following system of ordinary differential equations:

$$\begin{aligned}
e_h + \frac{\tilde{\lambda_h}}{v}(1 - e_h - i_h - r_h) - (q_h + \mu_h)\frac{e_h}{v} &= 0 \\
i_h + \frac{q_h}{v}eh - (\delta_h + \mu_h + v_h)\frac{i_h}{v} &= 0 \\
r_h + \frac{v_h}{v}i_h - (\mu_h + \alpha_h)\frac{r_h}{v} &= 0 \\
De_m^{''} - (k-v)e_m^{'} - \tilde{\lambda_m}(1 - e_m - i_m) + (\xi_m + \mu_m)e_m &= 0 \\
Di_m^{''} - (k-v)i_m^{'} - q_m e_m + (\xi_m + \mu_m)i_m &= 0
\end{aligned} \tag{50}$$

where the line denotes the derivative in relation to z. This system is equivalent to the following system of first-order differential equations:

$$\begin{aligned}
\frac{de_h}{dz} &= -\frac{\lambda_h}{v}(1 - e_h - i_h - r_h) + (q_h + \mu_h)\frac{e_h}{v} \\
\frac{di_h}{dz} &= -\frac{q_h}{v}eh + (\delta_h + \mu_h + v_h)\frac{i_h}{v} \\
\frac{dr_h}{dz} &= -\frac{v_h}{v}i_h + (\mu_h + \alpha_h)\frac{r_h}{v}
\end{aligned} \tag{51}$$

$$\begin{aligned}
\frac{de_m}{dz} &= u_1 \\
\frac{du_1}{dz} &= \frac{1}{D}(k-v)u_1 + \frac{\lambda_m}{D}(1 - e_m - i_m) - (\xi_m + \mu_m)\frac{e_m}{D} \\
\frac{di_m}{dz} &= u_2 \\
\frac{du_2}{dz} &= \frac{1}{D}(k-v)u_2 + \frac{1}{D}q_m e_m - (\xi_m + \mu_m)\frac{i_m}{D}
\end{aligned} \tag{52}$$

where

$$\frac{di_m}{dt} = i_m''(z), \frac{de_m}{dt} = e_m''(z)$$

There are two equilibrium point of the system above the first is $g_1 = (0,0,0,0,0,0,0)$ Which represents disease free equilibrium and the second, $g_2 = (E_h^o, I_h^o, R_h^o, E_m^o, I_m^o, 0, 0)$ which is the non trivial equilibrium that corresponds to infection The conditon for the existence of g_2 is $R_o > 1$ because $E_h^o, I_h^o, R_h^o, E_m^o, I_m^o$ are non-negative.We want to find the condition for a solution with a positive wave propagation speed v and with non-negative $E_h^o, I_h^o, R_h^o, E_m^o, I_m^o$ that the following boundary conditions holds.

$$\begin{gathered}
e_h(-\infty) = E_h{**} \text{ and } e_h(+\infty) = 0 \\
i_h(-\infty) = I_h{**} \text{ and } i_h(-\infty) = 0 \\
e_m(-\infty) = E_m{**} \text{ and } e_m(-\infty) = 0 \\
i_m(-\infty) = I_m{**} \text{ and } i_m(-\infty) = 0 \\
u(-\infty) = 0 \text{and } u(+\infty) = 0
\end{gathered}$$

we searched for solutions in the positive region,such that $E_h^o > 0, I_h^o > 0, R_h^o > 0, E_m^o > 0, I_m^o > 0$ and the solution cannot oscillate near the origin. The eigen value of the Jacobian matrix must be real and non complex values otherwise $E_h^o < 0, I_h^o < 0, R_h^o < 0, E_m^o < 0, I_m^o < 0$ for some z
The Jacobian matrix at g^* is given by

$$Dg_1 = \begin{pmatrix}
\frac{q_h+\mu_h}{v} - \psi & 0 & 0 & 0 & 0 & -\frac{b\phi\beta_{\text{hm}}}{v} & 0 \\
-\frac{q_h}{v} & \frac{v_h+\delta_h+\mu_h}{v} - \psi & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{v_h}{v} & -\psi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\psi & 1 & 0 & 0 \\
0 & 0 & 0 & -\mu_m - \xi_m & \frac{k-v}{D} - \psi & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1-\psi & 0 \\
0 & 0 & 0 & \frac{q_m}{D} & 0 & -\mu_m - \xi_m & \frac{k-v}{D} - \psi
\end{pmatrix}, \quad (53)$$

The corresponding eigenvalues are the roots of the polynomial

$$p(\Psi) = \Psi^6 + b_0\Psi^5 + b_1\Psi^4 + b_2\Psi^3 + b_3\Psi^2 + b_4\Psi + b_5 \quad (54)$$

where
$b_0 = \frac{2k}{D} + \frac{2v}{D} - \frac{q_h}{v} - \frac{\delta_h}{v} + \frac{2\mu_h}{v} - \frac{v_h}{v} + 1$
$b_1 = \frac{k^2}{D^2} - \frac{2kv}{D^2} + \frac{v^2}{D^2} - \frac{2\delta_h}{D} + \frac{2kq_h}{Dv} + \frac{2k\delta_h}{Dv} + \frac{4k\mu_h}{Dv} + \frac{2kv_h}{Dv} - \frac{4\mu_h}{D} - \frac{2q_h}{D} - \frac{2v_h}{D} - \frac{2k}{D} - \frac{2v}{D} + \frac{\delta_h q_h}{v^2} + \frac{\mu_h q_h}{v^2} + \frac{q_h v_h}{v^2} + \frac{q_h}{v} + \frac{\delta_h\mu_h}{v^2} + \frac{\mu_h^2}{v^2} + \frac{\mu_h v_h}{v^2} + \frac{\delta_h}{v} + \frac{2\mu_h}{v} + \frac{v_h}{v} + \mu_m - \xi_m$
$b_2 = \frac{k^2 q_h}{D^2 v} - \frac{k^2\delta_h}{D^2 v} - \frac{2k^2\mu_h}{D^2 v} + \frac{k^2 v_h}{D^2 v} + \frac{2k\delta_h}{D^2} + \frac{4k\mu_h}{D^2} + \frac{2kq_h}{D^2} + \frac{2kv_h}{D^2} - \frac{vq_h}{D^2} - \frac{v\delta_h}{D^2} + \frac{2v\mu_h}{D^2} - \frac{vv_h}{D^2} - \frac{k^2}{D^2} + \frac{2kv}{D^2} - \frac{v^2}{D^2} + \frac{2\delta_h}{D} - \frac{2k\delta_h q_h}{Dv^2} - \frac{2k\mu_h q_h}{Dv^2} - \frac{2kq_h v_h}{Dv^2} - \frac{2kq_h}{Dv} - \frac{2k\delta_h\mu_h}{Dv^2} - \frac{2k\mu_h^2}{Dv^2} - \frac{2k\mu_h v_h}{Dv^2} - \frac{2k\delta_h}{Dv} - \frac{4k\mu_h}{Dv} - \frac{2kv_h}{Dv} + \frac{4\mu_h}{D} + \frac{2\delta_h q_h}{Dv} + \frac{2\mu_h q_h}{Dv} + \frac{2q_h v_h}{Dv} + \frac{2q_h}{D} + \frac{2\delta_h\mu_h}{Dv} + \frac{2\mu_h^2}{Dv} + \frac{2\mu_h v_h}{Dv} + \frac{2v_h}{D} - \frac{k\mu_m}{D} - \frac{k\xi_m}{D} + \frac{v\mu_m}{D} + \frac{v\xi_m}{D} - \frac{q_h\mu_m}{v} - \frac{q_h\xi_m}{v} - \frac{\delta_h\mu_m}{v} - \frac{\delta_h\xi_m}{v} - \frac{2\mu_h\xi_m}{v} - \frac{v_h\mu_m}{v} - \frac{2\mu_h\mu_m}{v} - \frac{v_h\xi_m}{v} - \frac{\delta_h q_h}{v^2} - \frac{\mu_h q_h}{v^2} - \frac{q_h v_h}{v^2} - \frac{\delta_h\mu_h}{v^2} - \frac{\mu_h^2}{v^2} - \frac{\mu_h v_h}{v^2} + \mu_m + \xi_m$
$b_3 = -\frac{q_h k^2}{D^2 v} - \frac{v_h k^2}{D^2 v} - \frac{\delta_h k^2}{D^2 v} - \frac{2\mu_h k^2}{D^2 v} - \frac{\mu_h^2 k^2}{D^2 v^2} - \frac{q_h v_h k^2}{D^2 v^2} - \frac{q_h\delta_h k^2}{D^2 v^2} - \frac{q_h\mu_h k^2}{D^2 v^2} - \frac{v_h\mu_h k^2}{D^2 v^2} - \frac{\delta_h\mu_h k^2}{D^2 v^2} + \frac{2\mu_h^2 k}{D^2 v} + \frac{2q_h k}{D^2} + \frac{2q_h v_h k}{D^2 v} + \frac{2v_h k}{D^2} + \frac{2q_h\delta_h k}{D^2 v} + \frac{2\delta_h k}{D^2} + \frac{2q_h\mu_h k}{D^2 v} + \frac{2v_h\mu_h k}{D^2 v} + \frac{2\delta_h\mu_h k}{D^2 v} + \frac{4\mu_h k}{D^2} - \frac{\mu_m k}{D} - \frac{\xi_m k}{D} - \frac{q_h\mu_m k}{Dv} - \frac{v_h\mu_m k}{Dv} - \frac{\delta_h\mu_m k}{Dv} - \frac{2\mu_h\mu_m k}{Dv} - \frac{q_h\xi_m k}{Dv} - \frac{v_h\xi_m k}{Dv} - \frac{\delta_h\xi_m k}{Dv} - \frac{2\mu_h\xi_m k}{Dv} - \frac{2\mu_h^2 k}{Dv^2} - \frac{2q_h v_h k}{Dv^2} - \frac{2q_h\delta_h k}{Dv^2} - \frac{2q_h\mu_h k}{Dv^2} - \frac{2v_h\mu_h k}{Dv^2} - \frac{2\delta_h\mu_h k}{Dv^2} + \frac{2\mu_h^2}{Dv} + \frac{2q_h v_h}{Dv} + \frac{2q_h\delta_h}{Dv} + \frac{2q_h\mu_h}{Dv} + \frac{2v_h\mu_h}{Dv} + \frac{2\delta_h\mu_h}{Dv} + \frac{v\mu_m}{D} + \frac{q_h\mu_m}{D} + \frac{v_h\mu_m}{D} + \frac{\delta_h\mu_m}{D} + \frac{2\mu_h\mu_m}{D} + \frac{v\xi_m}{D} + \frac{q_h\xi_m}{D} + \frac{v_h\xi_m}{D} + \frac{\delta_h\xi_m}{D} + \frac{2\mu_h\xi_m}{D} - \frac{\mu_h^2}{D^2} - \frac{vq_h}{D^2} - \frac{vv_h}{D^2} - \frac{q_h v_h}{D^2} - \frac{v\delta_h}{D^2} - \frac{q_h\delta_h}{D^2} - \frac{2v\mu_h}{D^2} - \frac{q_h\mu_h}{D^2} - \frac{v_h\mu_h}{D^2} - \frac{\delta_h\mu_h}{D^2} - \frac{q_h\mu_m}{v} - \frac{v_h\mu_m}{v} - \frac{\delta_h\mu_m}{v} - \frac{2\mu_h\mu_m}{v} - \frac{q_h\xi_m}{v} - \frac{v_h\xi_m}{v} - \frac{\delta_h\xi_m}{v} - \frac{2\mu_h\xi_m}{v} - \frac{\mu_h^2\mu_m}{v^2} - \frac{q_h v_h\mu_m}{v^2} - \frac{q_h\delta_h\mu_m}{v^2} - \frac{q_h\mu_h\mu_m}{v^2} - \frac{v_h\mu_h\mu_m}{v^2} - \frac{\delta_h\mu_h\mu_m}{v^2} - \frac{\mu_h^2\xi_m}{v^2} - \frac{q_h v_h\xi_m}{v^2} - \frac{q_h\delta_h\xi_m}{v^2} - \frac{q_h\mu_h\xi_m}{v^2} - \frac{v_h\mu_h\xi_m}{v^2} - \frac{\delta_h\mu_h\xi_m}{v^2}$
$b_4 = \frac{\mu_h^2 k^2}{D^2 v^2} + \frac{q_h v_h k^2}{D^2 v^2} + \frac{q_h\delta_h k^2}{D^2 v^2} + \frac{q_h\mu_h k^2}{D^2 v^2} + \frac{v_h\mu_h k^2}{D^2 v^2} + \frac{\delta_h\mu_h k^2}{D^2 v^2} + \frac{\mu_h^2\mu_m k}{Dv^2} + \frac{q_h\mu_m k}{Dv} + \frac{q_h v_h\mu_m k}{Dv^2} + \frac{v_h\mu_m k}{Dv} +$

$\frac{q_h\delta_h\mu_m k}{Dv^2}+\frac{\delta_h\mu_m k}{Dv}+\frac{q_h\mu_h\mu_m k}{Dv^2}+\frac{v_h\mu_h\mu_m k}{Dv^2}+\frac{\delta_h\mu_h\mu_m k}{Dv^2}+\frac{2\mu_h\mu_m k}{Dv}+\frac{\mu_h^2\xi_m k}{Dv^2}$
$b_5=-\frac{k\delta_h q_h\mu_m}{Dv^2}-\frac{k\delta_h q_h\xi_m}{Dv^2}-\frac{k\mu_h q_h\xi_m}{Dv^2}-\frac{k\mu_h q_h\mu_m}{Dv^2}-\frac{kq_h v_h\mu_m}{Dv^2}-\frac{kq_h v_h\xi_m}{Dv^2}-\frac{k\delta_h\mu_h\xi_m}{Dv^2}-\frac{k\delta_h\mu_h\mu_m}{Dv^2}-\frac{k\mu_h^2\xi_m}{Dv^2}-\frac{k\mu_h v_h\xi_m}{Dv^2}-\frac{k\mu_h^2\mu_m}{Dv^2}-\frac{k\mu_h v_h\mu_m}{Dv^2}+\frac{\delta_h q_h\mu_m}{Dv}+\frac{\delta_h q_h\xi_m}{Dv}+\frac{\mu_h q_h\xi_m}{Dv}+\frac{\mu_h q_h\mu_m}{Dv}+\frac{q_h v_h\mu_m}{Dv}+\frac{q_h v_h\xi_m}{Dv}+\frac{\delta_h\mu_h\xi_m}{Dv}+\frac{\delta_h\mu_h\mu_m}{Dv}+\frac{\mu_h^2\xi_m}{Dv}+\frac{\mu_h v_h\xi_m}{Dv}+\frac{\mu_h^2\mu_m}{Dv}+\frac{\mu_h v_h\mu_m}{Dv}$
From Routh Hurwitz criterion, the root of the polynomial $g(x)$ are either negative or have a negative real part if and only if

1. $b_i(i=1,2,3,4,5,6)$
2. $b_1b_2>b_3$
3. $b_1b_2b_3-b_1^2b_4>b_3^2+b_1b_5$
4. $b_3^2b_4+2b_1b_4b_5>b_1b_5b_2^2+b_1b_3b_4b_2+b_3b_5b_2+b_1^2b_4^2+b_5^2$

However, as $R_o>1$, $\psi_5>0$, then $\lim_{\Psi}\longrightarrow\pm\infty$, $b(\Psi)=+\infty$. To provide the minimum wave propagation speed, the polynomial have 6 complex roots, 3 conjugated complex roots and 3 real roots. In equation (54), the trajectories must approach the disease -free equilibrium and they cannot oscillate because the positive region is invariant.Thus the polynomial must have negative real roots.

However, we consider the case without advection ($K=0$). If $R_o1=1$, then $p(0)=0$, The polynomial $p(\Psi)$ always has a negative real root if the $\lim_{\Psi}\longrightarrow-\infty=+\infty$. Therefore,for $R_o1=1$, we obtain that $v_{min}=0$, and then, nonwave speed exists. For $R_o1>1$,the existence of three real root depends on the value of v.For $v<v_{min}$,the roots are complex.For $v=v_min$, we have a double negative real root and for $v>v_{min}$ three negative real roots. However,the minimal speed at the double root of the polynomial is $p(v_min)=p'(v_min)=0$ Secondly, we consider the advection movement(i.e. $k\neq 0), k$ appear adding the value of v, from the expression of the polynomial coefficients b_0,b_1,b_2,b_3andb_4. Hence, advection increases the wave speed when it is in the same direction of the wave front and decreases it when it is in the opposite direction.

Remark The determination of the wave speed, v will be determine by estimating the fly speed of the mosquito per day which we term as diffusion (the D value). We will thereafter use appropriate curves for the explicit meaning of the wave speed and its epidemiological impact on the malaria transmission.

4 Numerical simulations/sensitivity analysis

The following Table 3 gives a summary of the parameter values used herein:

$$D=\frac{L^2}{2T}$$

where L is the average distance covered by mosquito in a day and T is the time taken to travel [10].

The following graphical representation of the formulated model are obtained from Table 2 parameters' values.

Table 2: Description of the model parameters' values

Parameter	Value	Sensitivity Index	Reference
μ_h	0.00004643/day	+0.500000	Kenya National Bureau of Statistics
β_{hm}	0.24/day	+0.230400	[5]
β_{mh}	0.14/day	+0.001000	[5]
q_h	0.083/day	+0.000300	[5]
α_h	0.000017/day	Not applicable	[4]
υ_h	0.00265/day	-0.089600	[5]
K_p	1	+0.002100	Variable
ϵ	0.5	-0.015220	Variable
γ	0.6	+0.001000	Variable
δ_h	0.0003454/day	-0.489000	[4]
b	0.5	+1.000000	Variable
θ_m	0.343	+0.043210	[5]
q_m	0.091	+0.000500	[4]
g	1.84	+0.120000	[5]
μ_m	0.1041	-0.001600	[4]
h	0.05	Not applicable	Variable
D	3.125	Not applicable	Computed
k	4.5	Not applicable	[6]
L	2.5	Not applicable	[6]

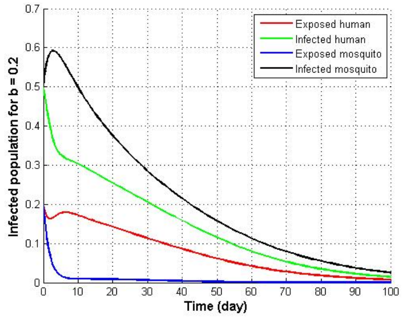

Figure 1: Plot of the infected population over time

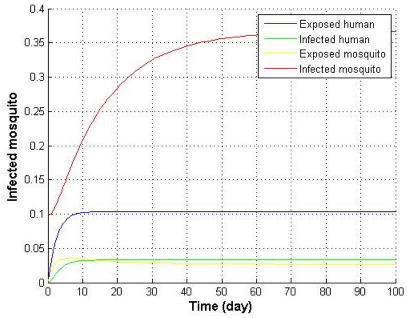

Figure 5: Plot of the infected compartment of Equations (4) with initial conditions $e_h(0) = 0, i_h(0) = 0, e_m(0) = 0$ and $i_m(0) = 0.1$

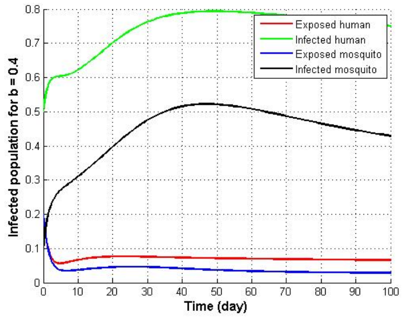

Figure 2: Plot of the infected population over time

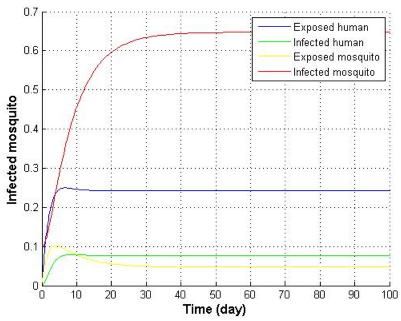

Figure 6: Plot of the infected compartment of Equations (4) with initial conditions $e_h(0) = 0, i_h(0) = 0, e_m(0) = 0$ and $i_m(0) = 0.3$

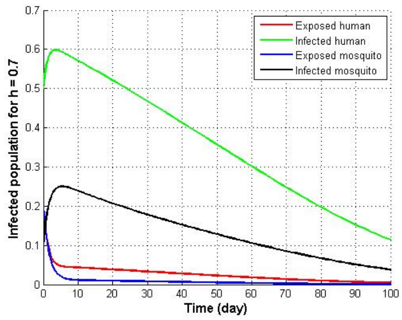

Figure 3: Plot of the infected population over time

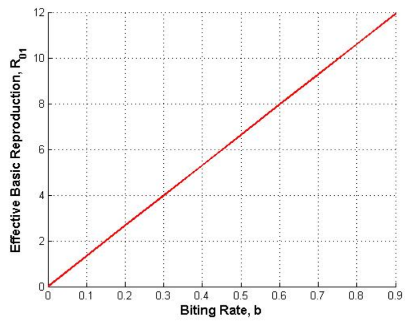

Figure 7: Plot of the effective reproduction number, R_{01} over the biting rate, b

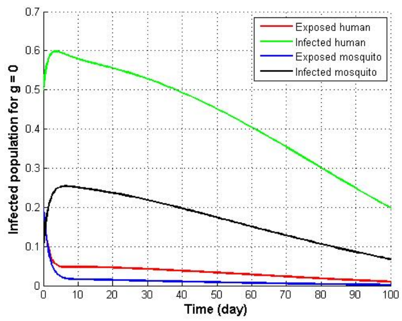

Figure 4: Plot of the infected population over time

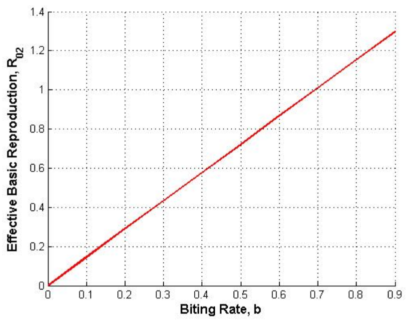

Figure 8: Plot of the effective reproduction number, R_{02} over the biting rate, b

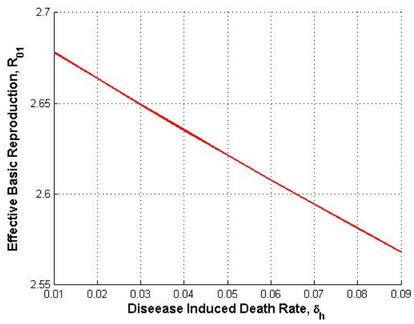

Figure 9: Plot of the effective reproduction number, R_{01} over the disease induced death rate, δ_h

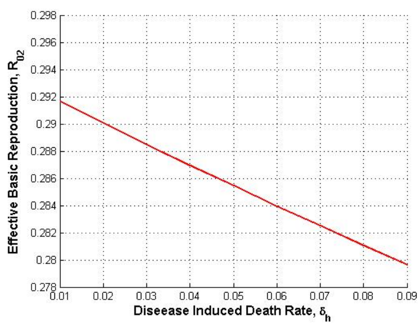

Figure 10: Plot of the effective reproduction number, R_{02} over the disease induced death rate, δ_h

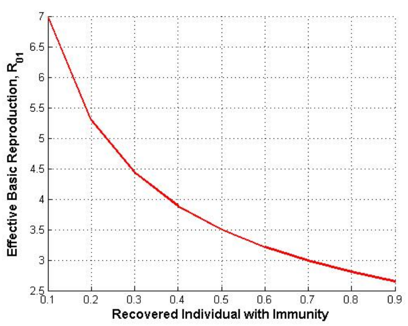

Figure 11: Plot of the effective reproduction number, R_{01} over the recovered individuals with immunity, v_h

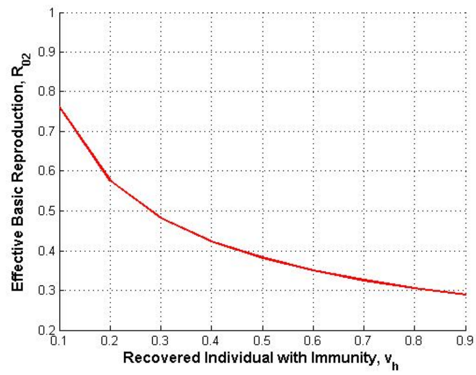

Figure 12: Plot of the effective reproduction number, R_{02} over the immune individuals, v_h

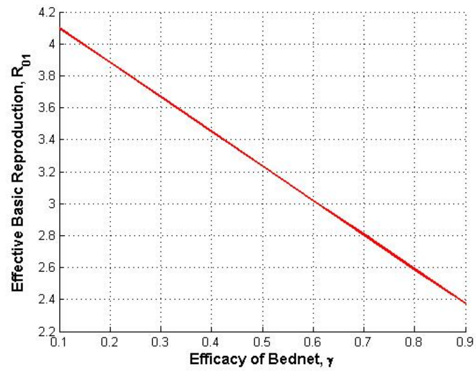

Figure 13: Plot of the effective reproductive number, R_{01} over the bed net efficacy, γ

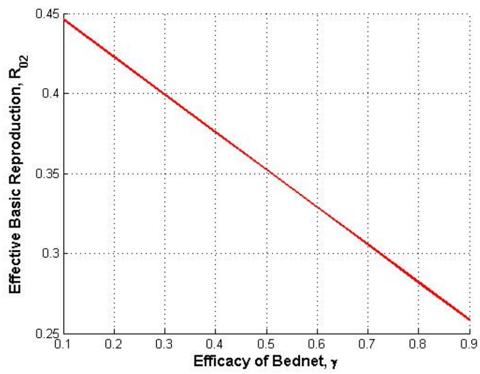

Figure 14: Plot of the effective reproductive number, R_{02} over the bed net efficacy, γ

Figure 15: Plot of the effective reproductive number, R_{01} over the invasive plants control rate, h

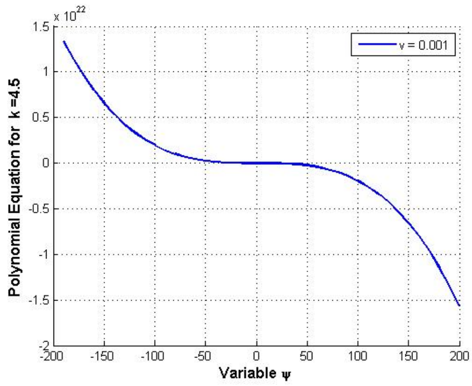

Figure 18: Graph of the polynomial $m(\psi)$ for the parameters outlined in Table 3

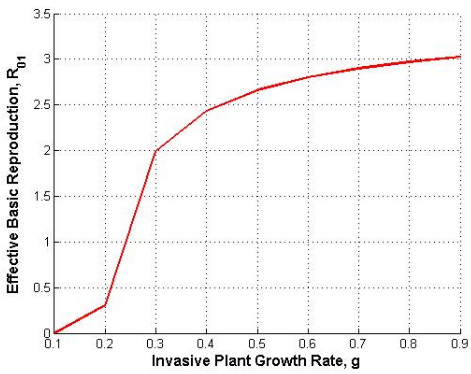

Figure 16: Plot of the effective reproductive number, R_{01} over the invasive plants growth rate, g

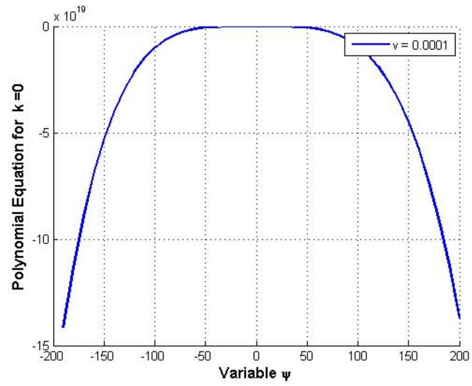

Figure 19: Graph of the polynomial $m(\psi)$ for the parameters outlined in Table 3

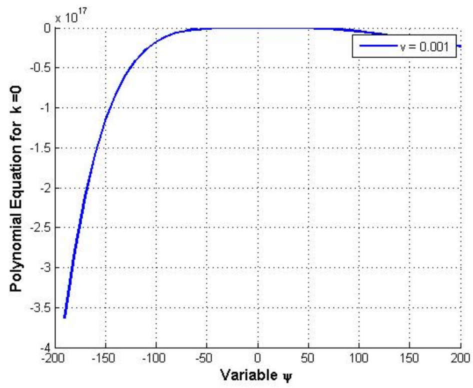

Figure 17: Graph of the polynomial $m(\psi)$ for the parameters outlined in Table 3

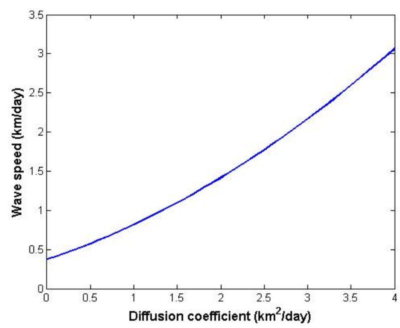

Figure 20: Graph of the propagation speed as a function of the diffusion coefficient D when $k = 0$. The values used for the other parameters are outlined in Table 3

5 Discussions and conclusion

The equilibrium,E_1 is the equilibrium state where there is plant in human environment with mosquito present.In this equilibrium there is a contact between human and mosquito population and breeding site for mosquito population to thrive. This may result to malaria in human population as time progresses. The Equilibrium E_2 is a Disease Free Equilibrium at which there is no invasive plant in the host environment.The contact rate of mosquito and human is low as the mosquito do not have a resting place to reside around the human environment after bite. Malaria transmission is very low and it can easily be controlled by mere use of mosquito net. The disease will persist in the host environment provided that those that are infected with the disease keep on transmitting the disease. The less than or equal ($\leq$) sign of the lyapunov function shows the decrease of the susceptible population. The numerical analysis of the homogeneous model shows that the decrease in disease induced death rate, increase in bednet efficacy, and the control rate of alien invasive plant will drastically reduced malaria disease.

In conclusion, in this paper, we have presented a non-linear mathematical model that studies the impact of invasive alien plants on the dynamics of malaria transmission and its analyses. The non-linear homogeneous equations part of the model was analysed. The model had two basic reproduction numbers, R_{01} and R_{02} under two cases: where there is plant and where there is no plant respectively. Different analyses such as: equilibrium points, stabilities, backward bifurcation, travelling wave solutions were carried out in detailed for possible understanding of malaria transmission. Sensitivity analysis were carried out, and the result shows that it is quite sensitive to the biting rate, disease induced death rate, recovered rate and bed-net efficacy.

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