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Complex Ebola Virus Disease fractional order model in the Caputo sense with Laplace Adomian Decomposition Method model solution

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Abstract

Ebola Virus Disease is a life-threatening disease that is transmitted between humans and animals. The aim of this study is to develop a fractional order Ebola Virus Disease model for the dynamics of Ebola considering the control measures;quarantine,isolation,treatment, vaccine and protection (condom use) with a view to study the effects of these control measures on the spread of Ebola in a given population of humans and animals in the Caputo Sense. The fractional order Ebola Virus Disease model in the Caputo sense to control the spread of the disease with eighteen compartments is considered in this paper.This model considered vaccination, condom use, quarantine, use of treatment drugs and isolation as control measures combined together. We analyzed the model where the validity of the model was proven by establishing a region of invariance and the positivity of solutions. We also established the model disease-free equilibrium (DFE)and endemic equilibrium (EE). The reproduction number (a threshold value) (B) was obtained using the next generation matrix. The result of the study produced an expression for B, where if the reproduction number B, is less than 1 then the model DFE point is stable, so the Ebola virus disease dies out. If the reproduction number B is greater than 1, then the model DFE point is unstable, so the Ebola Virus disease persists in the populations.We also analysed the local stability of the DFE point and found out it will be stable when the reproduction number $B < 1$. The novel application of the Laplace–Adomian decomposition method to the model obtained a result of the complex fractional model in an infinite series form which converges further to its exact value. Comparing the solutions of the fractional Ebola virus disease model with that of the classical case using simulation plots we found out that the case of fractional order has more degree of freedom in such a way that can be varied as the fraction (α) could be varied to get different results. We proved convergence of the Laplace Adomian Decomposition method using the Cauchy-Kovalevskaya theorem for differential equations with analytic vector fields and then obtained a new result on the convergence rate of the Laplace Adomian Decomposition Method.

Keywords and phrases: Ebola virus Disease; Laplace–Adomian decomposition method; Fractional order model; Mathematical model; Approximate (Analytical) Solution

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1 Introduction

Ebola Virus Disease (EVD) is a disease that is also called Ebola haemorrhagic fever which is deadly in nature[1]. The disease was discovered near Ebola River in The Democratic Republic of Congo in 1976 [2, 3]. The disease infects and affects human and non – human pirates. Five species of the virus namely; Bundibugyo Ebola Virus, Tai Forest Ebola Virus, Reston Ebola Virus, Su-dan Ebola Virus, and Zaire Ebola Virus have been identified [4]. Since the birthing and naming of the disease so many outbreaks of the disease have been witnessed in Africa including the 2014 Ebola outbreak in West Africa which in history was called the largest of its kind because it affected many countries at the same time. A group of viruses known as orthoebolaviruses causes Ebola virus disease. This virus is formally known as ebolavirus. The disease has an incubation period of 2 to 21 days after first contact with the symptoms that may include fever, aches, pains, fatigue diarrhea, vomiting, and unexplained bleeding[3, 5]. Scientists believe that human can initially get infected with Ebola virus disease through effective contact with an infected animal which may be fruit bat or non human primate. Research shows that hosts of Ebola Virus may be bats, arthropods and rodents [6].

Ebola virus disease can infect human beings through direct contact with body fluids or blood an Ebola Virus infectious or sick person, infected object and surfaces and dead person. Vaccine drugs and treatment drugs for Ebola Virus disease against the virus are now in use such that those dying from the disease are far less than those recovering from the disease. These drugs include rVSV- EBOV which is 70% to 100% effective to protect against the virus [7, 8]. Some cases have been traced to sexual transmission due to virus residual being persistent in the sperm of men that recovered from the disease[9, 10] . In applied sciences Frac-tional Calculus has so many applications hence in mathematical modelling. From many studies we have seen that fractional calculus can develop the process of some Phenomena in natural science [11, 12] . Fractional-order differential systems have been found to have the advantage of sample modelling, clear parameter meaning and accurate description of some materials and processes with memory and genetic characteristics [13]. Now within the circle of researchers there is a high growing interest into the role of fractional calculus in many areas of modelling real- world problems like Ebola Virus Disease outbreaks particularly in Africa [14, 15].

Laplace–Adomian decomposition method (LADM) is a technique that requires the combination of the Laplace Transforms and the Adomian decomposition method (ADM) [16]. Adomian's is a good effective technique for the solutions of an Ebola virus disease model or a disease model system of ordinary differential equations. On the other hand, Laplace transform is a perfect method applied in applied sciences and engineering. The Laplace–Adomian decomposition method (LADM) is the combination of these two methods. LADM has been applied to so many problems in physics, engineering, biology and applied mathematics [17]. The main idea of the method is to assume an infinite solution of the form: $Z = \sum_{n=0}^{\infty} Z_n$ then apply Laplace transformation to the differential equation. The non-linear terms of the model equation are then decomposed in terms of the Adomian polynomials, and then an iterative procedure is formulated for the determination of the (Z_n) in a recursive form. This method can be used for a system of linear and non-linear ordinary and partial differential equations of the classical and fractional order model. This method does not require any perturbation and also has no need for a predefined step size. This method is an effective method for explicit and numerical solutions of a system of differential equations representing all kinds of physical problems [17].

The Laplace Adomian Decomposition Method has been used by many to obtain the approximate solution of frac-tional order models, The approximate solution of a model for HIV infection was obtained using the Laplace–Adomian decomposition method by Ongun [11]. Adejoh and Mbah [17] in their work also obtained the approximate and numerical solution of their fractional order cancer disease model using Laplace–Adomian decomposition method. The numerical solution of a fractional order smoking model was obtained via the Laplace–Adomian decomposition method by Fazal et al. [5]. Fazal et al. [5] also determined the numerical solution of a fractional order epidemic model of a childhood disease using the Laplace–Adomian decomposition method. Recent studies have employed the Laplace-Adomian decomposition method (LADM) to solve fractional-order models of Ebola virus dynamics.Atokolo et al [18] considered fractional mathematical model of Lassa Fever where Adams

Bashforth-Moultan method was used to investigate the model approximate solution. In 2020, Akeem Olarewaju Yunus and Morufu Oyedunsi Olayiwola utilized LADM to analyze a co-dynamic Ebola and malaria transmission model with Caputo fractional-order [19]. Similarly, Queeneth Ojoma Ahman et al. applied LADM to approximate the solution of a large fractional-order Ebola virus disease model with control measures [20].

The LADM has also been used to solve various nonlinear fractional differential equations. Shah et al. (2020) employed a new analytical technique to analyze the fractional view of third-order Kortewege–De Vries equations [21]. Ganji and Jafari (2020) developed a new approach using LADM to solve non-linear Volterra integro-differential equations with Mittag-Leffler kernel [22]. Furthermore, Shah et al. (2020) utilized LADM to solve the family of Kuramoto–Sivashinsky equations semi-analytically [23]. Additionally, Shah et al. (2019) proposed a novel method using LADM to obtain the analytical solution of fractional Zakharov–Kuznetsov equations [24]. These studies demonstrate the effectiveness and efficiency of LADM in solving fractional-order models and nonlinear differential equations, highlighting its potential as a valuable tool for researchers in the field. The Laplace–Adomian decomposition method unlike other numerical methods requires no linearization and discretization and as such the results obtained from it are more realistic and effective[25, 26, 27]. Un-like the classical integer model that describes only the local properties of a system, The fractional order model provides a better description of the whole space of a model system; it also provides a better explanation of a real-life system with memory effects [28, 12, 29, 30, 31, 32, 33] The Riemann–Liouville derivative and the Caputo derivative are regarded as sin-gular kernels fractional derivative relative to biological problems, Others that are also non singular are; Mittag-Lefler and the Atangana–Baleanu operators [31]. The classical order and fractional order derivatives have been used in studying transmission dynamics of Ebola disease models [25, 26, 27] and the likes, but to the best of our knowledge it has not been adopted and applied in a large eighteen equation compartmental Ebola model. Also, an eighteen equation Ebola Virus model solution has not been determined using the LADM. To the best of our knowledge, no author has presented an Ebola model that has up to eighteen (18) compartments to study the transmission dynamics of Ebola. Above all we considered a non-integer Ebola model which has more degree of freedom and as such can be varied to obtain more preferred responses which also to the best of our knowledge has not been given adequate attention by researchers. Thus, in this work, we considered a system of eighteen equation compartmental fractional order Ebola model (using the Caputo derivative) for the control of Ebola disease presented with analysis and the analytical solution would be obtained using the Laplace–Adomian decomposition method (LADM).

2 Methodology/Model formulation

We consider two communities of human and animal interacting with each other. The human population is divided into sixteen compartments of Susceptible human population $(S(t))$,Vaccinated human population $(S_v(t))$, unvaccinated human population $(S_u(t))$, vaccinated condom using human population $(S_{vc}(t))$, vaccinated non using condom human population $(S_{vn}(t))$, unvaccinated condom using human population $(S_{uc}(t))$, unvaccinated non using condom human population $(S_{un}(t))$, Exposed human population$(E(t))$, Exposed quarantined human population $(E_Q(t))$, Exposed treated human population $(E_T(t))$,Infectious human population $(I(t))$, Infectious treated human population$(I_T(t))$, Infectious isolated human population $(I_i(t))$, Infectious not treated human population $(I_N(t))$, Recovered human population $(R(t))$ and Dead unburied human population $(D_u(t))$ while the animal population is divided into three compartments of susceptible animal population $(S_r(t))$, Exposed animal population $(E_r(t))$ and Infectious animal population $(I_r(t))$. The animal population is recruited into the animal susceptible population at the rate (Λ) while the animal susceptible population is reduced through exposure to the disease at the rate (ω) and entering the animal exposed population and animal natural death at the rate (μ_r),the exposed animal population reduces by becoming infectious and entering the animal infectious population at the rate (ϕ) and animal natural death at the rate (μ_r) and the infectious population reduces due to animal natural death at the rate (μ_r) and animal disease death at the rate (δ_4). The human population is recruited into the human susceptible population at the rate (P) while the human susceptible population reduces due to human natural death at the rate (μ), through those going for vaccination at the proportion (r) and those not going for vaccination at the proportion $(X = 1 - r)$ with its modification parameter (K_1). The human susceptible vaccinated population reduces due to human natural death at the rate (μ), through those using condom at the proportion (λ) and those not using condom at the proportion $(Z = 1 - \lambda)$ with its parameter modification (K_2).The human susceptible unvaccinated population reduces due to human natural death at the rate (μ), through those using condom at the proportion (σ) and those not using condom at the proportion $(Y = 1 - \sigma)$ with its parameter modification (K_3). The human susceptible vaccinated condom using population reduces due to human natural death at the rate (μ) and through exposure to the disease at the rate (β_1) and entering the human exposed population.The human susceptible vaccinated not using condom population reduces due to human natural death at the rate (μ) and through exposure to the disease at the rate (β_2) and entering the human exposed population. The human susceptible unvaccinated condom using population reduces due to human natural death at the rate (μ) and through exposure to the disease at the rate (β_3) and entering the human exposed population.The human susceptible unvaccinated not using condom population reduces due to human natural death at the rate (μ) and through exposure to the disease at the rate (β_4) and entering the human exposed population. The exposed human population reduces due to human natural death at the rate (μ) and through those exposed going into quarantine at the rate (α_1) and those exposed going for treatment after exposure at the rate (α_2). The exposed quarantined human population reduces due to human natural death at the rate (μ) and by becoming infectious and entering the human infectious population at the rate (θ_1). The exposed treated human population reduces due to human natural death at the rate (μ) and by becoming infectious and entering the human infectious population at the rate (θ_2) with those recovered entering the recovered population at the rate (ρ). The infectious human population reduces due to human natural death at the rate (μ) and through those infectious going for treatment at the rate (S_1) and those infectious going into isolation at the rate (S_2) with those not going for any form of treatment or isolation at the rate (S_3). The infectious treated human population reduces due to human natural death at the rate (μ), by those recovering from the disease and are entering the recovered population at the rate (J_2) and through those that die from the disease and are entering the dead unburied population at the rate (δ_1).The infectious isolated human population reduces due to human natural death at the rate (μ), by those recovering from the disease and are entering the recovered population at the rate (J_1) and through those that die from the disease and are entering the dead unburied population at the rate (δ_2).The infectious not treated human population reduces due to human natural death at the rate (μ) and through those that die from the disease and are entering the dead unburied population at the rate (δ_3).The recovered human population reduces due to human natural death at the rate (μ) and through those that are losing their temporal immunity and becoming susceptible to the disease again at the rate (τ). The dead unburied human population reduces due to proper burial of the corps at the rate (q).

2.1 The assumptions of the model

(1) There is no vertical transmission of the infection i.e. there is no infection from mother to unborn baby.

(2) There is no herd immunity in the population.

(3) The population is not constant and there is no inherent immunity in the population.

(4) The infectious compartment is a transition point i.e. Infected and infectious individuals do not stay long before joining isolated, treated or not treated compartments.

(5) The disease recovered individuals are still infectious through semen.

(6) Quarantined and isolated individuals are under close surveillance and do not contribute to the transmission of the disease.

(7) Death resulting from the disease only takes place in the infectious compartments.

(8) Ebola infected and infectious animals in the population have interactions with the susceptible human population.

(9) The animals do not recover from the disease.

The schematic diagram of the Ebola Mathematical model is shown in Figure 1 below.

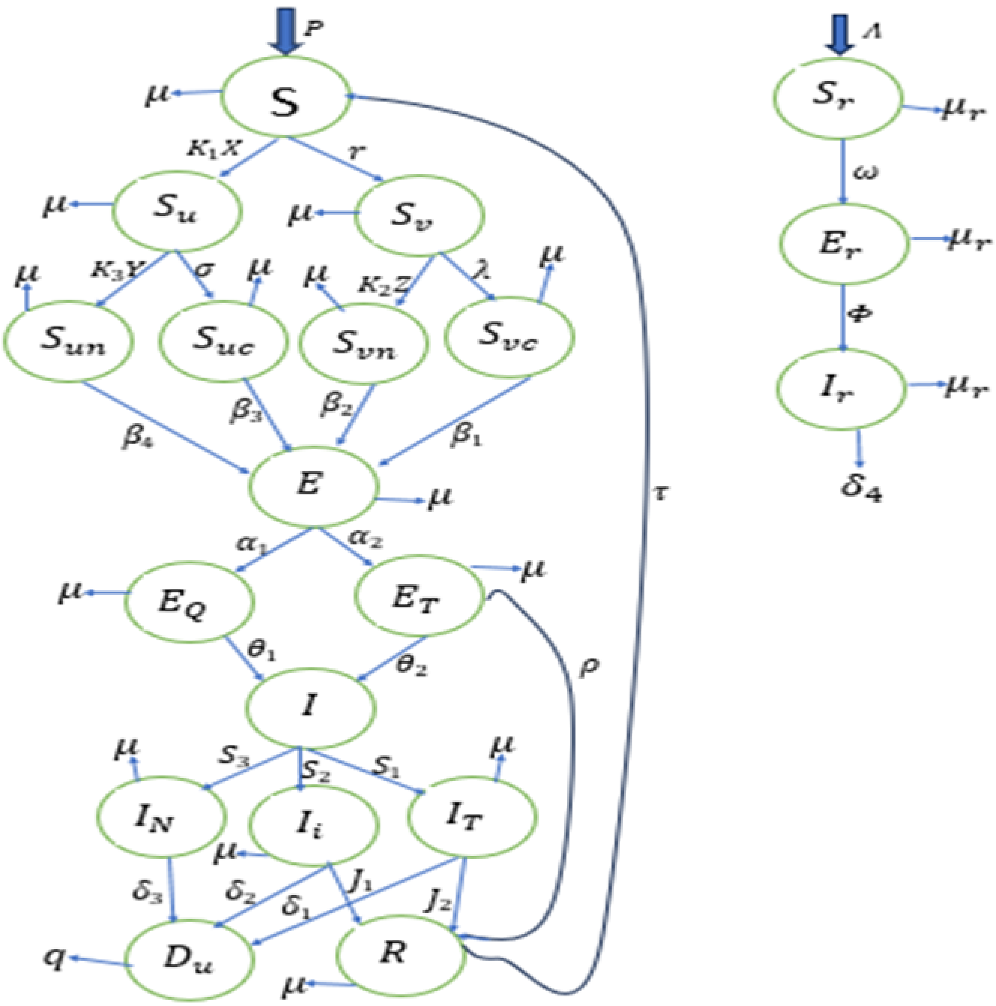

Figure 1: THE EBOLA VIRUS DISEASE FLOW DIAGRAM

2.2 Ebola mathematical model equations

incorporating the above assumptions, formulations and Figure 1 we produce the mathematical model equations as follows:

$$
\left.\begin{aligned}
(1)\frac{dS}{dt} &= P\ \breve{}\ (\mu + r + K_1 X)S + \tau R\\
(2)\frac{dS_v}{dt} &= rS\breve{}(\mu + \lambda + K_2 Z)S_v\\
(3)\frac{dS_u}{dt} &= K_1 X S\breve{}(\mu + \sigma + K_3 Y)S_u\\
(4)\frac{dS_{vc}}{dt} &= \lambda S_v - (\mu + \beta_1)S_{vc}\\
(5)\frac{dS_{vn}}{dt} &= K_2 Z S_v - (\mu + \beta_2)S_{vn}\\
(6)\frac{dS_{uc}}{dt} &= \sigma S_u - (\mu + \beta_3)S_{uc}\\
(7)\frac{dS_{un}}{dt} &= K_3 Y S_u - (\mu + \beta_4)S_{un}\\
(8)\frac{dE}{dt} &= \beta_1 S_{vc} + \beta_2 S_{vn} +\\
&\beta_3 S_{uc} + \beta_4 S_{un} - (\mu + \alpha_1 + \alpha_2)E\\
(9)\frac{dE_Q}{dt} &= \alpha_1 E - (\mu + \theta_1)E_Q\\
(10)\frac{dE_T}{dt} &= \alpha_2 E - (\mu + \theta_2 + \rho)E_T\\
(11)\frac{dI_T}{dt} &= \theta_1 S_1 E_Q + \theta_2 S_1 E_T - (\mu + \delta_1 + J_2)I_T\\
(12)\frac{dI_i}{dt} &= \theta_1 S_2 E_Q + \theta_2 S_2 E_T - (\mu + \delta_2 + J_1)I_i\\
(13)\frac{dI_N}{dt} &= \theta_1 S_3 E_Q + \theta_2 S_3 E_T - (\mu + \delta_3)I_N\\
(14)\frac{dR}{dt} &= J_1 I_i + J_2 I_T + \rho E_T\breve{}(\mu + \tau)R\\
(15)\frac{dD_u}{dt} &= \delta_3 I_N + \delta_2 I_i + \delta_1 I_T - qD_u\\
(16)\frac{dS_r}{dt} &= \Lambda - (\mu_r + \omega)S_r\\
(17)\frac{dE_r}{dt} &= \omega S_r - (\mu_r + \phi)E_r\\
(18)\frac{dI_r}{dt} &= \phi E_r - (\mu_r + \delta_4)I_r
\end{aligned}\right\} \tag{1}
$$

With initial conditions given as follows;

$$
\left.\begin{aligned}
S(0) = g_1 \geq 0, S_v(0) = g_2 \geq 0, S_u(0) = g_3 \geq 0,\\
S_{vc}(0) = g_4 \geq 0, S_{vn}(0) = g_5 \geq 0, S_{uc}(0) = g_6 \geq 0,\\
S_{un}(0) = g_7 \geq 0, E(0) = g_8 \geq 0, E_Q(0) = g_9 \geq 0, E_T(0) = g_{10} \geq 0,\\
I_T(0) = g_{11} \geq 0, I_i(0) = g_{12} \geq 0, I_N(0) = g_{13} \geq 0, R(0) = g_{14} \geq 0,\\
D_u(0) = g_{15} \geq 0, S_r(0) = g_{16} \geq 0, E_r(0) = g_{17} \geq 0\ and\ I_r(0) = g_{18} \geq 0.
\end{aligned}\right\} \tag{2}
$$

where;

$$\left.\begin{aligned} \beta_1 &= \frac{m_1 I_N + m_2 I_T + m_3 I_r + m_4 D_u}{N} \\ \beta_2 &= \frac{n_1 I_N + n_2 I_T + n_3 I_r + n_4 D_u}{N} \\ \beta_3 &= \frac{e_1 I_N + e_2 I_T + e_3 I_r + e_4 D_u}{N} \\ \beta_4 &= \frac{t_1 I_N + t_2 I_T + t_3 I_r + t_4 D_u}{N} \\ T_s(t) &= S + S_v + S_u + S_{vc} + S_{vn} + S_{uc} + S_{un} + \\ & E + E_Q + E_T + I_T + I_i + I_N + R + D_u \\ T_r(t) &= S_r + E_r + I_r \\ N(t) &= T_s + T_r \end{aligned}\right\} \tag{3}$$

Applying assumption (4) the model equation became 18 instead of 19 as the compartments are 19 but the infectious compartment is a transition point such that no one stays there for long. The model variables and parameters descriptions are presented in Table 1 and Table 2 respectively.

2.3 Fractional order Ebola Virus Disease model

In this model the Caputo derivative is considered as a differential operator.When using the Caputo fractional initial value problem, we can express the initial condition with an initial integer order with physical interpretation that is very easy for real life interpretation, and thus is suitable for the Ebola Virus Disease model. In this section, we now present some well known definitions and results that will be used throughout this paper.

Definition 1

The Caputo fractional order derivative of a function (f) on the interval $[0, T]$ is defined as follows:

$$[{}^c D_0^\alpha f(t)] = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} f^n(s) ds \tag{4}$$

where $n = [\alpha] + 1$ *and* $[\alpha]$ represents the integer part of α. In particular, for $0 < \alpha < 1$ the Caputo derivative becomes;

$$[{}^c D_0^\alpha f(t)] = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f(s)}{(t-s)^\alpha} ds \tag{5}$$

Definition 2

The Laplace transform of Caputo derivatives is defined as follows:

$$\mathcal{L}[{}^c D^\alpha v(t)] = M^\alpha h(M) \sum_{k=0}^n M^{\alpha-i-1} x^k(0), n-1 < \alpha < n, n \in N \tag{6}$$

for arbitrary $b_i \in R, i = 0, 1, 2, ..., n-1$ where $n = [\alpha] + 1$ *and* $[\alpha]$ represents the non - integer part of α.

Lemma 1.1

The following result stands for fractional differential equations;

$$I^\alpha[{}^c D^\alpha r(t)] = r(t) + \sum_{i=0}^{n-1} \frac{r^{(i)}(0)}{i!} t^i \tag{7}$$

for arbitrary $\alpha > 0, i = 0, 1, 2, ..., n-1$ where $n = [\alpha] + 1$ *and* $[\alpha]$ represents the integer parts of α. Introducing the fractional order and substituting equation (3) into the model (1), we have a new set of model equation which is described by the following set of fractional differential equations of order

(α) for $0 < \alpha < 1$.

$$\left.\begin{aligned}
(1)D^{\alpha}S(t) &= P - (\mu + r + K_1X)S + \tau R\\
(2)D^{\alpha}S_v(t) &= rS - (\mu + \lambda + K_2Z)S_v\\
(3)D^{\alpha}S_u(t) &= K_1XS - (\mu + \sigma + K_3Y)S_u\\
(4)D^{\alpha}S_{vc}(t) &= \lambda S_v - [\mu S_{vc} + 1/N(m_1I_NS_{vc}+\\
&\quad m_2I_TS_{vc} + m_3I_rS_{vc} + m_4D_uS_{vc})]\\
(5)D^{\alpha}S_{vn}(t) &= K_2ZS_v - [\mu S_{vn} + 1/N(n_1I_NS_{vn}+\\
&\quad n_2I_TS_{vn} + n_3I_rS_{vn} + n_4D_uS_{vn})]\\
(6)D^{\alpha}S_{uc}(t) &= \sigma S_u - [\mu S_{uc} + 1/N(e_1I_NS_{uc}+\\
&\quad e_2I_TS_{uc} + e_3I_rS_{uc} + e_4D_uS_{uc})]\\
(7)D^{\alpha}S_{un}(t) &= K_3YS_u - [\mu S_{un} + 1/N(t_1I_NS_{un}+\\
&\quad t_2I_TS_{un} + t_3I_rS_{un} + t_4D_uS_{un})]\\
(8)D^{\alpha}E(t) &= 1/N[m_1I_NS_{vc} + m_2I_TS_{vc}+\\
&\quad m_3I_rS_{vc} + m_4D_uS_{vc} + n_1I_NS_{vn}+\\
&\quad n_2I_TS_{vn} + n_3I_rS_{vn} + n_4D_uS_{vn}+\\
&\quad e_1I_NS_{uc} + e_2I_TS_{uc} + e_3I_rS_{uc}+\\
&\quad e_4D_uS_{uc} + t_1I_NS_{un} + t_2I_TS_{un}+\\
&\quad t_3I_rS_{un} + t_4D_uS_{un}] - (\mu + \alpha_1 + \alpha_2)E\\
(9)D^{\alpha}E_Q(t) &= \alpha_1E - (\mu + \theta_1)E_Q\\
(10)D^{\alpha}E_T(t) &= \alpha_2E - (\mu + \theta_2 + \rho)E_T\\
(11)D^{\alpha}I_T(t) &= \theta_1S_1E_Q + \theta_2S_1E_T - (\mu + \delta_1 + J_2)I_T\\
(12)D^{\alpha}I_i(t) &= \theta_1S_2E_Q + \theta_2S_2E_T - (\mu + \delta_2 + J_1)I_i\\
(13)D^{\alpha}I_N(t) &= \theta_1S_3E_Q + \theta_2S_3E_T - (\mu + \delta_3)I_N\\
(14)D^{\alpha}R(t) &= J_1I_i + J_2I_T + \rho E_T - (\mu + \tau)R\\
(15)D^{\alpha}D_u(t) &= \delta_3I_N + \delta_2I_i + \delta_1I_T - q\\
(16)D^{\alpha}S_r(t) &= \Lambda - (\mu_r + \omega)S_r\\
(17)D^{\alpha}E_r(t) &= \omega S_r - (\mu_r + \phi)E_r\\
(18)D^{\alpha}I_r(t) &= \phi E_r - (\mu_r + \delta_4)I_r
\end{aligned}\right\} \tag{8}$$

The Table1 gives a description summary of the Variables used herein:
The Table 2 gives a description summary of the Parameters used herein:

3 Analysis of the model

In this section, we herein analyse the system of fractional Ebola model(8) as follows;

3.1 Positively invariant Region

In this section we show that the closed set $\Omega = \{(S, S_v, S_u, S_{vc}, S_{vn}, S_{uc}, S_{un}, E, E_Q, E_T, I_T, I_i, I_N, R, D_u, S_r, E_r, I_r) \in R^{18}_{+} : 0 \leq N\}$ provides the Fractional Ebola Virus model system (8) positively invariant feasible region.

Lemma 3.1

The solutions of the Fractional Ebola Virus model system (8) are non-negative and bounded if they start in Ω.

Proof

Table 1: **Ebola model variables and descriptions**

Variables	Descriptions
$S(t)$	Ebola Susceptible Population
$S_v(t)$	Ebola Susceptible Vaccinated Population
$S_u(t)$	Ebola Susceptible Unaccinated Population
$S_{vc}(t)$	Ebola Susceptible Vaccinated Condom using Population
$S_{vn}(t)$	Ebola Susceptible Vaccinated not using Condom Population
$S_{uc}(t)$	Ebola Susceptible Unaccinated Condom using Population
$S_{un}(t)$	Ebola Susceptible Unaccinated not using Condom Population
$E(t)$	Ebola Exposed Population
$E_Q(t)$	Ebola Exposed Quarantined Population
$E_T(t)$	Ebola Exposed Treated Population
$I(t)$	Ebola Infectious Population
$I_T(t)$	Ebola Infectious Treated Population
$I_i(t)$	Ebola Infectious Isolated Population
$I_N(t)$	Ebola Infectious no Treatment Population
$R(t)$	Ebola Recovered Population
$D_u(t)$	Ebola Disease Dead Unburied Population
$S_r(t)$	Ebola Susceptible Animal Population
$E_r(t)$	Ebola Exposed Animal Population
$I_r(t)$	Ebola Susceptible Animal Population
$T_s(t)$	Total Human Population
$T_r(t)$	Total Animal Population
$N(t)$	Total Human and Animal Population

Table 2: **Ebola model parameters and descriptions**

Parameters	Descriptions
P	Human rate of recruitment
μ	Human rate of natural death
r	Human rate of Vaccination
K_1	Human vaccination modification parameter
λ	Human Vaccinated rate of condom use
K_2	Human Vaccinated condom use modification parameter
σ	Human Unaccinated rate of condom use
K_3	Human Unaccinated condom use modification parameter
β_1	Human Vaccinated condom using population force of infection
β_2	Human Vaccinated non condom using population force of infection
β_3	Human Unaccinated condom using population force of infection
β_4	Human Unvaccinated non condom using population force of infection
α_1	Human exposed rate of quarantine
α_2	Human exposed rate of treatment
θ_1	Human exposed quarantined rate of becoming infectious
θ_2	Human exposed treated rate of becoming infectious
ρ	Human exposed treated rate of recovery
S_1	Human infectious rate of treatment
S_2	Human infectious rate of isolation
S_3	Human infectious rate of no treatment or isolation
J_1	Human infectious isolated rate of recovery
J_2	Human infectious treatment rate of recovery
δ_1	Human infectious treated disease death rate
δ_2	Human infectious isolated disease death rate
δ_3	Human infectious no treatment disease death rate
τ	Human Recovered rate of immunity loss
q	Disease dead unburied rate of proper burial
Λ	Animal Population recruitment rate
μ_r	Animal Population natural death rate
ω	Animal Population rate of exposure to disease
ϕ	Animal Population rate of becoming infectious
δ_4	Animal Population disease death rate

from (8) we obtain the following;

$$\left.\begin{aligned} D^\alpha S(t)|_{s=0} = P - \tau R \geq 0 \\ D^\alpha S_v(t)|_{S_v=0} = rS \geq 0 \\ D^\alpha S_u(t)|_{S_u=0} = K_1XS \geq 0 \\ D^\alpha S_{vc}(t)|_{S_{vc}=0} = \lambda S_v \geq 0 \\ D^\alpha S_{vn}(t)|_{S_{vn}=0} = K_2ZS_v \geq 0 \\ D^\alpha S_{uc}(t)|_{S_{uc}=0} = \sigma S_u \geq 0 \\ D^\alpha S_{un}(t)|_{S_{un}=0} = K_3YS_u \geq 0 \\ D^\alpha E(t)|_{E=0} = \beta_1 S_{vc} + \beta_2 S_{vn} + \beta_3 S_{uc} + \beta_4 S_{un} \geq 0 \\ D^\alpha E_Q(t)|_{E_Q=0} = \alpha_1 E \geq 0 \\ D^\alpha E_T(t)|_{E_T=0} = \alpha_2 E \geq 0 \\ D^\alpha I_T(t)|_{I_T=0} = \theta_1 s_1 E_Q + \theta_2 s_1 E_T \geq 0 \\ D^\alpha I_i(t)|_{I_i=0} = \theta_1 s_2 E_Q + \theta_2 s_2 E_T \geq 0 \\ D^\alpha I_N(t)|_{I_N=0} = \theta_1 s_3 E_Q + \theta_2 s_3 E_T \geq 0 \\ D^\alpha R(t)|_{R=0} = J_1 I_i + J_2 I_T + \rho E_T \geq 0 \\ D^\alpha D_u(t)|_{D_u=0} = \delta_3 I_N + \delta_2 I_i + \delta_1 I_T \geq 0 \\ D^\alpha S_r(t)|_{S_r=0} = \Lambda \geq 0 \\ D^\alpha E_r(t)|_{E_r=0} = \omega S_r \geq 0 \\ D^\alpha I_r(t)|_{I_r=0} = \phi E_r \geq 0 \end{aligned}\right\} \tag{9}$$

Hence, the sub-populations $S, S_v, S_u, S_{vc}, S_{vn}, S_{uc}, S_{un}, E, E_Q, E_T, I_T, I_i, I_N, R, D_u,$ S_r, E_r *and* I_r are non-negative. Also, as $N = T_s + T_r$ wherever N is considered to be constant each of the sub-population will lie in $[0, N]$. Thus, the sub-populations $S, S_v, S_u, S_{vc}, S_{vn}, S_{uc}, S_{un}, E, E_Q, E_T, I_T, I_i, I_N, R,$ D_u, S_r, E_r *and* I_r are also bounded.

3.2 Fractional Ebola Virus model equilibrium points and basic reproduction number

At equilibrium;

$$\left.\begin{aligned} D^\alpha S = D^\alpha S_v = D^\alpha S_u = D^\alpha S_{vc} = D^\alpha S_{vn} = D^\alpha S_{uc} = D^\alpha S_{un} = 0 \\ D^\alpha E = D^\alpha E_Q = D^\alpha E_T = D^\alpha I_T = D^\alpha I_i = D^\alpha I_N = 0 \\ D^\alpha R = D^\alpha D_u = D^\alpha S_r = D^\alpha E_r = D^\alpha I_r = 0 \end{aligned}\right\} \tag{10}$$

The Fractional Ebola Virus Disease free equilibrium $(C^f) = \{S^f, S_v^f, S_u^f, S_{vc}^f, S_{vn}^f, S_{uc}^f, S_{un}^f, E^f, E_Q^f, E_T^f, I_T^f, I_i^f, I_N^f,$ $R^f, D_u^f, S_r^f, E_r^f, I_r^f\} = \{\frac{P}{(\mu+r+K_1X)}, \frac{rS^f}{(\mu+\lambda+K_2Z)},$
$\frac{K_1XS^f}{(\mu+\sigma+K_3Y)}, \frac{\lambda S_v^f}{\mu}, \frac{K_2ZS_v^f}{\mu}, \frac{\sigma S_u^f}{\mu},$
$\frac{K_3YS_u^f}{\mu}, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\Lambda}{\mu_r}, 0, 0\}.$
while
The Fractional Ebola Virus Disease endemic equilibrium $(C^h) = \{S^h, S_v^h, S_u^h, S_{vc}^h, S_{vn}^h, S_{uc}^h, S_{un}^h, E^h, E_Q^h, E_T^h, I_T^h,$ $I_i^h, I_N^h, R^h, D_u^h, S_r^h, E_r^h, I_r^h\}$
$= \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9,$
$A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{15}, A_{16}, A_{17}, A_{18}\}.$

where

$$\left.\begin{aligned}
A_1 &= \frac{P+\tau R^h}{(\mu+r+K_1X)},\\
A_2 &= \frac{rS^h}{(\mu+\lambda+K_2Z)},\\
A_3 &= \frac{K_1XS^h}{(\mu+\sigma+K_3Y)},\\
A_4 &= \frac{\lambda rS^h}{(\mu+\lambda+K_2Z)(\mu+\beta_1)},\\
A_5 &= \frac{K_2ZrS^h}{(\mu+\lambda+K_2Z)(\mu+\beta_2)},\\
A_6 &= \frac{\sigma K_1XS^h}{(\mu+\sigma+K_3Y)(\mu+\beta_3)},\\
A_7 &= \frac{K_3YK_1XS^h}{(\mu+\sigma+K_3Y)(\mu+\beta_4)},\\
A_8 &= \frac{\beta_1S^f_{vc}+\beta_2S^f_{vn}+\beta_3S^f_{uc}+\beta_4S^f_{un}}{(\mu+\alpha_1+\alpha_2)},\\
A_9 &= \frac{\alpha_1E^h}{(\mu+\theta_1)},\\
A_{10} &= \frac{\alpha_2E^h}{(\mu+\theta_2+\rho)},\\
A_{11} &= \frac{\theta_1s_1E^h_Q+\theta_2s_1E^h_T}{(\mu+\delta_1+J_2)}, A_{12} = \frac{\theta_1s_2E^h_Q+\theta_2s_3E^h_T}{(\mu+\delta_3)},\\
A_{13} &= \frac{\theta_1s_3E^h_Q+\theta_2s_1E^h_T}{(\mu+\delta_1+J_2)}, A_{14} = \frac{J_1I^h_i+J_2I^h_T+\rho E^h_T}{(\mu+\tau)},\\
A_{15} &= \frac{\delta_3I^h_N+\delta_2I^h_i+\delta_1I^h_T}{q},\\
A_{16} &= \frac{\Lambda}{(\mu_r)+\omega},\\
A_{17} &= \frac{\omega S^h_r}{(\mu_r+\phi)}, A_{18} = \frac{\phi E^h_r}{(\mu_r+\delta_4)}
\end{aligned}\right\} \tag{11}$$

The basic reproduction number B can be computed for the model using the next-generation matrix method as described in [28], we calculate B as follows; considering the newly occurring infections $E, E_Q, E_T, I_T, I_i, I_N, R, D_u, S_r, E_r$ and I_r we calculate the corresponding Jacobian matrices F and V at the Ebola model disease-free equilibrium point C_f

$$\left.\begin{aligned} F &= \begin{bmatrix} 0 & 0 & 0 & a_1 & 0 & a_2 & 0 & a_3 & 0 & a_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ V &= \begin{bmatrix} b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -a & b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -b & 0 & b_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -c & -d & b_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -e & -f & 0 & b_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & -g & -h & 0 & 0 & b_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & -w & -k & -x & 0 & b_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & -m & -n & -p & 0 & b_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -y & b_{10} \end{bmatrix} \end{aligned}\right\} \tag{12}$$

where;

$$\left.\begin{aligned} a_1 &= \frac{m_2 S^e_{vc} + n_2 S^e_{vn} + e_2 S^e_{uc} + t_2 S^e_{un}}{N} \\ a_2 &= \frac{m_1 S^e_{vc} + n_1 S^e_{vn} + e_1 S^e_{uc} + t_1 S^e_{un}}{N} \\ a_3 &= \frac{m_4 S^e_{vc} + n_4 S^e_{vn} + e_4 S^e_{uc} + t_4 S^e_{un}}{N} \\ a_4 &= \frac{m_3 S^e_{vc} + n_3 S^e_{vn} + e_3 S^e_{uc} + t_3 S^e_{un}}{N} \end{aligned}\right\} \tag{13}$$

and

$$\left.\begin{aligned} a_1 &= \frac{P[\frac{r(m_2\lambda+n_2K_2Z)}{(\mu+\lambda+K_2Z)} + \frac{K_1X(e_2\sigma+t_2K_3Y)}{(\mu+\sigma+K_3Y)}]}{\mu N(\mu+r+K_1X)} \\ a_2 &= \frac{P[\frac{r(m_1\lambda+n_1K_2Z)}{(\mu+\lambda+K_2Z)} + \frac{K_1X(e_1\sigma+t_1K_3Y)}{(\mu+\sigma+K_3Y)}]}{\mu N(\mu+r+K_1X)} \\ a_3 &= \frac{P[\frac{r(m_4\lambda+n_4K_2Z)}{(\mu+\lambda+K_2Z)} + \frac{K_1X(e_4\sigma+t_4K_3Y)}{(\mu+\sigma+K_3Y)}]}{\mu N(\mu+r+K_1X)} \\ a_4 &= \frac{P[\frac{r(m_3\lambda+n_3K_2Z)}{(\mu+\lambda+K_2Z)} + \frac{K_1X(e_3\sigma+t_3K_3Y)}{(\mu+\sigma+K_3Y)}]}{\mu N(\mu+r+K_1X)} \\ b_1 = (\mu+\alpha_1+\alpha_2)&, b_2 = (\mu+\theta_1), \\ b_3 = (\mu+\theta_2+\rho)&, b_4 = (\mu+\delta_1+J_2), \\ b_5 = (\mu+\delta_2+J_1)&, b_6 = (\mu+\delta_3), \\ b_7 = (\mu+\tau), b_8 = q&, b_9 = (\mu_r+\phi), \\ b_{10} = (\mu_r+\delta_4), a = \alpha_1&, b = \alpha_2, c = \theta_2 s_1, \\ d = \theta_1 s_1, e = \theta_1 s_2&, f = \theta_2 s_2, g = \theta_1 s_3, \\ h = \theta_2 s_3, w = \rho, k = J_2&, x = J_1, m = \delta_1, \\ n = \delta_2&, p = \delta_3, y = \phi. \end{aligned}\right\} \tag{14}$$

With,

$$eig(FV^{-1}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{b_1b_2b_3b_4b_5b_6b_8}[b_8b_5(a_1b_6(dbb_2+cab_3)+a_2b_4(hbb_2+gab_3))+ \\ a_3(b_2b_5b(phb_4+mdb_6)+b_4(nfbb_2b_6+pgab_3b_5)+ \\ b_3b_6a(neb_4+mcb_5)] \end{bmatrix} \tag{15}$$

Thus, the basic reproduction number of our Ebola Virus model which is given as the largest eigenvalue $\rho(FV^{-1})$ is given as follows;
$\rho(FV^{-1}) = \frac{1}{b_1b_2b_3b_4b_5b_6b_8}[b_8b_5(a_1b_6(dbb_2+cab_3)+a_2b_4(hbb_2+gab_3))+a_3(b_2b_5b(phb_4+mdb_6)+b_4(nfbb_2b_6+pgab_3b_5)+b_3b_6a(neb_4+mcb_5)]$
$\Rightarrow B = \frac{1}{b_1b_2b_3b_4b_5b_6b_8}[b_8b_5(a_1b_6(dbb_2+cab_3)+a_2b_4(hbb_2+gab_3))+a_3(b_2b_5b(phb_4+mdb_6)+b_4(nfbb_2b_6+pgab_3b_5)+$
$b_3b_6a(neb_4+mcb_5)]$

$$\left.\begin{aligned} &\therefore B \\ &= \frac{P}{\mu N(\mu+r+K_1X)(\mu+\alpha_1+\alpha_2)(\mu+\theta_1)(\mu+\theta_2+\rho)(\mu+\delta_1+J_2)(\mu+\delta_2+J_1)(\mu+\delta_3)q} \\ &[q(\mu+\delta_2+J_1)(\frac{r(m_2\lambda+n_2K_2Z)}{(\mu+\lambda+K_2Z)}+\frac{K_1X(e_2\sigma+t_2K_3Y)}{(\mu+\sigma+K_3Y)}) \\ &(\mu+\delta_3)(\theta_1s_1\alpha_2(\mu+\theta_1)+\theta_2s_1\alpha_1(\mu+\theta_2+\rho))+(\frac{r(m_1\lambda+n_1K_2Z)}{(\mu+\lambda+K_2Z)}+\frac{K_1X(e_1\sigma+t_1K_3Y)}{(\mu+\sigma+K_3Y)}) \\ &(\mu+\delta_1+J_2)(\theta_2s_3\alpha_2(\mu+\theta_1)+\theta_1s_3\alpha_1(\mu+\theta_2+\rho))) \\ &+(\frac{r(m_4\lambda+n_4K_2Z)}{(\mu+\lambda+K_2Z)}+\frac{K_1X(e_4\sigma+t_4K_3Y)}{(\mu+\sigma+K_3Y)})((\mu+\theta_1)(\mu+\delta_2+J_1)\alpha_2 \\ &(\delta_3\theta_2s_3(\mu+\delta_1+J_2)+\delta_1\theta_1s_1(\mu+\delta_3))+(\mu+\delta_1+J_2)(\delta_2\theta_2s_2\alpha_2(\mu+\theta_1) \\ &(\mu+\delta_3)+\delta_3\theta_1s_3\alpha_1(\mu+\theta_2+\rho)(\mu+\delta_2+J_1))+(\mu+\theta_2+\rho)(\mu+\delta_3)\alpha_1 \\ &(\delta_2\theta_1s_2(\mu+\delta_1+J_2)+\delta_1\theta_2s_1(\mu+\delta_2+J_1))] \end{aligned}\right\} \tag{16}$$

$B = \frac{(a_1b_6(dbb_2+cab_3)+a_2b_4(hbb_2+gab_3))}{b_1b_2b_3b_4b_6} + \frac{a_3(b_2b_5b(phb_4+mdb_6)+b_4(nfbb_2b_6+pgab_3b_5)+b_3b_6a(neb_4+mcb_5)}{b_1b_2b_3b_4b_5b_6b_8}$
$B = B_r + B_i$
where
$B_r = \frac{(a_1b_6(dbb_2+cab_3)+a_2b_4(hbb_2+gab_3))}{b_1b_2b_3b_4b_6}$
$B_r = B_{r1} + B_{r2}$
$B_{r1} = \frac{(a_1(dbb_2+cab_3)}{b_1b_2b_3b_4}$,
$B_{r2} = \frac{a_2(hbb_2+gab_3)}{b_1b_2b_3b_6}$
and
$B_i = \frac{a_3(b_2b_5b(phb_4+mdb_6)+b_4(nfbb_2b_6+pgab_3b_5)+b_3b_6a(neb_4+mcb_5)}{b_1b_2b_3b_4b_5b_6b_8}$

with
$B_r = \frac{a_1b_6dbb_2}{b_1b_2b_3b_4b_6} + \frac{a_1b_6cab_3}{b_1b_2b_3b_4b_6} +$
$\frac{a_2b_4hbb_2}{b_1b_2b_3b_4b_6} + \frac{a_2b_4gab_3}{b_1b_2b_3b_4b_6}$

$B_r = B_{r1a} + B_{r1b} + B_{r2a} + B_{r2b}$
$B_{r1a} = \frac{a_1 b_6 d b b_2}{b_1 b_2 b_3 b_4 b_6} = \frac{a_1 b_6 d b}{b_1 b_3 b_4 b_6} = \frac{a_1 d b}{b_1 b_3 b_4}$
$B_{r1b} = \frac{a_1 b_6 c a b_3}{b_1 b_2 b_3 b_4 b_6} = \frac{a_1 b_6 c a}{b_1 b_2 b_4 b_6} = \frac{a_1 c a}{b_1 b_2 b_4}$
$B_{r2a} = \frac{a_2 b_4 h b b_2}{b_1 b_2 b_3 b_4 b_6} = \frac{a_2 b_4 h b}{b_1 b_3 b_4 b_6} = \frac{a_2 h b}{b_1 b_3 b_6}$
$B_{r2b} = \frac{a_2 b_4 g a b_3}{b_1 b_2 b_3 b_4 b_6} = \frac{a_2 b_4 g a}{b_1 b_2 b_4 b_6} = \frac{a_2 g a}{b_1 b_2 b_6}$

The basic reproductive number B is a crucial tool that helps in evaluating Ebola infectious disease transmission potential in a given susceptible population free of the disease at the time. It helps to quantify the mean number of the secondary Ebola disease infections generated by a single Ebola infected individual in the population that is wholly susceptible to Ebola disease. If the Basic reproduction number $B < 1$, the Ebola Virus infection quickly come to an end as the disease will not spread. However, the disease will spread through the population if $B > 1$.

3.3 Local stability analysis of the model DFE (C^f)

Here, we analyze local stability of the model DFE points (C^f) and show its local stability by showing that all the eigenvalues of the Jacobian matrix obtained from the linearization of the model system (8) and evaluated at C^f are negative. This is stated as a theorem and proven as follows;

Theorem 2

The DFE of the Fractional bola Model system (8) is locally asymptotically stable in Ω, if $B < 1$ and unstable $B > 1$.

Proof

The Jacobian matrix J of the model at the DFE point is obtained as follows:

$$J = \begin{bmatrix}
-h_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau & 0 & 0 & 0 & 0 \\
r & -h_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K_1X & 0 & -h_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & -y_1 & 0 & -y_2 & 0 & -y_3 & 0 & 0 & -y_4 \\
0 & K_2Z & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & -x_1 & 0 & -x_2 & 0 & -x_3 & 0 & 0 & -x_4 \\
0 & 0 & \sigma & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & -c_1 & 0 & -c_2 & 0 & -c_3 & 0 & 0 & -c_4 \\
0 & 0 & K_3Y & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & -d_1 & 0 & -d_2 & 0 & -d_3 & 0 & 0 & -d_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_1 & 0 & 0 & a_1 & 0 & a_2 & 0 & a_3 & 0 & 0 & a_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a & -b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b & 0 & -b_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c & d & -b_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e & f & 0 & -b_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & h & 0 & 0 & -b_6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w & k & x & 0 & -b_7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_8 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_9 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_{10}
\end{bmatrix} \tag{17}$$

where;

$$\left.\begin{aligned}
h_1 = (\mu + r + K_1X), h_2 = (\mu + \lambda + K_2Z), h_3 = (\mu + \sigma + K_3Y), \\
y_1 = \frac{m_2 S_{vc}^e}{N}, y_2 = \frac{m_1 S_{vc}^e}{N}, y_3 = \frac{m_4 S_{vc}^e}{N}, y_4 = \frac{m_3 S_{vc}^e}{N} \\
x_1 = \frac{n_2 S_{vn}^e}{N}, x_2 = \frac{n_1 S_{vn}^e}{N}, x_3 = \frac{n_4 S_{vn}^e}{N}, x_4 = \frac{n_3 S_{vn}^e}{N} \\
c_1 = \frac{e_2 S_{uc}^e}{N}, c_2 = \frac{e_1 S_{uc}^e}{N}, c_3 = \frac{e_4 S_{uc}^e}{N}, c_4 = \frac{e_3 S_{uc}^e}{N} \\
d_1 = \frac{t_2 S_{un}^e}{N}, d_2 = \frac{t_1 S_{un}^e}{N}, d_3 = \frac{t_4 S_{un}^e}{N}, d_4 = \frac{t_3 S_{un}^e}{N}
\end{aligned}\right\} \tag{18}$$

with others as described in equations (13) and (14).

Applying matrix operations to (17) we obtain the following eigenvalues;

$$\left.\begin{array}{r}\lambda_1=-\mu,\lambda_2=-\mu,\lambda_3=-\mu,\lambda_4=-\mu,\lambda_5=-\mu_r,\\ \lambda_6=-b_8,\lambda_7=-b_9,\lambda_8=-b_{10},\lambda_9=-h_2,\lambda_{10}=-h_3,\\ \lambda_{11}=-h_1,\lambda_{12}=-b_7\ and\ \lambda_{13}=-b_5\end{array}\right\} \tag{19}$$

which are all negative
With these obtained eigenvalues the matrix (17) reduces to;

$$J_e=\begin{bmatrix} -b_1 & 0 & 0 & a_1 & a_2 \\ a & -b_2 & 0 & 0 & 0 \\ 0 & 0 & -b_3 & 0 & 0 \\ 0 & c & d & -b_4 & 0 \\ 0 & g & h & 0 & -b_6 \end{bmatrix} \tag{20}$$

Using MATLAB software, we obtained the characteristic equation of (20) as follows;
$\lambda^5+G_1\lambda^4+G_2\lambda^3+G_3\lambda^2+G_4\lambda+G_5$
Where;
$G_1=(b_1+b_2+b_3+b_4+b_6)>0$
$G_2=b_1(b_2+b_3+b_4+b_6)+b_2(b_3+b_4+b_6)$
$+b_3(b_4+b_6)+b_4b_6>0$
$G_3=b_1b_2b_3+b_2b_3(b_4+b_6)+b_4b_6(b_1+b_2+b_3)+$
$b_1b_2b_4(1-B_{r1b})+b_1b_2b_6(1-B_{r2b})+$
$b_1b_3b_4(1-B_{r1a})+b_1b_3b_6(1-B_{r2a})>0\ if\ B<1$
$G_4=b_2b_3b_4b_6+b_1b_2b_3b_4(1-B_{r1})+b_1b_2b_3b_6(1-B_{r2})+$
$b_1b_3b_4b_6(1-(B_{r1a}+B_{r2a}))+$
$b_1b_2b_4b_6(1-(B_{r1b}+B_{r2b}))>0\ \ if\ \ B<1)$
$G_5=b_1b_2b_3b_4b_6(1-Br)>0\ \ if\ \ B<1$
By applying the Routh-Hurwitz criteria [34], the characteristic equation has a real root that is strictly negative if $G_1>0,G_2>0,G_3>0,G_4>0\ and\ G_5>0$. Which we have shown that $G_1>0,G_2>0\ with\ G_3,G_4\ and\ G_5$ all greater than zero if $B<1$.
Thus, we conclude that the fractional Ebola virus Model disease free-equilibrium point U^e of system (8) is locally asymptotically stable in Ω if $B<1$.

4 Approximate solution of the Ebola Model applying Laplace-Adomian Decomposition Method(LADM)

In this section, we apply the Laplace - Adomian Decomposition Method (LADM) technique to the fractional order Ebola model (8)[20].

Applying Laplace transform to both sides of (8) we have;

$$\left.\begin{aligned}
(1)S^{\alpha}\mathcal{L}\{S\}-S^{(\alpha-1)}S(0) &= \mathcal{L}\{P-(\mu+r+K_1X)S+\tau R\}\\
(2)S^{\alpha}\mathcal{L}\{S_v\}-S^{(\alpha-1)}S_v(0) &= \mathcal{L}\{rS-(\mu+\lambda+K_2Z)S_v\}\\
(3)S^{\alpha}\mathcal{L}\{S_u\}-S^{(\alpha-1)}S_u(0) &= \mathcal{L}\{K_1XS-(\mu+\sigma+K_3Y)S_u\}\\
(4)S^{\alpha}\mathcal{L}\{S_{vc}\}-S^{(\alpha-1)}S_{vc}(0) &= \mathcal{L}\{\lambda S_v-[\mu S_{vc}+1/N(m_1I_NS_{vc}+\\
&\quad m_2I_TS_{vc}+m_3I_rS_{vc}+m_4D_uS_{vc})]\}\\
(5)S^{\alpha}\mathcal{L}\{S_{vn}\}-S^{(\alpha-1)}S_{vn}(0) &= \mathcal{L}\{K_2ZS_v-[\mu S_{vn}+1/N(n_1I_NS_{vn}+\\
&\quad n_2I_TS_{vn}+n_3I_rS_{vn}+n_4D_uS_{vn})]\}\\
(6)S^{\alpha}\mathcal{L}\{S_{uc}\}-S^{(\alpha-1)}S_{uc}(0) &= \mathcal{L}\{\sigma S_u-[\mu S_{uc}+1/N(e_1I_NS_{uc}+\\
&\quad e_2I_TS_{uc}+e_3I_rS_{uc}+e_4D_uS_{uc})]\}\\
(7)S^{\alpha}\mathcal{L}\{S_{un}\}-S^{(\alpha-1)}S_{un}(0) &= \mathcal{L}\{K_3YS_u-[\mu S_{un}+1/N(t_1I_NS_{un}+\\
&\quad t_2I_TS_{un}+t_3I_rS_{un}+t_4D_uS_{un})]\}\\
(8)S^{\alpha}\mathcal{L}\{E\}-S^{(\alpha-1)}E(0) &= \mathcal{L}\{1/N[m_1I_NS_vc+m_2I_TS_vc+\\
&\quad m_3I_rS_vc+m_4D_uS_vc+n_1I_NS_vn+\\
&\quad n_2I_TS_vn+n_3I_rS_vn+n_4D_uS_vn+\\
&\quad e_1I_NS_uc+e_2I_TS_uc+e_3I_rS_uc+\\
&\quad e_4D_uS_uc+t_1I_NS_un+t_2I_TS_un+\\
&\quad t_3I_rS_un+t_4D_uS_un]-(\mu+\alpha_1+\alpha_2)E\}\\
(9)S^{\alpha}\mathcal{L}\{E_Q\}-S^{(\alpha-1)}E_Q(0) &= \mathcal{L}\{\alpha_1E-(\mu+\theta_1)E_Q\}\\
(10)S^{\alpha}\mathcal{L}\{E_T\}-S^{(\alpha-1)}E_T(0) &= \mathcal{L}\{\alpha_2E-(\mu+\theta_2+\rho)E_T\}\\
(11)S^{\alpha}\mathcal{L}\{I_T\}-S^{(\alpha-1)}I_T(0) &= \mathcal{L}\{\theta_1S_1E_Q+\theta_2S_1E_T-(\delta_1+J_2)I_T\}\\
(12)S^{\alpha}\mathcal{L}\{I_i\}-S^{(\alpha-1)}I_i(0) &= \mathcal{L}\{\theta_1S_2E_Q+\theta_2S_2E_T-(\delta_2+J_1)I_i\}\\
(13)S^{\alpha}\mathcal{L}\{I_N\}-S^{(\alpha-1)}I_N(0) &= \mathcal{L}\{\theta_1S_3E_Q+\theta_2S_3E_T-\delta_3I_N\}\\
(14)S^{\alpha}\mathcal{L}\{R\}-S^{(\alpha-1)}R(0) &= \mathcal{L}\{J_1I_i+J_2I_T+\rho E_T-\tau R\}\\
(15)S^{\alpha}\mathcal{L}\{D_u\}-S^{(\alpha-1)}D_u(0) &= \mathcal{L}\{\delta_3I_N+\delta_2I_i+\delta_1I_T\}\\
(16)S^{\alpha}\mathcal{L}\{S_r\}-S^{(\alpha-1)}S_r(0) &= \mathcal{L}\{\Lambda-\mu_rS_r\}\\
(17)S^{\alpha}\mathcal{L}\{E_r\}-S^{(\alpha-1)}E_r(0) &= \mathcal{L}\{\omega S_r-\mu_rE_r\}\\
(18)S^{\alpha}\mathcal{L}\{I_r\}-S^{(\alpha-1)}I_r(0) &= \mathcal{L}\{\phi E_r-(\mu_r+\delta_4)I_r\}
\end{aligned}\right\} \tag{21}$$

Applying the initial conditions (2) to (8) and dividing by S^{α} gives the following;

$$\left.\begin{aligned}
(1)\mathcal{L}\{S\} &= \frac{g_1}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{P-(\mu+r+K_1X)S+\tau R\}]\\
(2)\mathcal{L}\{S_v\} &= \frac{g_2}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{rS-(\mu+\lambda+K_2Z)S_v\}]\\
(3)\mathcal{L}\{S_u\} &= \frac{g_3}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{K_1XS-(\mu+\sigma+K_3Y)S_u\}]\\
(4)\mathcal{L}\{S_{vc}\} &= \frac{g_4}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\lambda S_v-[\mu S_{vc}+1/N(m_1I_NS_{vc}+\\
&\quad m_2I_TS_{vc}+m_3I_rS_{vc}+m_4D_uS_{vc})]\}]\\
(5)\mathcal{L}\{S_{vn}\} &= \frac{g_5}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{K_2ZS_v-[\mu S_{vn}+1/N(n_1I_NS_{vn}+\\
&\quad n_2I_TS_{vn}+n_3I_rS_{vn}+n_4D_uS_{vn})]\}]\\
(6)\mathcal{L}\{S_{uc}\} &= \frac{g_6}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\sigma S_u-[\mu S_{uc}+1/N(e_1I_NS_{uc}+\\
&\quad e_2I_TS_{uc}+e_3I_rS_{uc}+e_4D_uS_{uc})]\}]\\
(7)\mathcal{L}\{S_{un}\} &= \frac{g_7}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{K_3YS_u-[\mu S_{un}+1/N(t_1I_NS_{un}+\\
&\quad t_2I_TS_{un}+t_3I_rS_{un}+t_4D_uS_{un})]\}]\\
(8)\mathcal{L}\{E\} &= \frac{g_8}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{1/N[m_1I_NS_vc+m_2I_TS_vc+\\
&\quad m_3I_rS_vc+m_4D_uS_vc+n_1I_NS_vn+\\
&\quad n_2I_TS_vn+n_3I_rS_vn+n_4D_uS_vn+\\
&\quad e_1I_NS_uc+e_2I_TS_uc+e_3I_rS_uc+\\
&\quad e_4D_uS_uc+t_1I_NS_un+t_2I_TS_un+\\
&\quad t_3I_rS_un+t_4D_uS_un]-(\mu+\alpha_1+\alpha_2)E\}]\\
(9)\mathcal{L}\{E_Q\} &= \frac{g_9}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\alpha_1E-(\mu+\theta_1)E_Q\}]\\
(10)\mathcal{L}\{E_T\} &= \frac{g_{10}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\alpha_2E-(\mu+\theta_2+\rho)E_T\}]\\
(11)\mathcal{L}\{I_T\} &= \frac{g_{11}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\theta_1S_1E_Q+\theta_2S_1E_T-(\delta_1+J_2)I_T\}]\\
(12)\mathcal{L}\{I_i\} &= \frac{g_{12}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\theta_1S_2E_Q+\theta_2S_2E_T-(\delta_2+J_1)I_i\}]\\
(13)\mathcal{L}\{I_N\} &= \frac{g_{13}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\theta_1S_3E_Q+\theta_2S_3E_T-\delta_3I_N\}]\\
(14)\mathcal{L}\{R\} &= \frac{g_{14}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{J_1I_i+J_2I_T+\rho E_T-\tau R\}]\\
(15)\mathcal{L}\{D_u\} &= \frac{g_{15}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\delta_3I_N+\delta_2I_i+\delta_1I_T\}]\\
(16)\mathcal{L}\{S_r\} &= \frac{g_{16}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\Lambda-\mu_rS_r\}]\\
(17)\mathcal{L}\{E_r\} &= \frac{g_{17}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\omega S_r-\mu_rE_r\}]\\
(18)\mathcal{L}\{I_r\} &= \frac{g_{18}}{S} + \frac{1}{S^{\alpha}}[\mathcal{L}\{\phi E_r-(\mu_r+\delta_4)I_r\}]
\end{aligned}\right\} \tag{22}$$

Assuming that the solutions of
$S(t), S_v(t), S_u(t), S_{vc}(t), S_{vn}(t), S_{uc}(t), S_{un}(t), E(t), E_Q(t),$
$E_T(t), I_T(t), I_i(t), I_N(t), R(t), D_u(t), S_r(t), E_r(t), I_r(t)$

are in the form of infinite series given by;

$$\left.\begin{aligned}
S(t) = \sum_{(n=0)}^{\infty} S_n, S_v(t) = \sum_{(n=0)}^{\infty} S_{v_n}\\
S_u(t) = \sum_{(n=0)}^{\infty} S_{u_n}, S_{vc}(t) = \sum_{(n=0)}^{\infty} S_{vc_n}\\
S_{vn}(t) = \sum_{(n=0)}^{\infty} S_{vn_n}, S_{uc}(t) = \sum_{(n=0)}^{\infty} S_{uc_n}\\
S_{un}(t) = \sum_{(n=0)}^{\infty} S_{un_n}, E(t) = \sum_{(n=0)}^{\infty} E_n\\
E_Q(t) = \sum_{(n=0)}^{\infty} E_{Q_n}, E_T(t) = \sum_{(n=0)}^{\infty} E_{T_n}\\
I_T(t) = \sum_{(n=0)}^{\infty} I_{T_n}, I_i(t) = \sum_{(n=0)}^{\infty} I_{i_n}\\
I_N(t) = \sum_{(n=0)}^{\infty} I_{N_n}, R(t) = \sum_{(n=0)}^{\infty} R_n\\
D_u(t) = \sum_{(n=0)}^{\infty} D_{u_n}, S_r(t) = \sum_{(n=0)}^{\infty} S_{r_n}\\
E_r(t) = \sum_{(n=0)}^{\infty} E_{r_n}, I_r(t) = \sum_{(n=0)}^{\infty} I_{r_n}
\end{aligned}\right\} \tag{23}$$

We then decompose the nonlinear terms of the Ebola fractional order model (8) before we proceed as follows;

$$\left.\begin{aligned}
I_N(t)S_{vc}(t) = \sum_{(n=0)}^{\infty} A_n\\
I_T(t)S_{vc}(t) = \sum_{(n=0)}^{\infty} B_n\\
I_r(t)S_{vc}(t) = \sum_{(n=0)}^{\infty} F_n\\
D_u(t)S_{vc}(t) = \sum_{(n=0)}^{\infty} G_n\\
I_N(t)S_{vn}(t) = \sum_{(n=0)}^{\infty} H_n\\
I_T(t)S_{vn}(t) = \sum_{(n=0)}^{\infty} L_n\\
I_r(t)S_{vn}(t) = \sum_{(n=0)}^{\infty} Q_n\\
D_u(t)S_{vn}(t) = \sum_{(n=0)}^{\infty} T_n\\
I_N(t)S_{uc}(t) = \sum_{(n=0)}^{\infty} U_n\\
I_T(t)S_{uc}(t) = \sum_{(n=0)}^{\infty} V_n\\
I_r(t)S_{uc}(t) = \sum_{(n=0)}^{\infty} W_n\\
D_u(t)S_{uc}(t) = \sum_{(n=0)}^{\infty} X_n\\
I_N(t)S_{un}(t) = \sum_{(n=0)}^{\infty} Y_n\\
I_T(t)S_{un}(t) = \sum_{(n=0)}^{\infty} Z_n\\
I_r(t)S_{un}(t) = \sum_{(n=0)}^{\infty} N_n\\
D_u(t)S_{un}(t) = \sum_{(n=0)}^{\infty} M_n
\end{aligned}\right\} \tag{24}$$

Where the Adomian polynomials in (24) are defined as follows;

$$\left.\begin{aligned}
A_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{Nk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{vck}(t)\}|_{(\lambda=0)}\\
B_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{Tk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{vck}(t)\}|_{(\lambda=0)}\\
F_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{rk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{vck}(t)\}|_{(\lambda=0)}\\
G_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k D_{uk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{vck}(t)\}|_{(\lambda=0)}\\
H_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{Nk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{vnk}(t)\}|_{(\lambda=0)}\\
L_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{Tk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{vnk}(t)\}|_{(\lambda=0)}\\
Q_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{rk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{vnk}(t)\}|_{(\lambda=0)}\\
T_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k D_{uk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{vnk}(t)\}|_{(\lambda=0)}\\
U_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{Nk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{uck}(t)\}|_{(\lambda=0)}\\
V_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{Tk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{uck}(t)\}|_{(\lambda=0)}\\
W_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{rk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{uck}(t)\}|_{(\lambda=0)}\\
X_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k D_{uk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{uck}(t)\}|_{(\lambda=0)}\\
Y_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{Nk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{unk}(t)\}|_{(\lambda=0)}\\
Z_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{Tk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{unk}(t)\}|_{(\lambda=0)}\\
N_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k I_{rk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{unk}(t)\}|_{(\lambda=0)}\\
M_n &= \frac{1}{\Gamma(n+1)}\frac{d^n}{d\lambda^n}\{\sum\nolimits_{(k=0)}^{n}\lambda^k D_{uk}(t)\sum\nolimits_{(k=0)}^{n}\lambda^k S_{unk}(t)\}|_{(\lambda=0)}
\end{aligned}\right\} \tag{25}$$

From the definitions in (25) when $n = 0$ we have the following;

$$\left.\begin{aligned}
A_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{N0}][\lambda^0 S_{vc0}] = I_{N0}S_{vc0}\\
B_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{T0}][\lambda^0 S_{vc0}] = I_{T0}S_{vc0}\\
F_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{r0}][\lambda^0 S_{vc0}] = I_{r0}S_{vc0}\\
G_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 D_{u0}][\lambda^0 S_{vc0}] = D_{u0}S_{vc0}\\
H_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{N0}][\lambda^0 S_{vn0}] = I_{N0}S_{vn0}\\
L_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{T0}][\lambda^0 S_{vn0}] = I_{T0}S_{vn0}\\
Q_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{r0}][\lambda^0 S_{vn0}] = I_{r0}S_{vn0}\\
T_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 D_{u0}][\lambda^0 S_{vn0}] = D_{u0}S_{vn0}\\
U_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{N0}][\lambda^0 S_{uc0}] = I_{N0}S_{uc0}\\
V_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{T0}][\lambda^0 S_{uc0}] = I_{T0}S_{uc0}\\
W_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{r0}][\lambda^0 S_{uc0}] = I_{r0}S_{uc0}\\
X_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 D_{u0}][\lambda^0 S_{uc0}] = D_{u0}S_{uc0}\\
Y_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{N0}][\lambda^0 S_{un0}] = I_{N0}S_{un0}\\
Z_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{T0}][\lambda^0 S_{un0}] = I_{T0}S_{un0}\\
N_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 I_{r0}][\lambda^0 S_{un0}] = I_{r0}S_{un0}\\
M_0 &= \frac{1}{\Gamma(1)}\frac{d^0}{d\lambda^0}[\lambda^0 D_{u0}][\lambda^0 S_{un0}] = D_{u0}S_{un0}
\end{aligned}\right\} \tag{26}$$

Similarly, when $n = 1$ we have the following;

$$\left.\begin{aligned}
A_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{N0} + \lambda I_{N1}][S_{vc0} + \lambda S_{vc1}]|_{(\lambda=0)} = I_{N0}S_{vc1} + I_{N1}S_{vc0}\\
B_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{T0} + \lambda I_{T1}][S_{vc0} + \lambda S_{vc1}]|_{(\lambda=0)} = I_{T0}S_{vc1} + I_{T1}S_{vc0}\\
F_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{r0} + \lambda I_{r1}][S_{vc0} + \lambda S_{vc1}]|_{(\lambda=0)} = I_{r0}S_{vc1} + I_{r1}S_{vc0}\\
G_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[D_{u0} + \lambda D_{u1}][S_{vc0} + \lambda S_{vc1}]|_{(\lambda=0)} = D_{u0}S_{vc1} + D_{u1}S_{vc0}\\
H_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{N0} + \lambda I_{N1}][S_{vn0} + \lambda S_{vn1}]|_{(\lambda=0)} = I_{N0}S_{vn1} + I_{N1}S_{vn0}\\
L_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{T0} + \lambda I_{T1}][S_{vn0} + \lambda S_{vn1}]|_{(\lambda=0)} = I_{T0}S_{vn1} + I_{T1}S_{vn0}\\
Q_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{r0} + \lambda I_{r1}][S_{vn0} + \lambda S_{vn1}]|_{(\lambda=0)} = I_{r0}S_{vn1} + I_{r1}S_{vn0}\\
T_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[D_{u0} + \lambda D_{u1}][S_{vn0} + \lambda S_{vn1}]|_{(\lambda=0)} = D_{u0}S_{vn1} + D_{u1}S_{vn0}\\
U_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{N0} + \lambda I_{N1}][S_{uc0} + \lambda S_{uc1}]|_{(\lambda=0)} = I_{N0}S_{uc1} + I_{N1}S_{uc0}\\
V_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{T0} + \lambda I_{T1}][S_{uc0} + \lambda S_{uc1}]|_{(\lambda=0)} = I_{T0}S_{uc1} + I_{T1}S_{uc0}\\
W_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{r0} + \lambda I_{r1}][S_{uc0} + \lambda S_{uc1}]|_{(\lambda=0)} = I_{r0}S_{uc1} + I_{r1}S_{uc0}\\
X_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[D_{u0} + \lambda D_{u1}][S_{uc0} + \lambda S_{uc1}]|_{(\lambda=0)} = D_{u0}S_{uc1} + D_{u1}S_{uc0}\\
Y_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{N0} + \lambda I_{N1}][S_{un0} + \lambda S_{un1}]|_{(\lambda=0)} = I_{N0}S_{un1} + I_{N1}S_{un0}\\
Z_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{T0} + \lambda I_{T1}][S_{un0} + \lambda S_{un1}]|_{(\lambda=0)} = I_{T0}S_{un1} + I_{T1}S_{un0}\\
N_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[I_{r0} + \lambda I_{r1}][S_{un0} + \lambda S_{un1}]|_{(\lambda=0)} = I_{r0}S_{un1} + I_{r1}S_{un0}\\
M_1 &= \frac{1}{\Gamma(2)}\frac{d}{d\lambda}[D_{u0} + \lambda D_{u1}][S_{un0} + \lambda S_{un1}]|_{(\lambda=0)} = D_{u0}S_{un1} + D_{u1}S_{un0}
\end{aligned}\right\} \tag{27}$$

We substitute (22) for $n = 0$ into (23) and (24) to have the following;

$$\left.\begin{aligned}
\mathcal{L}\{S_0\} = \frac{g_1}{S}, \mathcal{L}\{S_{v0}\} = \frac{g_2}{S}, \mathcal{L}\{S_{u0}\} = \frac{g_3}{S}, \mathcal{L}\{S_{vc0}\} = \frac{g_4}{S}, \mathcal{L}\{S_{vn0}\} = \frac{g_5}{S}, \mathcal{L}\{S_{uc0}\} = \frac{g_6}{S},\\
\mathcal{L}\{S_{un0}\} = \frac{g_7}{S}, \mathcal{L}\{E_0\} = \frac{g_8}{S}, \mathcal{L}\{E_{Q0}\} = \frac{g_9}{S}, \mathcal{L}\{E_{T0}\} = \frac{g_{10}}{S}, \mathcal{L}\{I_{T0}\} = \frac{g_{11}}{S}, \mathcal{L}\{I_{i0}\} = \frac{g_{12}}{S},\\
\mathcal{L}\{I_{N0}\} = \frac{g_{13}}{S}, \mathcal{L}\{R_0\} = \frac{g_{14}}{S}, \mathcal{L}\{D_{u0}\} = \frac{g_{15}}{S}, \mathcal{L}\{S_{r0}\} = \frac{g_{16}}{S}, \mathcal{L}\{E_{r0}\} = \frac{g_{17}}{S}, \mathcal{L}\{I_{r0}\} = \frac{g_{18}}{S}
\end{aligned}\right\} \tag{28}$$

Similarly, for $n = 1$ to $n = n + 1$, we obtain the following:

$$\left.\begin{aligned}
\mathcal{L}\{S_1\} &= \frac{1}{S^\alpha}[\mathcal{L}\{P\check{}(\mu + r + K_1X)S_0 + \tau R_0\}] \\
\mathcal{L}\{S_{v1}\} = \frac{1}{S^\alpha}[\mathcal{L}\{rS_0\check{}(\mu + \lambda + K_2Z)S_{v0}\}], \mathcal{L}\{S_{u1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{K_1XS_0\check{}(\mu + \sigma + K_3Y)S_{u0}\}] \\
\mathcal{L}\{S_{vc1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\lambda S_{v0} - [\mu S_{vc0} + 1/N(m_1A_0 + m_2B_0 + m_3F_0 + m_4G_0)]\}] \\
\mathcal{L}\{S_{vn1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{K_2ZS_{v0} - [\mu S_{vn0} + 1/N(n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0)]\}] \\
\mathcal{L}\{S_{uc1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\sigma S_{u0} - [\mu S_{uc0} + 1/N(e_1U_0 + e_2V_0 + e_3W_0 + e_4X_0)]\}] \\
\mathcal{L}\{S_{un1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{K_3YS_{u0} - [\mu S_{un0} + 1/N(t_1Y_0 + t_2Z_0 + t_3N_0 + t_4M_0)]\}] \\
\mathcal{L}\{E_1\} = \frac{1}{S^\alpha}[\mathcal{L}\{1/N[m_1A_0 &+ m_2B_0 + m_3F_0 + m_4G_0 + n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0 + e_1U_0 + e_2V_0 \\
&+e_3W_0 + e_4X_0 + t_1Y_0 + t_2Z_0 + t_3N_0 + t_4M_0] - (\mu + \alpha_1 + \alpha_2)E_0\}] \\
\mathcal{L}\{E_{Q1}\} = \frac{1}{S^\alpha}[\mathcal{L}\{\alpha_1E_0\check{}(\mu + \theta_1)E_{Q0}\}], \mathcal{L}\{E_{T1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\alpha_2E_0\check{}(\mu + \theta_2 + \rho)E_{T0}\}] \\
\mathcal{L}\{I_{T1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\theta_1S_1E_{Q0} + \theta_2S_1E_{T0}\check{}(\mu + \delta_1 + J_2)I_{T0}\}] \\
\mathcal{L}\{I_{i1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\theta_1S_2E_{Q0} + \theta_2S_2E_{T0}\check{}(\mu + \delta_2 + J_1)I_{i0}\}] \\
\mathcal{L}\{I_{N1}\} = \frac{1}{S^\alpha}[\mathcal{L}\{\theta_1S_3E_{Q0} + \theta_2S_3E_{T0}\check{}(\mu + \delta_3)I_{N0}\}], \mathcal{L}\{R_1\} &= \frac{1}{S^\alpha}[\mathcal{L}\{J_1I_{i0} + J_2I_{T0} + \rho E_{T0} - (\mu - \tau)R_0\}] \\
\mathcal{L}\{D_{u1}\} = \frac{1}{S^\alpha}[\mathcal{L}\{\delta_3I_{N0} + \delta_2I_{i0} + \delta_1I_{T0} - qD_{u0}\}], \mathcal{L}\{S_{r1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\Lambda\check{}(\mu_r + \omega)S_{r0}\}] \\
\mathcal{L}\{E_{r1}\} = \frac{1}{S^\alpha}[\mathcal{L}\{\omega S_{r0}\check{}(\mu_r + \phi)E_{r0}\}], \mathcal{L}\{I_{r1}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\phi E_{r0}\check{}(\mu_r - \delta_4I_{r0}\}] \\
\ldots = \ldots, \mathcal{L}\{S_{(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{P\check{}(\mu + r + K_1X)S_n + \tau R_n\}] \\
\mathcal{L}\{S_{v(n+1)}\} = \frac{1}{S^\alpha}[\mathcal{L}\{rS_n\check{}(\mu + \lambda + K_2Z)S_{v_n}\}], \mathcal{L}\{S_{u(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{K_1XS_n\check{}(\mu + \sigma + K_3Y)S_{u_n}\}] \\
\mathcal{L}\{S_{vc(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\lambda S_{v_n} - [\mu S_{vc_n} + 1/N(m_1A_n + m_2B_n + m_3F_n + m_4G_n)]\}] \\
\mathcal{L}\{S_{vn(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{K_2ZS_{v_n} - [\mu S_{vn_n} + 1/N(n_1H_n + n_2L_n + n_3Q_n + n_4T_n)]\}] \\
\mathcal{L}\{S_{uc(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\sigma S_{u_n} - [\mu S_{uc_n} + 1/N(e_1U_n + e_2V_n + e_3W_n + e_4X_n)]\}] \\
\mathcal{L}\{S_{un(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{K_3YS_{u_n} - [\mu S_{un_n} + 1/N(t_1Y_n + t_2Z_n + t_3N_n + t_4M_n)]\}] \\
\mathcal{L}\{E_{(n+1)}\} = \frac{1}{S^\alpha}[\mathcal{L}\{1/N[m_1A_n &+ m_2B_n + m_3F_n + m_4G_n + n_1H_n + n_2L_n + n_3Q_n + n_4T_n + e_1U_n + e_2V_n \\
&+e_3W_n + e_4X_n + t_1Y_n + t_2Z_n + t_3N_n + t_4M_n] - (\mu + \alpha_1 + \alpha_2)E_n\}] \\
\mathcal{L}\{E_{Q(n+1)}\} = \frac{1}{S^\alpha}[\mathcal{L}\{\alpha_1E_n\check{}(\mu + \theta_1)E_{Q_n}\}], \mathcal{L}\{E_{T(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\alpha_2E_n\check{}(\alpha + \theta_2 + \rho)E_{T_n}\}] \\
\mathcal{L}\{I_{T(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\theta_1S_1E_{Q_n} + \theta_2S_1E_{T_n}\check{}(\mu + \delta_1 + J_2)I_{T_n}\}] \\
\mathcal{L}\{I_{i(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\theta_1S_2E_{Q_n} + \theta_2S_2E_{T_n}\check{}(\mu + \delta_2 + J_1)I_{i_n}\}] \\
\mathcal{L}\{I_{N(n+1)}\} = \frac{1}{S^\alpha}[\mathcal{L}\{\theta_1S_2E_{Q_n} + \theta_2S_3E_{T_n}\check{}(\mu + \delta_3)I_{N_n}\}], \mathcal{L}\{R_{(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{J_1I_{i_n} + J_2I_{T_n} + \rho E_{T_n} - \rho R_n\}] \\
\mathcal{L}\{D_{u(n+1)}\} = \frac{1}{S^\alpha}[\mathcal{L}\{\delta_3I_{N_n} + \delta_2I_{i_n} + \delta_1I_{T_n} - qD_n\}], \mathcal{L}\{S_{r(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\Lambda\check{}(\mu_r + \omega)S_{r_n}\}] \\
\mathcal{L}\{E_{r(n+1)}\} = \frac{1}{S^\alpha}[\mathcal{L}\{\omega S_{r_n}\check{}(/mu_r + \phi)E_{r_n}\}], \mathcal{L}\{I_{r(n+1)}\} &= \frac{1}{S^\alpha}[\mathcal{L}\{\phi E_{r_n}\check{}(\mu + \delta_4)I_{r_n}\}]
\end{aligned}\right\} \quad (29)$$

Evaluating the Laplace transforms on the right hand side of (29) we have the following;

$$\left.\begin{gathered}
\mathcal{L}\{S_1\} = \{P\check{}(\mu + r + K_1X)S_0 + \tau R_0\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{v1}\} = \{rS_0\check{}(\mu + \lambda + K_2Z)S_{v0}\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{S_{u1}\} = \{K_1XS_0\check{}(\mu + \sigma + K_3Y)S_{u0}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{vc1}\} = \{\lambda S_{v0} - [\mu S_{vc0} + 1/N(m_1A_0 + m_2B_0 + m_3F_0 + m_4G_0)]\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{vn1}\} = \{K_2ZS_{v0} - [\mu S_{vn0} + 1/N(n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0)]\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{uc1}\} = \{\sigma S_{u0} - [\mu S_{uc0} + 1/N(e_1U_0 + e_2V_0 + e_3W_0 + e_4X_0)]\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{un1}\} = \{K_3YS_{u0} - [\mu S_{un0} + 1/N(t_1Y_0 + t_2Z_0 + t_3N_0 + t_4M_0)]\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{E_1\} = \{1/N[m_1A_0 + m_2B_0 + m_3F_0 + m_4G_0 + n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0 + e_1U_0 + e_2V_0 \\
+e_3W_0 + e_4X_0 + t_1Y_0 + t_2Z_0 + t_3N_0 + t_4M_0] - (\mu + \alpha_1 + \alpha_2)E_0\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{E_{Q1}\} = \{\alpha_1E_0\check{}(\mu + \theta_1)E_{Q0}\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{E_{T1}\} = \{\alpha_2E_0\check{}(\mu + \theta_2 + \rho)E_{T0}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{I_{T1}\} = \{\theta_1S_1E_{Q0} + \theta_2S_1E_{T0}\check{}(\mu + \delta_1 + J_2)I_{T0}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{I_{i1}\} = \{\theta_1S_2E_{Q0} + \theta_2S_2E_{T0}\check{}(\mu + \delta_2 + J_1)I_{i0}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{I_{N1}\} = \{\theta_1S_3E_{Q0} + \theta_2S_3E_{T0}\check{}(\mu + \delta_3)I_{N0}\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{R_1\} = \{J_1I_{i0} + J_2I_{T0} + \rho E_{T0} - (\mu - \tau)R_0\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{D_{u1}\} = \{\delta_3I_{N0} + \delta_2I_{i0} + \delta_1I_{T0} - qD_{u0}\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{S_{r1}\} = \{\Lambda\check{}(\mu_r + \omega)S_{r0}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{E_{r1}\} = \{\omega S_{r0}\check{}(\mu_r + \phi)E_{r0}\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{I_{r1}\} = \{\phi E_{r0}\check{}(\mu_r - \delta_4I_{r0}\}\frac{1}{S^{\alpha+1}} \\
\ldots = \ldots, \mathcal{L}\{S_{(n+1)}\} = \{P\check{}(\mu + r + K_1X)S_n + \tau R_n\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{v(n+1)}\} = \{rS_n\check{}(\mu + \lambda + K_2Z)S_{v_n}\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{S_{u(n+1)}\} = \{K_1XS_n\check{}(\mu + \sigma + K_3Y)S_{u_n}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{vc(n+1)}\} = \{\lambda S_{v_n} - [\mu S_{vc_n} + 1/N(m_1A_n + m_2B_n + m_3F_n + m_4G_n)]\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{vn(n+1)}\} = \{K_2ZS_{v_n} - [\mu S_{vn_n} + 1/N(n_1H_n + n_2L_n + n_3Q_n + n_4T_n)]\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{uc(n+1)}\} = \{\sigma S_{u_n} - [\mu S_{uc_n} + 1/N(e_1U_n + e_2V_n + e_3W_n + e_4X_n)]\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{S_{un(n+1)}\} = \{K_3YS_{u_n} - [\mu S_{un_n} + 1/N(t_1Y_n + t_2Z_n + t_3N_n + t_4M_n)]\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{E_{(n+1)}\} = \{1/N[m_1A_n + m_2B_n + m_3F_n + m_4G_n + n_1H_n + n_2L_n + n_3Q_n + n_4T_n + e_1U_n + e_2V_n+ \\
e_3W_n + e_4X_n + t_1Y_n + t_2Z_n + t_3N_n + t_4M_n] - (\mu + \alpha_1 + \alpha_2)E_n\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{E_{Q(n+1)}\} = \{\alpha_1E_n\check{}(\mu + \theta_1)E_{Q_n}\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{E_{T(n+1)}\} = \{\alpha_2E_n\check{}(\alpha + \theta_2 + \rho)E_{T_n}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{I_{T(n+1)}\} = \{\theta_1S_1E_{Q_n} + \theta_2S_1E_{T_n}\check{}(\mu + \delta_1 + J_2)I_{T_n}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{I_{i(n+1)}\} = \{\theta_1S_2E_{Q_n} + \theta_2S_2E_{T_n}\check{}(\mu + \delta_2 + J_1)I_{i_n}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{I_{N(n+1)}\} = \{\theta_1S_2E_{Q_n} + \theta_2S_3E_{T_n}\check{}(\mu + \delta_3)I_{N_n}\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{R_{(n+1)}\} = \{J_1I_{i_n} + J_2I_{T_n} + \rho E_{T_n} - \rho R_n\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{D_{u(n+1)}\} = \{\delta_3I_{N_n} + \delta_2I_{i_n} + \delta_1I_{T_n} - qD_n\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{S_{r(n+1)}\} = \{\Lambda\check{}(\mu_r + \omega)S_{r_n}\}\frac{1}{S^{\alpha+1}} \\
\mathcal{L}\{E_{r(n+1)}\} = \{\omega S_{r_n}\check{}(/mu_r + \phi)E_{r_n}\}\frac{1}{S^{\alpha+1}}, \mathcal{L}\{I_{r(n+1)}\} = \{\phi E_{r_n}\check{}(\mu + \delta_4)I_{r_n}\}\frac{1}{S^{\alpha+1}}
\end{gathered}\right\} \tag{30}$$

Taking the Laplace inverse transform of (30) we have;

$$
\left.
\begin{aligned}
S_1 &= \{P\breve{}(\mu + r + K_1X)S_0 + \tau R_0\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
S_{v1} &= \{rS_0\breve{}(\mu + \lambda + K_2Z)S_{v0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}, S_{u1} = \{K_1XS_0\breve{}(\mu + \sigma + K_3Y)S_{u0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
S_{vc1} &= \{\lambda S_{v0} - [\mu S_{vc0} + 1/N(m_1A_0 + m_2B_0 + m_3F_0 + m_4G_0)]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
S_{vn1} &= \{K_2ZS_{v0} - [\mu S_{vn0} + 1/N(n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0)]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
S_{uc1} &= \{\sigma S_{u0} - [\mu S_{uc0} + 1/N(e_1U_0 + e_2V_0 + e_3W_0 + e_4X_0)]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
S_{un1} &= \{K_3YS_{u0} - [\mu S_{un0} + 1/N(t_1Y_0 + t_2Z_0 + t_3N_0 + t_4M_0)]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
E_1 &= \{1/N[m_1A_0 + m_2B_0 + m_3F_0 + m_4G_0 + n_1H_0 + n_2L_0 + n_3Q_0 + n_4T_0 + e_1U_0 + e_2V_0\\
&\quad + e_3W_0 + e_4X_0 + t_1Y_0 + t_2Z_0 + t_3N_0 + t_4M_0] - (\mu + \alpha_1 + \alpha_2)E_0\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
E_{Q1} &= \{\alpha_1E_0\breve{}(\mu + \theta_1)E_{Q0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}, E_{T1} = \{\alpha_2E_0\breve{}(\mu + \theta_2 + \rho)E_{T0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
I_{T1} &= \{\theta_1S_1E_{Q0} + \theta_2S_1E_{T0}\breve{}(\mu + \delta_1 + J_2)I_{T0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
I_{i1} &= \{\theta_1S_2E_{Q0} + \theta_2S_2E_{T0}\breve{}(\mu + \delta_2 + J_1)I_{i0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
I_{N1} &= \{\theta_1S_3E_{Q0} + \theta_2S_3E_{T0}\breve{}(\mu + \delta_3)I_{N0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}, R_1 = \{J_1I_{i0} + J_2I_{T0} + \rho E_{T0} - (\mu - \tau)R_0\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
D_{u1} &= \{\delta_3I_{N0} + \delta_2I_{i0} + \delta_1I_{T0} - qD_{u0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}, S_{r1} = \{\Lambda\breve{}(\mu_r + \omega)S_{r0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
E_{r1} &= \{\omega S_{r0}\breve{}(\mu_r + \phi)E_{r0}\}\frac{t^\alpha}{\Gamma(\alpha+1)}, I_{r1} = \{\phi E_{r0}\breve{}(\mu_r - \delta_4I_{r0}\}\\
\ldots &= \ldots, S_{(n+1)} = \{P\breve{}(\mu + r + K_1X)S_n + \tau R_n\}\frac{t^\alpha}{\Gamma(\alpha+1)}S_{v(n+1)} = \{rS_n\breve{}(\mu + \lambda + K_2Z)S_{v_n}\}\\
S_{u(n+1)} &= \{K_1XS_n\breve{}(\mu + \sigma + K_3Y)S_{u_n}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
S_{vc(n+1)} &= \{\lambda S_{v_n} - [\mu S_{vc_n} + 1/N(m_1A_n + m_2B_n + m_3F_n + m_4G_n)]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
S_{vn(n+1)} &= \{K_2ZS_{v_n} - [\mu S_{vn_n} + 1/N(n_1H_n + n_2L_n + n_3Q_n + n_4T_n)]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
S_{uc(n+1)} &= \{\sigma S_{u_n} - [\mu S_{uc_n} + 1/N(e_1U_n + e_2V_n + e_3W_n + e_4X_n)]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
S_{un(n+1)} &= \{K_3YS_{u_n} - [\mu S_{un_n} + 1/N(t_1Y_n + t_2Z_n + t_3N_n + t_4M_n)]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
E_{(n+1)} &= \{1/N[m_1A_n + m_2B_n + m_3F_n + m_4G_n + n_1H_n + n_2L_n + n_3Q_n + n_4T_n + e_1U_n + e_2V_n + e_3W_n\\
&\quad + e_4X_n + t_1Y_n + t_2Z_n + t_3N_n + t_4M_n] - (\mu + \alpha_1 + \alpha_2)E_n\}\frac{t^\alpha}{\Gamma(\alpha+1)}, E_{Q(n+1)} = \{\alpha_1E_n\breve{}(\mu + \theta_1)E_{Q_n}\}\\
&\frac{t^\alpha}{\Gamma(\alpha+1)}, E_{T(n+1)} = \{\alpha_2E_n\breve{}(\alpha + \theta_2 + \rho)E_{T_n}\}\frac{t^\alpha}{\Gamma(\alpha+1)}, I_{T(n+1)} = \{\theta_1S_1E_{Q_n} + \theta_2S_1E_{T_n}\breve{}(\mu + \delta_1 + J_2)I_{T_n}\}\\
&\frac{t^\alpha}{\Gamma(\alpha+1)}, I_{i(n+1)} = \{\theta_1S_2E_{Q_n} + \theta_2S_2E_{T_n}\breve{}(\mu + \delta_2 + J_1)I_{i_n}\}\frac{t^\alpha}{\Gamma(\alpha+1)}, I_{N(n+1)} =\\
&\{\theta_1S_2E_{Q_n} + \theta_2S_3E_{T_n}\breve{}(\mu + \delta_3)I_{N_n}\}\frac{t^\alpha}{\Gamma(\alpha+1)}, R_{(n+1)} = \{J_1I_{i_n} + J_2I_{T_n} + \rho E_{T_n} - \rho R_n\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
D_{u(n+1)} &= \{\delta_3I_{N_n} + \delta_2I_{i_n} + \delta_1I_{T_n} - qD_n\}\frac{t^\alpha}{\Gamma(\alpha+1)}, S_{r(n+1)} = \{\Lambda\breve{}(\mu_r + \omega)S_{r_n}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\
E_{r(n+1)} &= \{\omega S_{r_n}\breve{}(/mu_r + \phi)E_{r_n}\}\frac{t^\alpha}{\Gamma(\alpha+1)}, I_{r(n+1)} = \{\phi E_{r_n}\breve{}(\mu + \delta_4)I_{r_n}\}\frac{t^\alpha}{\Gamma(\alpha+1)}
\end{aligned}
\right\}
\tag{31}
$$

Recalling and applying the initial conditions (2) we have the following;

$$\left.\begin{aligned} S_0 = g_1, S_{v0} = g_2, S_{u0} = g_3, S_{vc0} = g_4,\\ S_{vn0} = g_5, S_{uc0} = g_6, S_{un0} = g_7, E_0 = g_8, E_{Q0} = g_9,\\ E_{T0} = g_{10}, I_{T0} = g_{11}, I_{i0} = g_{12}, I_{N0} = g_{13}, R_0 = g_{14},\\ D_{u0} = g_{15}, S_{r0} = g_{16}, E_{r0} = g_{17}, I_{r0} = g_{18}. \end{aligned}\right\} \tag{32}$$

Substituting all these conditions we finally obtain the Ebola fractional model solution in the form of infinite series (23)using LADM as follows;

$$\left.\begin{aligned} S(t) = S_0 + S_1 + S_2 + \dots\\ S(t) = g_1 + \{P\check{}(\mu + r + K_1X)g_1 + \tau g_{14}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{P\check{}(\mu + r + K_1X)S_1 + \tau R_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{33}$$

$$\left.\begin{aligned} S_v(t) = S_{v0} + S_{v1} + S_{v2} + \dots\\ S_v(t) = g_2 + \{rg_1\check{}(\mu + \lambda + K_2Z)g_2\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{rS_1\check{}(\mu + \lambda + K_2Z)S_{v1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{34}$$

$$\left.\begin{aligned} S_u(t) = S_{u0} + S_{u1} + S_{u2} + \dots\\ S_u(t) = g_3 + \{K_1Xg_1\check{}(\mu + \sigma + K_3Y)g_3\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{K_1XS_1\check{}(\mu + \sigma + K_3Y)S_{u1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{35}$$

$$\left.\begin{aligned} S_{vc}(t) = S_{vc0} + S_{vc1} + S_{vc2} + \dots\\ S_{vc}(t) = g_4 +\\ \{\lambda g_2 - [\mu g_4 + 1/N(m_1g_4g_{13} + m_2g_4g_{11} + m_3g_4g_{18} + m_4g_4g_{15})]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{\lambda S_{v1} - [\mu S_{vc1} + 1/N(m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1)]\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{36}$$

$$\left.\begin{aligned} S_{vn}(t) = S_{vn0} + S_{vn1} + S_{vn2} + \dots\\ S_{vn}(t) = g_5 +\\ \{K_2Zg_2 - [\mu g_5 + 1/N(n_1g_5g_{13} + n_2g_5g_{11} + n_3g_5g_{18} + n_4g_5g_{15})]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{K_2ZS_{v1} - [\mu S_{vn1} + 1/N(n_1H_1 + n_2L_1 + n_3Q_1 + n_4T_1)]\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{37}$$

$$\left.\begin{aligned} S_{uc}(t) = S_{uc0} + S_{uc1} + S_{uc2} + \dots\\ S_{uc}(t) = g_6 +\\ \{\sigma g_3 - [\mu g_6 + 1/N(e_1g_6g_{13} + e_2g_6g_{11} + e_3g_6g_{18} + e_4g_6g_{15})]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{\sigma S_{u1} - [\mu S_{uc1} + 1/N(e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1)]\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{38}$$

$$\left.\begin{aligned}S_{un}(t) = S_{un0} + S_{un1} + S_{un2} + \dots\\ S_{un}(t) = g_7 +\\ \{K_3 Y g_3 - [\mu g_7 + 1/N(t_1 g_7 g_{13} + t_2 g_7 g_{11} + t_3 g_7 g_{18} + t_4 g_7 g_{15})]\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{K_3 Y S_{u1} - [\mu S_{un1} + 1/N(t_1 Y_1 + t_2 Z_1 + t_3 N_1 + t_4 M_1)]\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots\end{aligned}\right\} \tag{39}$$

$$\left.\begin{aligned}E(t) = E_0 + E_1 + E_2 + \dots\\ E(t) = g_8 + \{1/N[C_1 + C_2 + C_3 + C_4] - (\mu + \alpha_1 + \alpha_2)g_8\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{1/N[C_5 + C_6 + C_7 + C_8] - (\mu + \alpha_1 + \alpha_2)E_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots\end{aligned}\right\} \tag{40}$$

Where;

$$\left.\begin{aligned}C_1 = m_1 g_4 g_{13} + m_2 g_4 g_{11} + m_3 g_4 g_{18} + m_4 g_4 g_{15}\\ C_2 = n_1 g_5 g_{13} + n_2 g_5 g_{11} + n_3 g_5 g_{18} + n_4 g_5 g_{15}\\ C_3 = e_1 g_6 g_{13} + e_2 g_6 g_{11} + e_3 g_6 g_{18} + e_4 g_6 g_{15}\\ C_4 = t_1 g_7 g_{13} + t_2 g_7 g_{11} + t_3 g_6 g_{18} + t_4 g_7 g_{15}\\ C_5 = m_1 A_1 + m_2 B_1 + m_3 F_1 + m_4 G_1\\ C_6 = n_1 H_1 + n_2 L_1 + n_3 Q_1 + n_4 T_1\\ C_7 = e_1 U_1 + e_2 V_1 + e_3 W_1 + e_4 X_1\\ C_8 = t_1 Y_1 + t_2 Z_1 + t_3 N_1 + t_4 M_1\end{aligned}\right\} \tag{41}$$

$$\left.\begin{aligned}E_Q(t) = E_{Q0} + E_{Q1} + E_{Q2} + \dots\\ E_Q(t) = g_9 +\\ \{\alpha_1 g_8 \breve{} (\mu + \theta_1) g_9\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\alpha_1 E_1 \breve{} (\mu + \theta_1) E_{Q1}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots\end{aligned}\right\} \tag{42}$$

$$\left.\begin{aligned}E_T(t) = E_{T0} + E_{T1} + E_{T2} + \dots\\ E_T(t) = g_{10} +\\ \{\alpha_2 g_8 \breve{} (\mu + \theta_2 + \rho) g_{10}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{\alpha_2 E_1 \breve{} (\mu + \theta_2 + \rho) E_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots\end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned}I_T(t) = I_{T0} + I_{T1} + I_{T2} + \dots\\ I_T(t) = g_{11} +\\ \{\theta_1 S_1 g_9 + \theta_2 S_1 g_{10} \breve{} (\mu + \delta_1 + J_2) g_{11}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{\theta_1 S_1 E_{Q1} + \theta_2 S_1 E_{T1} \breve{} (\mu + \delta_1 + J_2) I_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots\end{aligned}\right\} \tag{44}$$

$$\left.\begin{aligned}I_i(t) = I_{i0} + I_{i1} + I_{i2} + \dots\\ I_i(t) = g_{12} +\\ \{\theta_1 S_2 g_9 + \theta_2 S_2 g_{10} \breve{} (\mu + \delta_2 + J_1) g_{12}\}\frac{t^\alpha}{\Gamma(\alpha+1)}\\ +\{\theta_1 S_2 E_{Q1} + \theta_2 S_2 E_{T1} \breve{} (\mu + \delta_2 + J_1) I_{i1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \dots\end{aligned}\right\} \tag{45}$$

$$\left.\begin{aligned} I_N(t) = I_{N0} + I_{N1} + I_{N2} + \dots \\ I_N(t) = g_{13} + \\ \{\theta_1 S_3 g_9 + \theta_2 S_3 g_{10} \breve{} (\mu + \delta_3) g_{13}\} \frac{t^\alpha}{\Gamma(\alpha+1)} \\ + \{\theta_1 S_3 E_{Q1} + \theta_2 S_3 E_{T1} \breve{} (\mu + \delta_3) I_{N1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{46}$$

$$\left.\begin{aligned} R(t) = R_0 + R_1 + R_2 + \dots \\ R(t) = g_{14} + \\ \{J_1 g_{12} + J_2 g_{11} + \rho g_{10} - \tau g_{14}\} \frac{t^\alpha}{\Gamma(\alpha+1)} \\ + \{J_1 I_{i1} + J_2 I_{T1} + \rho E_{T1} - \tau R_1\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{47}$$

$$\left.\begin{aligned} D_u(t) = D_{u0} + D_{u1} + D_{u2} + \dots \\ D_u(t) = g_{15} + \\ \{\delta_3 g_{13} + \delta_2 g_{12} + \delta_1 g_{11} - q g 15\} \frac{t^\alpha}{\Gamma(\alpha+1)} \\ + \{\delta_3 I_{N1} + \delta_2 I_{i1} + \delta_1 I_{T1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{48}$$

$$\left.\begin{aligned} S_r(t) = S_{r0} + S_{r1} + S_{r2} + \dots \\ S_r(t) = g_{16} + \\ \{\Lambda \breve{} (\mu_r + \omega) g_{16}\} \frac{t^\alpha}{\Gamma(\alpha+1)} \\ + \{\Lambda \breve{} (\mu_r + \omega) S_{r1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{49}$$

$$\left.\begin{aligned} E_r(t) = E_{r0} + E_{r1} + E_{r2} + \dots \\ E_r(t) = g_{17} + \\ \{\omega g_{16} \breve{} (\mu_r + \phi) g_{17}\} \frac{t^\alpha}{\Gamma(\alpha+1)} \\ + \{\omega S_{r1} \breve{} (\mu_r + \phi) E_{r1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{50}$$

$$\left.\begin{aligned} I_r(t) = I_{r0} + I_{r1} + I_{r2} + \dots \\ I_r(t) = g_{18} + \\ \{\phi g_{17} \breve{} (\mu_r + \delta_4) g_{18}\} \frac{t^\alpha}{\Gamma(\alpha+1)} \\ + \{\phi E_{r1} \breve{} (\mu + \delta_4) I_{r1}\} \frac{t^\alpha}{\Gamma(\alpha+1)} + \dots \end{aligned}\right\} \tag{51}$$

Thus, the solution in series form is given as follows;

$$
\left.\begin{aligned}
S(t) &= g_1 + \{P\breve{}(\mu + r + K_1X)g_1 + \tau g_{14}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{P\breve{}(\mu + r + K_1X)S_1 + \tau R_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
S_v(t) &= g_2 + \{rg_1\breve{}(\mu + \lambda + K_2Z)g_2\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{rS_1\breve{}(\mu + \lambda + K_2Z)S_{v1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
S_u(t) &= g_3 + \{K_1Xg_1\breve{}(\mu + \sigma + K_3Y)g_3\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{K_1XS_1\breve{}(\mu + \sigma + K_3Y)S_{u1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
S_{vc}(t) &= g_4 + \{\lambda g_2 - [\mu g_4 + 1/N(m_1g_4g_{13} + m_2g_4g_{11} + m_3g_4g_{18} + m_4g_4g_{15})]\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
&\quad + \{\lambda S_{v1} - [\mu S_{vc1} + 1/N(m_1A_1 + m_2B_1 + m_3F_1 + m_4G_1)]\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
S_{vn}(t) &= g_5 + \{K_2Zg_2 - [\mu g_5 + 1/N(n_1g_5g_{13} + n_2g_5g_{11} + n_3g_5g_{18} + n_4g_5g_{15})]\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
&\quad + \{K_2ZS_{v1} - [\mu S_{vn1} + 1/N(n_1H_1 + n_2L_1 + n_3Q_1 + n_4T_1)]\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
S_{uc}(t) &= g_6 + \{\sigma g_3 - [\mu g_6 + 1/N(e_1g_6g_{13} + e_2g_6g_{11} + e_3g_6g_{18} + e_4g_6g_{15})]\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
&\quad + \{\sigma S_{u1} - [\mu S_{uc1} + 1/N(e_1U_1 + e_2V_1 + e_3W_1 + e_4X_1)]\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
S_{un}(t) &= g_7 + \{K_3Yg_3 - [\mu g_7 + 1/N(t_1g_7g_{13} + t_2g_7g_{11} + t_3g_7g_{18} + t_4g_7g_{15})]\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
&\quad + \{K_3YS_{u1} - [\mu S_{un1} + 1/N(t_1Y_1 + t_2Z_1 + t_3N_1 + t_4M_1)]\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
E(t) &= g_8 + \{1/N[C_1 + C_2 + C_3 + C_4] - (\mu + \alpha_1 + \alpha_2)g_8\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
&\quad + \{1/N[C_5 + C_6 + C_7 + C_8] - (\mu + \alpha_1 + \alpha_2)E_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
E_Q(t) &= g_9 + \{\alpha_1g_8\breve{}(\mu + \theta_1)g_9\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \alpha_1E_1\breve{}(\mu + \theta_1)E_{Q1}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
E_T(t) &= g_{10} + \{\alpha_2g_8\breve{}(\mu + \theta_2 + \rho)g_{10}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{\alpha_2E_1\breve{}(\mu + \theta_2 + \rho)E_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
I_T(t) &= g_{11} + \{\theta_1S_1g_9 + \theta_2S_1g_{10}\breve{}(\mu + \delta_1 + J_2)g_{11}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
&\quad + \{\theta_1S_1E_{Q1} + \theta_2S_1E_{T1}\breve{}(\mu + \delta_1 + J_2)I_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
I_i(t) &= g_{12} + \{\theta_1S_2g_9 + \theta_2S_2g_{10}\breve{}(\mu + \delta_2 + J_1)g_{12}\}\frac{t^\alpha}{\Gamma(\alpha+1)} \\
&\quad + \{\theta_1S_2E_{Q1} + \theta_2S_2E_{T1}\breve{}(\mu + \delta_2 + J_1)I_{i1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
I_N(t) &= g_{13} + \{\theta_1S_3g_9 + \theta_2S_3g_{10}\breve{}(\mu + \delta_3)g_{13}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{\theta_1S_3E_{Q1} + \theta_2S_3E_{T1}\breve{}(\mu + \delta_3)I_{N1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
R(t) &= g_{14} + \{J_1g_{12} + J_2g_{11} + \rho g_{10} - \tau g_{14}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{J_1I_{i1} + J_2I_{T1} + \rho E_{T1} - \tau R_1\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
D_u(t) &= g_{15} + \{\delta_3g_{13} + \delta_2g_{12} + \delta_1g_{11} - qg15\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{\delta_3I_{N1} + \delta_2I_{i1} + \delta_1I_{T1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
S_r(t) &= g_{16} + \{\Lambda\breve{}(\mu_r + \omega)g_{16}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{\Lambda\breve{}(\mu_r + \omega)S_{r1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
E_r(t) &= g_{17} + \{\omega g_{16}\breve{}(\mu_r + \phi)g_{17}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{\omega S_{r1}\breve{}(\mu_r + \phi)E_{r1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots \\
I_r(t) &= g_{18} + \{\phi g_{17}\breve{}(\mu_r + \delta_4)g_{18}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \{\phi E_{r1}\breve{}(\mu + \delta_4)I_{r1}\}\frac{t^\alpha}{\Gamma(\alpha+1)} + \ldots
\end{aligned}\right\} \tag{52}
$$

The Laplace Adomian Decomposition method reduces a solution in form of an infinite series that further converges easily to the exact value. The method unlike other numerical techniques does not require any perturbation nor predefined step size. Also, the method does not require discretization and linearization and as such the results obtained from it are more effective and realistic.

5 Numerical simulations

In this section, we consider the numerical simulation of the Ebola fractional model. Using the initial values and the parameter values presented in Table 3 and Table 4 respectively which where also used in [2, 14, 15, 20]

Table 3: **Ebola Fractional Model Variable Initial Values**

Model Variables	Initial Values
S_0	4396521
S_{v0}	1758608
S_{u0}	2637913
S_{vc0}	527582
S_{vn0}	1231026
S_{uc0}	1582748
S_{un0}	1055165
E_0	0
E_{Q0}	74
E_{T0}	0
I_{T0}	500
I_{i0}	800
I_{N0}	400
R_0	0
D_{u0}	24
S_{r0}	6000
E_{r0}	100
I_{r0}	0

Table 4: **Ebola Fractional Model Variable Initial Values**

Parameters	Values	Parameters	Values
Λ	100	ω	0.8
ω	0.5	ϕ	0.6
μ_r	0.08	δ_4	0.3110
P	400	μ	0.0000246575
r	0.05	K_1	0.895
λ	0.02	K_2	0.765
σ	0.07	K_3	0.0886
S_1	0.2257	θ_1	0.08333
S_2	0.25	θ_2	0.014
S_3	0.148	δ_1	0.11386
J_1	0.105186	δ_2	0.0901
J_2	0.17	δ_3	0.2443
K_3	0.02	α_1	0.07143
ρ	0.0314862	α_2	0.02741
τ	0.06	q	0.5
m_1	0.03	m_2	0.00003
m_3	0.01	m_4	0.0003
m_5	0.003	n_1	0.04
n_2	0.00004	n_3	0.04
n_4	0.09	n_5	0.00009
e_1	0.07	e_2	0.00007
e_3	0.0001	e_4	0.0007
e_5	0.007	t_1	0.09
t_2	0.00009	t_3	0.0002
t_4	0.0009	t_5	0.009

6 Discussions and conclusion

Here, we discuss the numerical simulation of our Ebola fractional order model so as to compare with the classical order. All the variables of our Ebola fractional order model are presented in Figure 2, Figure 3, Figure 4, Figure 5, Figure 6 and Figure 7 with variation in the value of the model fractional

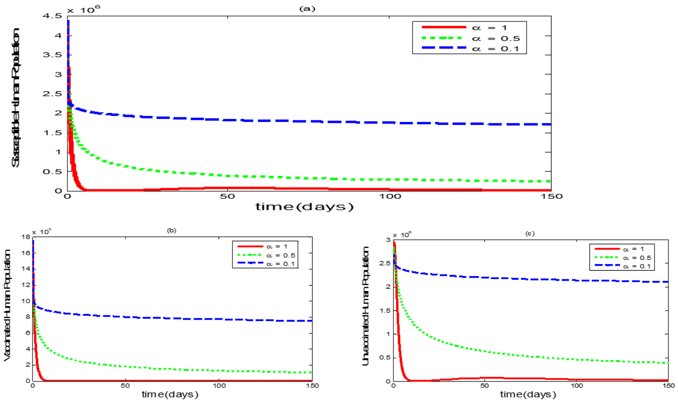

Figure 2: (a) Susceptible Human Population with Variation of α. (b) Vaccinated Human Population with Variation of α. (c) Unvaccinated Human Population with Variation of α.

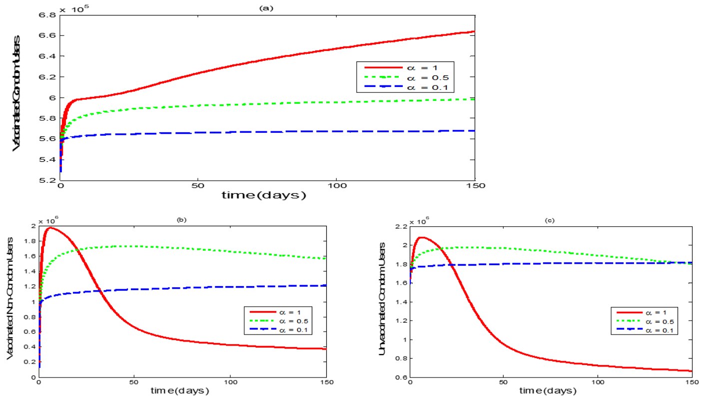

Figure 3: (a) Vaccinated Condom Users Population with Variation of α. (b) Vaccinated Non-Condom Users Population with Variation of α. (c) Unvaccinated Condom-Users Population with Variation of α.

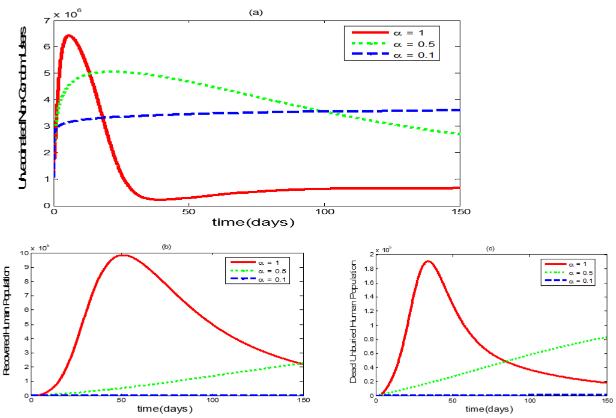

Figure 4: (a) Unvaccinated Non-Condom Users Population with Variation of α. (b) Recovered Human Population with Variation of α. (c) Dead Unburied Human Population with Variation of α.

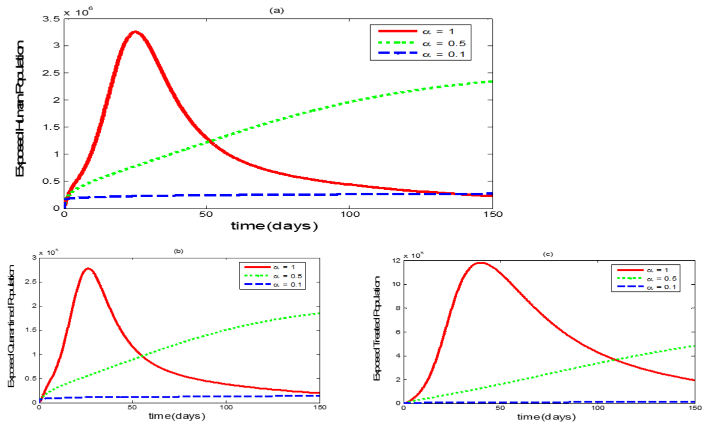

Figure 5: (a) Exposed Human Population with Variation of α. (b) Exposed Quarantined Human Population with Variation of α. (c) Exposed Treated Human Population with Variation of α.

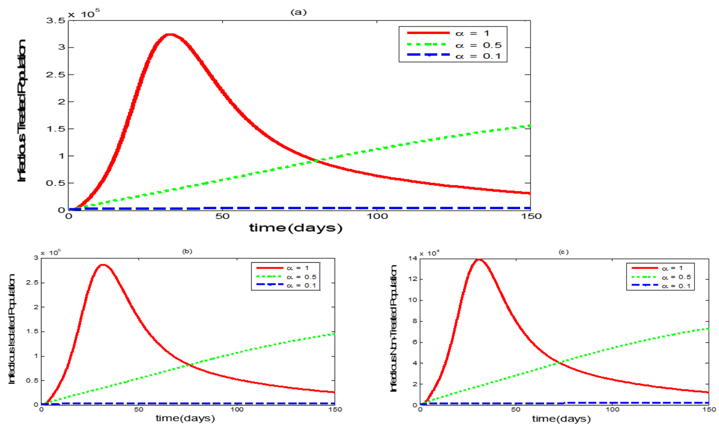

Figure 6: (a) Infectious Treated Human Population with Variation of α. (b) Infectious Isolated Human Population with Variation of α. (c) Infectious Non-Treated Human Population with Variation of α.

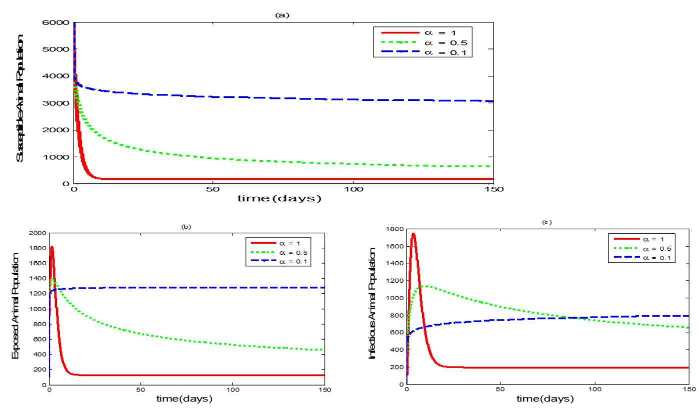

Figure 7: (a) Susceptible Animal Population with Variation of α. (b) Exposed Animal Population with Variation of α. (c) Infectious Animal Population with Variation of α.

order α. Figure 2 – Figure 7 shows that the Ebola fractional order model has more degrees of freedom as such the order α can then be varied in order to determine different degrees of responses of the different Variables that makes up the Ebola Fractional order model. This is seen when we compared the model fractional order (i.e.$\alpha = 0.5, 0.1$) with the model classical case where $\alpha = 1$. The graphs have shown the effect of α when $\alpha < 1$ and when $\alpha = 1$ which agrees with our earlier outcome that the Ebola fractional order model allows for determination of responses of α at different levels when compared with the Ebola classical order model. The Ebola fractional order model indeed assists us to obtain different outcomes in real time of the different classes of our Ebola fractional order model as shown in Figure 2 – Figure 7. We actually chose carefully our initial values and time to avoid obtaining negative population of human or animal which will not be meaningful biologically. The accuracy of a model numerical solution depends on the values of parameters and variables. To this end most of the parameter values employed in our analysis were not assumed but picked from existing literature on Ebola virus disease. Also, Since the main aim of this work is to determine the approximate solution of the formulated model, we did not boarder to calibrate to real data for Ebola virus. Secondly, getting real data for data fitting was challenging as the data was not always available on demand. So, we present in this work the approximate solution of the model using hypothetical data which the sources were acknowledged.

6.1 The Laplace-Adomian Decomposition Method (LADM) convergence analysis

The solution of (24) is expressed in form of infinite series as seen in equation (25) which converges uniformly to its exact solution. To verify the convergence of the series (24), we apply the method used in [25]. For sufficient conditions of convergence of the Laplace Adomian Decomposition Method, we present the following theorem:

Theorem 1 (see [33])

Let K be a Banach space and

$X :\to K$ be a constructive nonlinear operator such that for all

$(k), (k) \in K, \|X(k) - X(k')\| \leq m\|(k) - (k)'\|, 0 < m < 1$. Then, X has a unique point k such that $Xk = k$ where $k = (S, S_v, S_u, S_{vc}, S_{vn}, S_{uc}, S_{un}, E, E_Q, E_T, I_T, I_i,$

$I_N, R, D_u, S_r, E_r, I_r)$. The series can then be written by applying the Adomian decomposition method as follows:

$k_n = \sum_{i=1}^{n-1} k_i, n = 1, 2, 3 \ldots$

and assume that

$k_0 \in Z_r(k)$

where

$Z_r(k) = \{k' \in K : \|k' - k\| < r\}$

then we have as follows:

(i) $k_n \in Z_r(k)$

(ii) $\lim_{n\to\infty} k_n = k$

proof

To prove (i) we invoke mathematical induction

when $n = 1$ we have

$$\|k_0 - k\| = \|X(k_0) - X(k)\| \leq \|k_0 - k\| \tag{53}$$

if this is true for $y - 1$ then

$$\|k_0 - k\| \leq m^{y-1}\|k_0 - k\| \tag{54}$$

This gives the following

$$\|k_y - k\| = \|X(k_{y-1}) - X(k)\| \leq m\|k_{m-1} - k\| \leq m^n\|k_0 - k\| \tag{55}$$

Therefore,

$$\|k_m - k\| \leq m^n\|k_0 - k\| \leq m^n r < r \tag{56}$$

Which implies that $K_n \in Z_r(k)$.
Also, for condition (ii), we have that since

$$\|k_m - k\| \leq m^n \|k_0 - k\| \tag{57}$$

and

$$\lim_{n\to\infty} m^n = 0 \tag{58}$$

we can write

$$\lim_{n\to\infty} k_n = k \tag{59}$$

We thus prove convergence of the Laplace Adomian Decomposition method using the Cauchy-Kovalevskaya theorem for differential equations with analytic vector fields and then obtained a new result on the convergence rate of the Laplace Adomian Decomposition Method.

6.2 Conclusion

The Ebola Virus disease which is now preventable is still persisting and claiming the lives of many around some parts of the world. In this modified model, we studied the dynamics of Ebola transmission incorporating quarantine with contact tracing,Isolation, treatment, vaccine, and protection (condom use). We formulated an Ebola Virus fractional order mathematical model. through the analysis of the formulated model we found out that the disease (Ebola Virus) can go into extinction by maintaining the value of reproduction number (B) to be less than one through combined intervention of all our control measures put together. To solve the formulated model, we developed a numerical scheme that gives an approximate solution of our Ebola fractional order model using the Laplace–Adomian decomposition method. The solution obtained showed a good agreement with other numerical solutions. We also showed that approximate solution obtained converges to an exact solution.From our numerical simulation we finally, showed that the Ebola fractional order gives more degrees of freedom as the order can be varied to show responses by the different classes in real time. From the outcome it can also be seen that the reduction of the fractional order α gives a reduction in the population size of each of the considered classes, which by implication shows that the fractional order has a direct implication on our model . We herein have been able to show that the LADM can be used to solve an Ebola fractional order model with control measures of eighteen compartment after analysis which to the best of our knowledge has not been done before in literature. We also hope to compare our result with that of homotopy perturbation method and other methods in time for us to have a strong and better recommendation concerning the best method among them. We thus conclude that the Ebola fractional order model with eighteen equation set solved here using the Laplace–Adomian decomposition method gives a higher degree of freedom when compared to the Ebola classical order model. Therefore,linear and nonlinear ordinary and partial differential large equation set of fractional and classical orders can be solved using the Laplace–Adomian decomposition method.
From the result of our qualitative analysis, it was concluded that Ebola virus disease could be controlled provided the basic reproduction number could be maintained to a level below unity. The disease equilibrium points or state of the model was confirmed to be stable if the basic reproduction number is less than one, this epidemiologically implies that the disease will be eradicated from the population with respect to the initial population considered. It was further observed that Ebola virus disease would go into extinction through combined intervention of the control measures incorporated in our model. To this end, if all the control measures suggested in this research work were followed and implemented by medical health workers, the disease Ebola virus would be eradicated. To further understand the Ebola virus dynamics and return predictive accuracy, we recommend that both local and global sensitivity analysis should be carried out using the model basic reproduction number so as to know important parameters that should be targeted towards control intervention strategies. Secondly, real data could be fitted into the model so as to show real life scenario.

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