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Utility maximization and portfolio management with administrative fee and dividend

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Abstract

The study of utility maximization cannot be over emphasized in the study of portfolio management as it plays crucial role in the determination of investor's strategy over any investment. In this research work, we investigate some utility functions and their applications to portfolio optimization and risk management. Also, a portfolio consisting of one risk free asset and a risky asset which follows the geometric Brownian motion was considered. We took into consideration transaction cost, dividend and tax on invested funds. By applying Ito's lemma and maximum principle, we obtained an optimization problem which comprise of a nonlinear PDE (Hamilton Jacobi Bellman equation) and a control problem (optimal investment strategy). These problems are functions of the value function which is dependent on the utility of the investor at the expiration of such investment. Furthermore, the exponential and logarithm utility were used in solving for the optimal investment strategies and some numerical results also presented. It was observed for both cases that the optimal investment strategies (OIS) were directly proportional to appreciation rate, dividend and inversely proportional to the administrative charges, instantaneous volatility, tax and risk averse coefficient. Also, the OIS under logarithmic utility does not depend on the risk averse coefficient and tax while the optimal investment strategy under exponential utility does not depend on wealth.

Keywords and phrases: portfolio management; utility maximization; ito's lemma; geometric brownian motion; financial market.

1 Introduction

1.1 Background of the study

The study of Investment strategy and portfolio optimization has become increasingly of important in managing financial resources of individual investors and financial institutions for purpose of achieving optimal returns for both the investors and the fund managers. In years past, many authors such as [1] - [6] have spent time in developing robust investment plans and strategies to investment decision making. In developing investment strategies, one crucial aspect to be considered is the utility functions which evaluate or quantify investor's preferences and risk tolerance [7].

The idea of utility is rooted in the area of study involving economics; more specifically, the theory of consumer behaviour established by Daniel Bernoulli in the 18^{th} century which later evolves into expected utility theory by John von Neumann and Oskar Morgenstern in the mid-20^{th}century. According to [8], utility is a subjective psychological feeling or a trade off response of an investor to the anticipated gains (satisfaction) or losses (dissatisfaction) of specific risk events. It helps to measure investors' attitude and courage toward investments.

It serves as a mathematical illustration of an investor's satisfaction he or she derived from different investment outcomes [9]. By integrating utility functions into the investment decision making process, investors can align their investment choices with their distinct risk preferences, financial goals and attitude toward wealth. The utility functions help investors to make an informed investment decisions

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by carefully examining the risk-reward trade off and choosing investment combinations that maximize their expected utility. Utility functions can be linear, exponential, quadratic, logarithm, power, and other functional forms captures different aspect of investor preferences and risk aversion [10].
There are mainly two types of widely used utility goals in the study of optimal investment. one which maximizes the accumulated amount at the end of investment and the other is to balance the expected return and the risk .The first goal is made up of three types of utility function which include utility functions which exhibits Constant Relative Risk Aversion (CRRA) and example include logarithm utility [11] - [14] and power utility [15] - [18]. Also, utility functions which exhibits Constant Absolute Risk Aversion (CARA) and example which include exponential utility [19] - [23] and the quadratic loss function [24] - [27] While the second goals is made up of Value-at-risk (VAR) utility [28] and mean-variance utility [29] - [34].
As earlier stated, utility functions are tools which enable investor's decisions or investment choices. However, in solving optimal investment problems, some challenges exist owed to uncertainty in the market, behavioural indifference, volatility of the stock market prices, parameter determination, inadequate information and change in market structure, etc. It is of importance to be familiar with these challenges while choosing utility functions to solve optimal investments problems and to consider them together with other techniques in investment decision making process. The application of utility function to portfolio optimization has become very important as it helps investors to make informed investment decision. Hence in this research, we will be studying two classes of utility functions: one which exhibits constant absolute risk averse (CARA) and that which exhibits constant relative risk averse (CRRA), use it to maximize a given portfolio involving transaction cost, dividend and tax and use it to discuss the effect of some these parameters on the OIS and also compare the OIS for the two cases.

2 Methodology and financial market model

2.1 Methodology

In this section, we shall discuss the Itô process since it is the essential tool required in solving stochastic differential equations. Also, we will also formulate and modeled our required problem. So we begin with some definitions which will be of important to our study in this section.

Definition 2.1. *Let* $\mathrm{B_t}$ *be a one-dimensional Brownian motion on* $(\Omega, \mathrm{F}, \mathrm{p})$*. A (one-dimensional) Itô process (or stochastic integral) is a stochastic process* $X(t)$ *on* $(\Omega, \mathrm{F}, \mathrm{p})$ *of the form*

$$X(t) = X(0) + \int_S^T \alpha(s, \omega) ds + \int_S^T \sigma(s, \omega) dW(t) \tag{2.1}$$

where $\sigma \in \mathrm{W_H}$ *so that*

$$\mathrm{p}\left[\int_S^T \alpha(s, \omega)^2 ds < 8, \forall t = 0\right] = 1. \tag{2.2}$$

We also assume that u *is* $H(t)-$*adapted, where* $H(t)$ *is an increasing family of* σ*-algebras,* $\{H(t)\}\ t = 0$ *, such that* $\mathrm{B_t}$ *is a martingale with respect to* $H(t)$*, and*

$$\mathrm{p}\left[\int_S^T |\sigma(s, \omega)| ds < 8, \forall t = 0\right] = 1.$$

Assuming $X(t)$ *is an Itô process in the of form* (2.1)*, the differential form of* (1) *is given as*

$$dX(t) = \alpha dt + \sigma dW(t) \tag{2.3}$$

Where α *is the drift and* σ*, the standard deviation representing the instantaneous volatility.*

Remark 2.2. Let $X(t)$ be an Itô process given by (2.3)
Let $g(t,\ x)\ \in\ C^2([0,8)\ \times\ \mathfrak{R})$. Then $Y\ (t)\ =\ g(t, X(t))$ is again an Itô process, and

$$dY\ \ (t) = \frac{\partial g}{\partial t}\left(t, X\left(t\right)\right) dt\ + \frac{\partial g}{\partial x}\left(t, X\left(t\right)\right) dX\left(t\right) + \frac{1}{2}\frac{\partial^2 g}{\partial w^2}(t, X(t))(dX(t))^2, \tag{2.4}$$

Remark 2.3. For Ito processes $U(t)$ and $V\ (t)$ in $\mathfrak{R}$, Itô's product rule gives

$$d(U(t)V\ (t))\ =\ U(t)dV(t)\ +\ V\ (t)dU(t)\ +\ dU(t)dV(t).$$

where $(dX(t))^2 =\ dX(t)dX(t)$ is computed according to the rules

$$dtdt\ =\ dtdW(t)\ =\ dW(t)dt\ =\ 0;;\ dW(t)dW(t)\ =\ dt.$$

(2.4) can also be written as

$$dg(t, X(t))\ \ (t) = \frac{\partial g}{\partial t}\left(t, X\left(t\right)\right) dt\ + \frac{\partial g}{\partial x}\left(t, X\left(t\right)\right) dX\left(t\right) + \frac{1}{2}\frac{\partial^2 g}{\partial w^2}(t, X(t))(dX(t))^2, \tag{2.5}$$

2.2 Financial market model and formulation

Let us consider a portfolio which is made up of a risk free asset and a risky asset which follows the geometric Brownian motion in a financial market which is continuously open for a given period of time $t\ \in\ [0,\ T]$ where T is the expiration date of the investment. Let $\{W\ (t) : t =\ 0\}$ be standard Brownian motion defined on a complete probability space (Ω, F, p) where Ω is a real space and p is a probability measure and F is the filtration which represents the information generated by the three Brownian motions.

$$\frac{dA}{A} = rdt \quad A(0) = 1 \tag{2.6}$$

The price process is described in [2, 4, 35]

$$\frac{ds}{s} = \omega dt\ + kdW(t) \tag{2.7}$$

Where ω is the appreciation rate of the risky asset and r is is the interest rate of the risk free asset, W(t) is the standard Brownian motion defined on a complete probability space (Ω, f, p) .
Let π, be the amount an investor is allowed to invest in the risky asset. Hence the amount to be invested in the risk asset is given as $X - \pi$. where X is the wealth process of the investor.
Let $\tau, \kappa and \zeta$ represent the rate of taxes in the financial market, dividend income and transaction cost comprising of fees and stamp duties [24].
Also, we assume that $\omega + \kappa - \zeta - r > o$ the wealth process $X\ (t)$ corresponding to the investment strategy and the initial wealth of the investor $X_{O,}$ is given as

$$dX = \pi\frac{ds}{s} + \left(X - \pi\right)\frac{dA}{A} - \tau X\left(t\right)dt \tag{2.8}$$

Substitute (2.6) and (2.7) in (2.8), we have

$$dX = \pi\left[\omega dt + kdW\right] + \left(X - \pi\right) rdt - \tau X dt$$

$$= \pi\omega dt + \pi kdW + rXdt - \pi rdt - \tau Xdt$$

$$= \left[\pi\left(\omega - r\right) + \left(r - \tau\right)X\right]dt + \pi kdW \tag{2.9}$$

Considering the dividend and transaction cost, then (2.9) becomes

$$dX = \left[\pi\left(\omega + \kappa - \zeta - r\right) + \left(r - \tau\right)X\right]dt + \pi kdW \tag{2.10}$$

Next, we defined the value function of the investor's utility at the expiration of his or her investment as follows

$$G(T,x) = Max\ E\left[U\left(X\left(t\right)\right)/X = x\right] \tag{2.11}$$

From Ito's lemma Taylor series expansion, and maximum principle can obtain the (HJB) equation as follows

$$dG = G_t\ dt + G_x\ dx + \frac{1}{2}G_{xx}\left(dx\right)^2 = \left[\begin{array}{l} G_t\ dt + G_x\left[\left[\pi\left(\omega+\kappa-\zeta-r\right)+\left(r-\tau\right)X\right]dt + \pi k dW\right] \\ +\frac{1}{2}G_{xx}\left[\left[\pi\left(\omega+\kappa-\zeta-r\right)+\left(r-\tau\right)X\right]+\pi k dW\right]^2 \end{array}\right] = G_t\ dt + G_x\ [$$

According to Ito's lemma $dwdt = dtdt = 0,\ (dw)^2 = dt$

$$\frac{dG}{dt\to 0} = G_t + G_x\left[\pi\left(\omega+\kappa-\zeta-r\right)+\left(r-\tau\right)X\right] + \frac{1}{2}\pi^2k^2\,G_{xx} = 0$$

$$G_t + \left(r-\tau\right)X\,G_x + Sup\left\{\frac{1}{2}\pi^2k^2\,G_{xx} + \pi\left(\omega+\kappa-\zeta-r\right)G_x\right\} = 0$$

$$G(T,x) = U(x) \tag{2.12}$$

Where $G_t,\ G_x\ and\ G_{xx}$ are partial derivatives of the first and second order w.r.t time, and wealth. Differentiating (2.12) with respect to π, we have

$$\pi = \frac{-\left(\omega+\kappa-\zeta-r\right)G_x}{k^2G_{xx}} \tag{2.13}$$

Substituting (2.13) into (2.12), we have,

$$G_t + \left(r-\tau\right)xG_x + \frac{1}{2}k^2\left(\frac{\left(\omega+\kappa-\zeta-r\right)}{k^2}\frac{G_x}{G_{xx}}\right)^2 xG_{xx} - \frac{\left(\omega+\kappa-\zeta-r\right)G_x}{k^2G_{xx}}x\left(\omega+\kappa-\zeta-r\right)G_x = 0$$

$$\left[\begin{array}{l} G_t + \left(r-\tau\right)xG_x - \frac{1}{2}\left(\frac{\omega+\kappa-\zeta-r}{k}\right)^2\frac{G_x^2}{G_{xx}} = 0 \\ G\left(T,x\right) = U\left(x\right) \end{array}\right] \tag{2.14}$$

Next we will proceed to solve the nonlinear PDE in (2.14) for $G\left(T,x\right)$ under exponential utility, and logarithmic utility functions.

3 Analysis of the model

In this section we will be solving for the value function $G\left(T,x\right)$ from the HJB equation in (2.14) and the solution obtained will be used to obtain our investment strategies under exponential and logarithmic utility functions.
To achieve this, we will use the power transformation method similar to the used in [24].

3.1 Optimal investment strategy under logarithm utility function

The logarithmic utility function is given as

$$u\left(x\right) = \ln x \tag{3.1}$$

Next we conjecture a solution to (3.1) as follows

$$\left\{\begin{array}{c} G(t,x) = a(t)\ln x + b(t) \\ a(T)=1,\ b(t)=0 \end{array}\right. \tag{3.2}$$

Differentiating (3.2), we have

$$G_t = a'\left(t\right)\ln x + b'\left(t\right), G_x = \frac{a_t}{x}, G_{xx} = \frac{-a_t}{x^2} \tag{3.3}$$

Substituting (3.3) into (2.14), we have

$$a'(t)\ln x + b'(t) + (r-\tau)x\left(\frac{a(t)}{x}\right) - \frac{1}{2}\left(\frac{\omega+\kappa-\zeta-r}{k^2}\right)\left(\frac{a(t)}{x}\right)^2 \div \left(\frac{-a(t)}{x}\right) = 0$$

$$a'(t)\ln x + b'(t) + (r-\tau)a(t) + \frac{1}{2}\left(\frac{\omega+\kappa-\zeta-r}{k^2}\right)\frac{(a(t))^2}{a(t)} = 0$$

$$a'(t)\ln x + b'(t) + (r-\tau)a(t) + \frac{1}{2}\left(\frac{\omega+\kappa-\zeta-r}{k^2}\right)a(t) = 0 \tag{3.4}$$

Splitting

$$\left\{ {a'(t)=0 \atop a(T)=1} \right. \tag{3.5}$$

$$\left\{ {b'(t)+(r-\tau)a(t)+\frac{1}{2}\left(\frac{\omega+\kappa-\zeta-r}{k^2}\right)a(t)=0 \atop b(T)=0} \right. \tag{3.6}$$

Solving (3.5), we have

$$a(t) = 1 \tag{3.7}$$

Substituting (3.7) into (3.6)

$$b'(t) + (r-\tau) + \frac{1}{2}\left(\frac{\omega+\kappa-\zeta-r}{k^2}\right) = 0 \tag{3.8}$$

Solving (3.8), we have

$$b(t) = \left[(r-\tau) + \frac{1}{2}(\omega+\kappa-\zeta-r)\right](T-t) \tag{3.9}$$

Hence from (3.7) and (3.9) the value function in (3.2) becomes

$$G(t,x) = \ln x + \left[(r-\tau) + \frac{1}{2}(\omega+\kappa-\zeta-r)\right](T-t) \tag{3.10}$$

Also from (3.3)

$$G_x = \frac{a(t)}{x} = \frac{1}{x} \tag{3.11}$$

$$G_{xx} = \frac{-a(t)}{x^2} = \frac{-1}{x^2} \tag{3.12}$$

From equ (3.11) and (3.12), equation (2.13)

$$\pi = \frac{-(\omega+\kappa-\zeta-r)}{k^2}\left(\frac{1}{x} \div \frac{-1}{x^2}\right)$$

$$= \frac{(\omega+\kappa-\zeta-r)}{k^2}\left(\frac{1}{x} \times \frac{x^2}{1}\right)$$

Hence the optimal investment strategy under logarithmic utility is given as

$$\pi^* = \frac{x(\omega+\kappa-\zeta-r)}{k^2} \tag{3.13}$$

3.2 Optimal investment strategy under exponential utility function

The exponential utility function is given by

$$U(x) = \frac{-1}{q} e^{-qx}, \; q > 0 \tag{3.14}$$

Where q is the risk averse coefficient of the member and x is the investor's wealth
Next, we conjecture a solution to (3.14) as follows

$$G(x,t) = \frac{-1}{q} \exp\left\{-q\left[a(t)(x - d(t)) + g(t)\right]\right\} \tag{3.15}$$

$$a(T) = 1, \; d(T) = 0, \; g(T) = 0$$

Differentiating (3.15), we have

$$G_t = -q\left[q_t(x-d) - d_t a + g_t\right]G, G_x = -qag, \; G_{xx} = q^2 a^2 G \tag{3.16}$$

Substituting (3.16) into (2.14), we have

$$-qG\left\{\left[a_t + (r-\tau)a\right]x - \left(d_t + \frac{a_t}{a}d\right)a + \left[g_t - \frac{1}{2}\frac{(\omega + \kappa - \zeta - r)}{qk^2}\right]\right\} = 0 \tag{3.17}$$

Splitting (3.17), we have

$$\begin{cases} a_t + (r-\tau)a = 0 \\ a(T) = 1 \end{cases} \tag{3.18}$$

$$\begin{cases} d_t + \frac{a_t}{a} d_t = 0 \\ d(T) = 0 \end{cases} \tag{3.19}$$

$$\begin{cases} g_t - \frac{1}{2}\left(\frac{\omega+\kappa-\zeta-r}{qk^2}\right) = 0 \\ g(T) = 0 \end{cases} \tag{3.20}$$

From (3.18), we have

$$\frac{a_t}{a} = -(r-\tau) \tag{3.21}$$

Substituting (3.21) into (3.19), we have

$$\begin{cases} d_t - (r-\tau)d(t) = 0 \\ d(T) = 0 \end{cases} \tag{3.22}$$

Next, we solve (3.18) to obtain

$$a(t) = e^{(r-\tau)(T-t)} \tag{3.23}$$

Similarly, if we solve (3.22), we have

$$d(t) = 0 \tag{3.24}$$

Finally, we solve (3.20) for $g(t)$as follows

$$g(t) = \left(\frac{\omega + \kappa - \zeta - r}{2qk^2}\right)(t - T) \tag{3.25}$$

Substituting (3.23), (3.24) and (3.25) into (3.15) we obtain our value function as follows

$$G(t,x) = \frac{-1}{q} Exp\left[-q\left(xe^{(r-\tau)(T-t)}\right) + \left(\frac{\omega + \kappa - \zeta - r}{2qk^2}\right)(t - T)\right] \tag{3.26}$$

From (3.16),

$$\frac{G_x}{G_{xx}} = \frac{-qaG}{q^2 a^2 G} = \frac{-1}{qa} \tag{3.27}$$

Substitute (3.23) into (3.27), we have

$$\frac{G_x}{G_{xx}} = \frac{-1}{qe^{(r-\tau)(T-t)}} = \frac{-1}{q}e^{(r-\tau)(t-T)} \tag{3.28}$$

Therefore, (2.13) now becomes

$$\pi* = \frac{-(\omega+\kappa-\zeta-r)}{k^2}\left(\frac{-1}{q}e^{(r-\tau)(t-T)}\right)$$

$$\pi* = \frac{(\omega+\kappa-\zeta-r)}{qk^2}e^{(r-\tau)(t-T)} \tag{3.29}$$

Hence the optimal investment strategy under exponential utility is given in (3.29)

4 Numerical simulations/sensitivity analysis

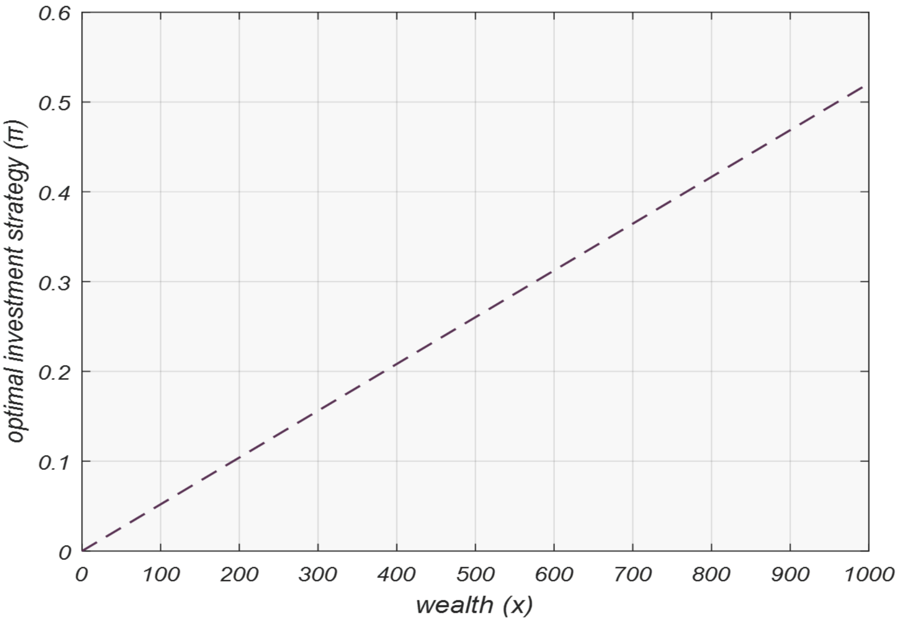

Figure 1: The variation of the optimal investment strategy with investor's wealth.

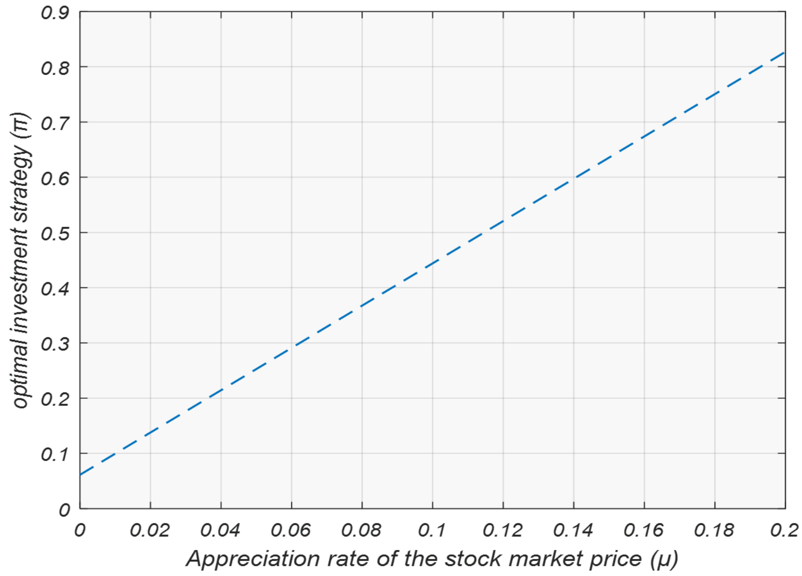

Figure 2: The variation of the optimal investment strategy with appreciation.

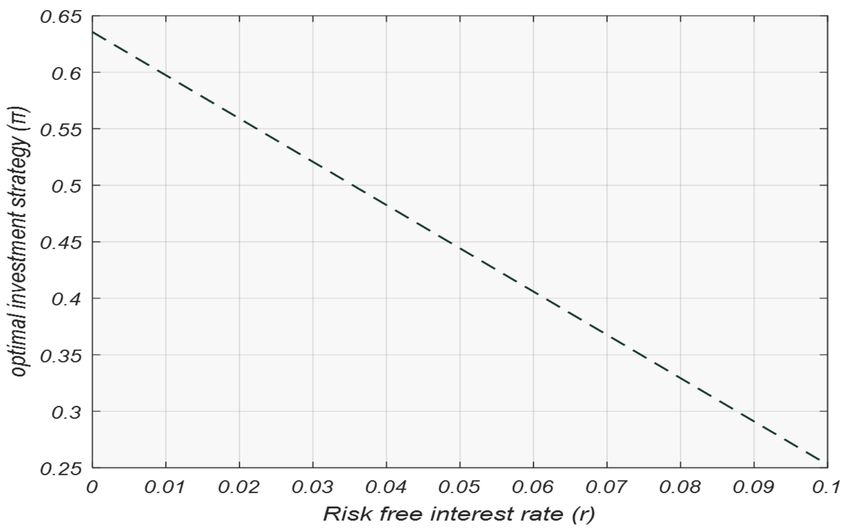

Figure 3: The variation of the optimal investment strategy with risk free interest rate.

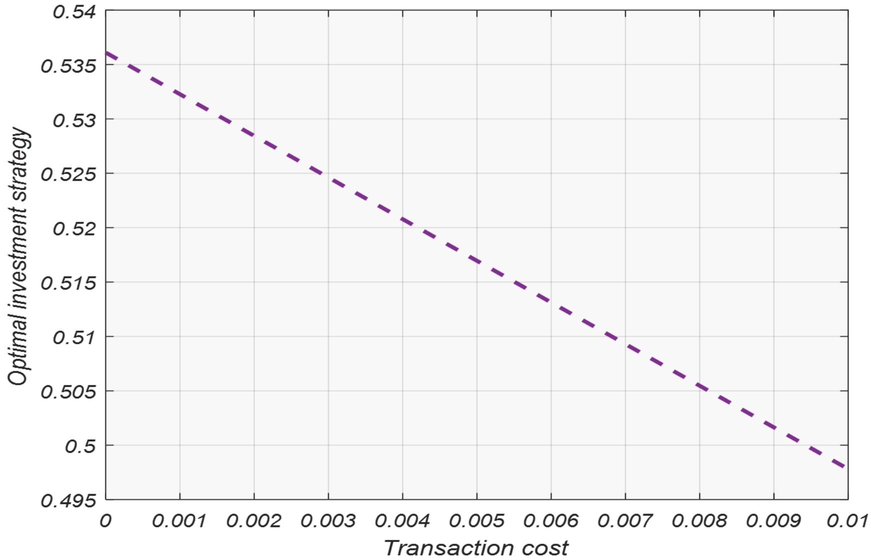

Figure 4: The variation of the optimal investment strategy with transaction cost.

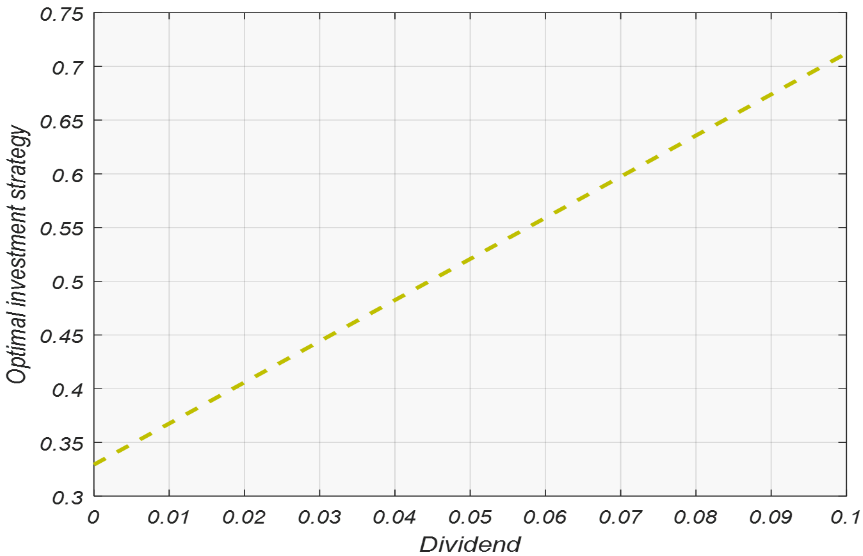

Figure 5: The variation of the optimal investment strategy with dividend.

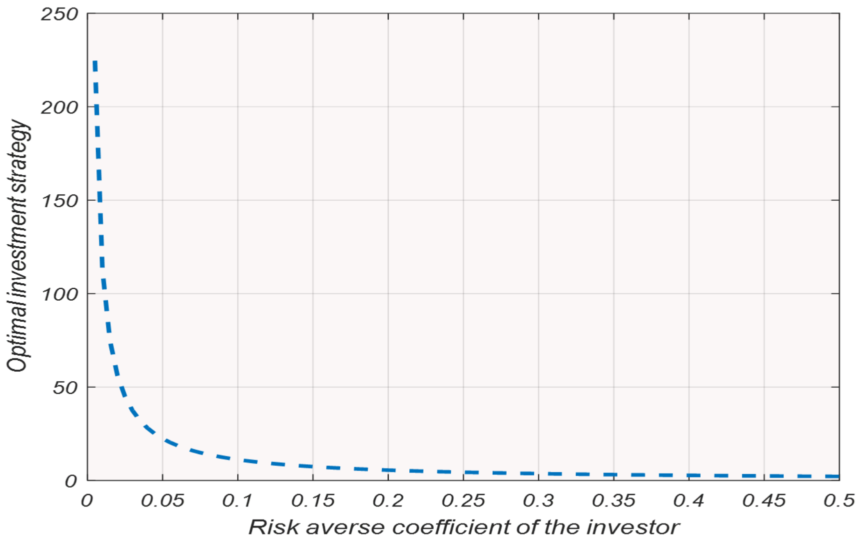

Figure 6: The variation of the optimal investment strategy with risk averse coefficient.

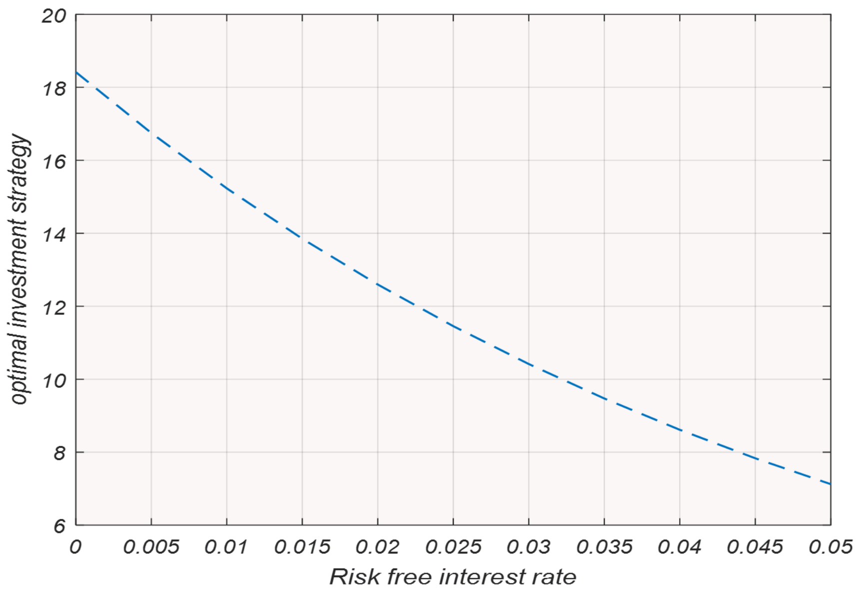

Figure 7: The variation of the optimal investment strategy with risk free interest.

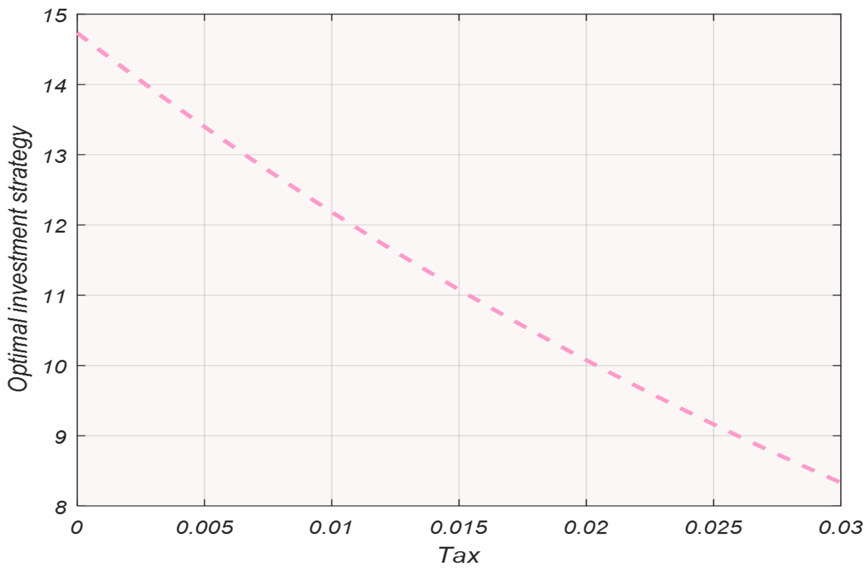

Figure 8: The variation of the optimal investment strategy with tax.

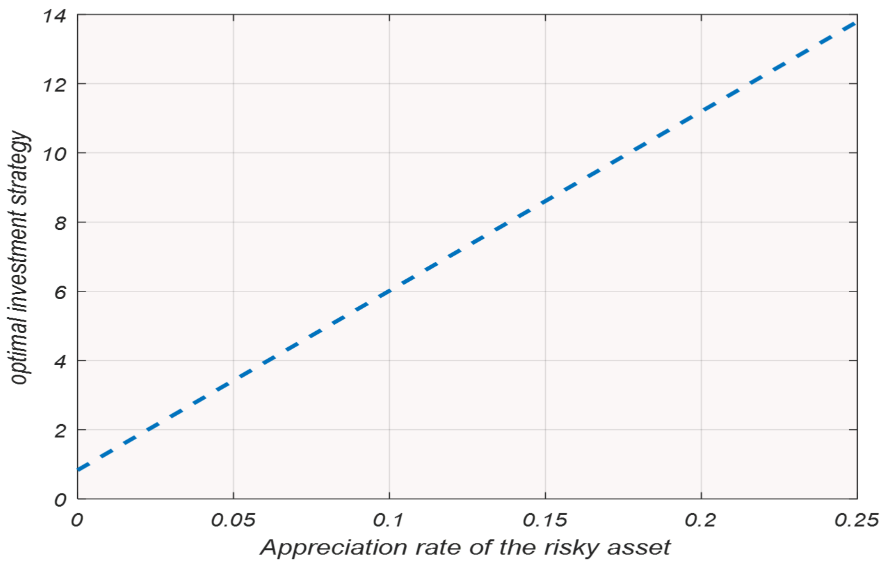

Figure 9: The variation of the optimal investment strategy with appreciation rate.

5 Discussions and conclusion

From (3.13), it is observed that the optimal investment strategy for an investor with logarithmic utility is a function of the wealth function, appreciation rate, dividend, transaction cost, risk free interest rate and instantaneous volatility. Also from fig 1, we observe that the investment strategy, is an increasing function of the wealth function. This implies that an investor will invest more when the wealth is more.

Figs. 2 and 5 show that the investment strategy is an increasing function of the appreciation rate; this is so, because the investor will be more interested in any asset with high expected rate of returns. Also, if the dividend from any given investment is attractive at the expiration of such investment year, the investor will be more interested in such asset and may be willing to increase the size of fund for such investment.

From (3.13), figs. 3 and 4, it can clearly be seen that the optimal investment strategy is a decreasing function of the transaction cost, risk free interest rate and instantaneous volatility. This is so because any business that attract high cost of operation, usually discourage the investor hence the reduction in funds allocated for such asset. Similarly, higher instantaneous volatility indicate the presence of higher risk and most investors get scared and worried over taking risk hence, this may lead to such investor reducing the funds allocation into such investment. This is also the reason why investors go for risk free assets more than risky assets when the interest of the risk free asset is high and attractive.

Furthermore, we observed the investment with strategy under logarithmic utility does not depend on the risk averse coefficient and tax.

From (3.29), it is observed that the optimal investment strategy for an investor with exponential utility is a function of the appreciation rate, dividend, transaction cost, risk free interest rate, tax and risk averse coefficient and instantaneous velocity of the risky asset. It is observed that the optimal investment strategy is an increasing function of the appreciation rate, and dividend but a decreasing function of transaction cost, risk free interest rate, tax on investment, risk averse coefficient and instantaneous volatility.

Fig. 6, discuss the effect of the risk aversion coefficient on the optimal investment strategy of the investor and we observed that the optimal control strategy for the risky asset, is inversely proportional to the risk aversion coefficient parameter. What we deduced from the graph in figure 4 is that an investor with higher risk aversion coefficient may invest a lesser percentage of their wealth in the risky asset (stock) while members with lower risk aversion coefficient may invest higher percentage of their wealth in the risky assets while reducing investment in the risk free asset. Fig. 7, shows that the optimal investment strategy decreases as the risk free interest rate increases and increases when the risk free interest rate decreases. This simply indicates that members will likely want to invest in risky asset when the interest rate from the risk free asset is not attractive. However, if the risk free interest rate is attractive enough, the investor may be advised by their fund administrators to invest more in the risk free asset, thereby reducing their investment in the risky asset. From fig. 8 and (3.29), the relationship between the optimal investment strategy, dividend and tax is established. We observed that the optimal investment strategy of the investor is an increasing function of the dividend and a decreasing function of tax. The consequence of (3.29) is that if the dividend from the risky asset is attractive, the investor may be tempted to invest more in risky asset and will invest less if the dividend from such investment is not attractive. Also, from figure 8, we observed that as the proportion of tax increases, the investor may tend to invest less in risky asset in other to balance his or wealth before retirement. This shows that higher tax rate could expose an investor to higher risk during investment planning and decision making. Also, we observed that the optimal investment strategy is a decreasing function of the administrative cost. The consequence of (3.29) is that if the administrative charges on investment of the risky asset is relatively low, the investor may be encouraged to invest more in risky asset and may invest less if otherwise.

From (3.29), the optimal investment strategy is a decreasing function of the instantaneous volatility. Since the instantaneous volatility represents the risk coefficient of the risky asset, risk averse members with high instantaneous volatility will be scared of investing much in the risky asset, thereby investing more in the risk free asset and vice versa.

Fig. 9, presents the graph of the optimal investment strategy against appreciation rate of the risky asset. It was observed that the investment strategy is an increasing function of the appreciation rate. This is so, because the investor will be more interested in any asset with high expected rate of returns. This is similar to the case under logarithm utility. Finally, we observed that the investment strategy does not depend on wealth.

In conclusion, this research presents the study of utility and its application to portfolio optimization and risk management. We discussed different utility functions such as, exponential and logarithm utilities. Also, a portfolio consisting of one risk free asset and a risky asset which follows the geometric Brownian motion was considered. We took into consideration transaction cost, dividend and tax on invested funds. Applying Ito's lemma and maximum principle, we obtained an optimization problem which comprise of a nonlinear PDE (Hamilton Jacobi Bellman equation) and a control problem (optimal investment strategy). These problems are functions of the value function which is dependent on the utility of the investor at the expiration of such investment. Furthermore, the exponential and logarithm utility were used in solving for the optimal investment strategies and some numerical results also presented. It was observed that the optimal investment strategy under logarithmic utility does not depend on the risk averse coefficient and tax while the optimal investment strategy under exponential utility does not depend on wealth.

Conflict of Interests

The authors declare that there is no conflict of interests regarding this paper.

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