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Revealing some of the hidden mathematical models of educational theories

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Abstract

This paper delves into the intersection of mathematical modeling and educational theory, exploring how mathematical frameworks have been instrumental in advancing our understanding of educational processes and pedagogical practices. This paper focused on such educational theories as information memory-processing theory, dynamic equilibrium theory, cooperative learning theory, Maslow motivation theory, Lewin's field theory and social learning theory. By employing mathematical models, researchers have been able to dissect complex educational phenomena, elucidate underlying mechanisms, and predict outcomes with a level of precision that traditional qualitative approaches often struggle to achieve.

Keywords and phrases: mathematical modeling; education; theories; educational theory

1 Introduction

Education as a field of study is interdisciplinary, drawing on insights and methodologies from multiple disciplines to address complex educational challenges and advance knowledge, policy, and practice. It plays a critical role in shaping individual opportunities, social mobility, economic development, and societal well-being by fostering intellectual growth, critical thinking, creativity, and citizenship. Educational theory encompasses a broad range of perspectives, principles, and frameworks that aim to understand and improve the process of teaching and learning [1]. It draws upon insights from various disciplines, including psychology, sociology, philosophy, and anthropology, to examine how individuals acquire knowledge, develop skills, and engage in educational experiences. It goes to show that educational theory informs the development of educational policies, practices, and interventions aimed at promoting equitable access to quality education and fostering lifelong learning [2]. By understanding the underlying principles and dynamics of teaching and learning, educators can adapt their approaches to meet the diverse needs of learners and create supportive learning environments that facilitate academic achievement, personal growth, and social development through mathematical modeling.

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Mathematical modeling is a method used to represent, analyze, and understand real-world phenomena or systems using mathematical structures and techniques. It involves creating mathematical descriptions of these systems based on observations, data, and theoretical principles. Mathematical modeling plays a crucial role in the field of education by providing a systematic framework for understanding, analyzing, and improving various aspects of teaching, learning, and educational systems. Advancing educational theories through mathematical modeling can be a powerful approach to understanding and improving learning processes. Mathematical modeling provides a systematic framework for studying educational theories, identifying factors that influence learning outcomes, and developing evidence-based practices for improving education. Advancing educational theories through mathematical modeling offers several significant benefits and opportunities. By mathematically formalizing educational theories, researchers can identify underlying patterns, predict outcomes, and test hypotheses with a higher degree of precision and clarity. Mathematical modeling allows researchers to simulate these complex systems, uncover emergent properties, and explore how various factors interact to influence educational outcomes [3]. By capturing the dynamic nature of education systems, mathematical models can provide insights into the underlying mechanisms driving learning, teaching, and educational change.

Mathematical models can be used to predict the outcomes of different educational interventions, policies, and practices, allowing policymakers, educators, and stakeholders to make informed decisions based on evidence and data. Advancing educational theories through mathematical modeling often involves interdisciplinary collaboration between researchers from diverse fields, including mathematics, computer science, psychology, sociology, and education. By bringing together expertise from multiple disciplines, interdisciplinary research teams can develop innovative modeling approaches, integrate insights from different domains, and address complex educational challenges from a holistic perspective [1, 4]. Interdisciplinary collaboration fosters cross-fertilization of ideas, methodologies, and best practices, leading to more robust and impactful research outcomes. Thus, advancing educational theories through mathematical modeling offers a powerful means of understanding, analyzing, and improving educational processes and outcomes. By harnessing the analytical power of mathematical modeling, researchers can contribute to the development of evidence-based educational theories, practices, and policies that promote student success, equity, and lifelong learning. This paper focused on such educational theories as information memory-processing theory, dynamic equilibrium theory, cooperative learning theory, Maslow motivation theory, Lewin's field theory and social learning theory.

2 Information memory-processing theory

The cognitive revolution of the 1950s and 1960s gave rise to the multi-store model of memory, or MSM. According to the theory put forth by Atkinson and Shiffrin in 1968 [5], environmental stimuli first arrive in sensory memory, where they can be registered for very short times before deteriorating or moving on to short-term memory (STM). The minimal amount of information that is truly in use at any given time is all that is contained in STM. At this point, verbal information is encoded in terms of sounds. According to Atkinson and Shiffrin, if memory traces in STM are not repeated (rehearsed), they can vanish in as little as 30 seconds. Practiced material is transferred to the long-term store, where it can stay indefinitely, albeit it may be lost there due to interference or deterioration. In view of the foregoing, we formulated a mathematical model of the information-memory processing approach as envisaged by Atkinson and Shiffrin. The mathematical model is based on the schematic diagram shown in Fig. 1 and the list of variables is given in Table 1.

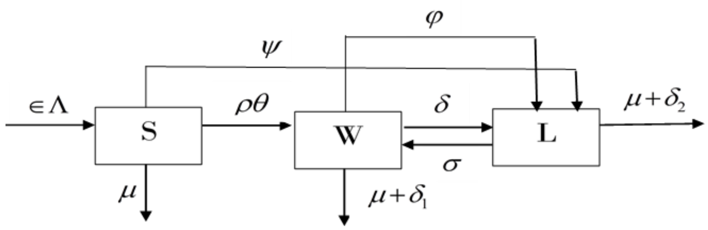

Fig. 1: Mathematical model flow diagram

Table 1: Model symbols and descriptions

Symbols	Description
S	Information in the sensory memory at time, t
W	Information in the short-term/working memory at time, t
L	Information in the long-term memory at time, t
Λ	Outside stimuli rate
θ	Initial processing rate
μ	Rate of forgetting, lost or interference
δ	Rate of transfer/progression/elaboration and organization into the LTM
σ	Rate of retrieval/reconstruction
ρ	Fraction of attention rate
ϵ	Fraction of outside stimuli rate
ψ	Attention rate
φ	Rehearsal rate
δ_1	Information loss rate due to displacement
δ_2	Information loss rate due to interference

Thus, putting the above formulations and assumptions together gives the following information memory-processing model, given by system of ordinary differential equations below:

$$\begin{cases} \dfrac{dS(t)}{dt} = \in \Lambda - \rho\theta S(t) - \psi S(t) - \mu S(t) \\ \dfrac{dW(t)}{dt} = (1-\rho)\theta S(t) - \sigma L(t) - (\delta + \delta_1 + \varphi + \mu) W(t) \\ \dfrac{dL(t)}{dt} = (\delta + \varphi) W(t) + \psi S(t) - (\sigma + \delta_2 + \mu) L(t) \end{cases} \tag{1.1}$$

For model (1.1) to be psychologically meaningful, it is important to prove that all its state variables are non-negative for all time (t). In other words, the solutions of model (1.1) with positive initial data will remain positive for all $t \geq 0$. Since model (1.1) monitors the human memory processing, all the state variables and parameters of the model are non-negative. Consider the psychologically feasible region

$$D = \left\{ S, W, L \in \mathbb{R}_+^3 : N \le \frac{\in \Lambda}{\mu} \right\}. \qquad (1.2)$$

It can be shown that the set D is a positively invariant set and a global attractor of this system. That is, any phase trajectory initiated anywhere in the non-negative region $\mathbb{R}_+^3$ of the phase space eventually enters the region D and remains in D thereafter. The region D is positively invariant for model (1.1). It follows from the first equation of the system (1.1), that

$$\begin{cases} \dfrac{dS(t)}{dt} = \in \Lambda - \rho\theta S(t) - \psi S(t) - \mu S(t) \\ \dfrac{d}{dt}\left\{ S(t)\exp\left[(\rho\theta+\psi+\mu)t\right] \right\} = (\in \Lambda)\exp\left[(\rho\theta+\psi+\mu)t\right] \\ S(t_1)\exp\left[(\rho\theta+\psi+\mu)t\right] - S(0) = \int_0^{t_1} (\in \Lambda)\exp\left[(\rho\theta+\psi+\mu)p\right]dp \\ S(t_1) = S(0)\exp\left[-(\rho\theta+\psi+\mu)t_1\right] + \exp\left[-(\rho\theta+\psi+\mu)t_1\right] \\ \times\int_0^{t_1} (\in \Lambda)\exp\left[(\rho\theta+\psi+\mu)p\right]dp > 0 \end{cases}$$

It can similarly be shown that $W > 0$ and $L > 0$ for all $t > 0$. Now, the rate of change of the total memory ($N = S + W + L$) is given by

$$\frac{dN(t)}{dt} = \in \Lambda - (2\rho - 1)\theta S(t) - \delta_1 W(t) - \delta_2 L(t) - \mu N(t) \qquad (1.3)$$

Since the right-hand side of (1.3) is bounded $\in \Lambda - \mu N(t)$, a standard comparison theorem can be used to show that

$$N(t) \le N(0)e^{-\mu t} + \frac{\in \Lambda}{\mu}\left(1 - e^{-\mu t}\right). \qquad (1.4)$$

In particular, if $N(0) \le \mu$, then $N(t) \le \mu$. Thus, D is positively invariant. Hence, no solution path leaves through any boundary of D and it is sufficient to consider the dynamics of model (1.1) in D. In this region, the model can be considered as being mathematically and psychologically well posed. Equilibrium analysis gives the fixed points, or equilibrium solutions, the values of state variables for which the system (1.1) will no longer change. At that time the rate of change are equated to zero. On the other hand, local stability analysis helps us to determine the dynamics of different memory stages near the equilibrium solutions. To achieve this, we compute the linearization of the system (1.1), which we obtained from the Jacobian matrix of the system (1.1). The Jacobian of the system (1.1) is given as:

$$J = \begin{pmatrix} -(\rho\theta+\psi+\mu) & 0 & 0 \\ (1-\rho)\theta & -(\delta+\delta_1+\varphi+\mu) & \sigma \\ \psi & (\delta+\varphi) & -(\sigma+\delta_2+\mu) \end{pmatrix} \qquad (1.5)$$

A memory becomes information-free at the point $(S_0, T_0, L_0) = \left(\frac{\epsilon\Lambda}{\mu}, 0, 0\right)$. We refer to this point as the information-free equilibrium (IFE). There is neither outside stimuli (information) in the STM nor LTM. The expected information size at this state will simply be the solution of $\frac{dS(t)}{dt} = \epsilon\Lambda - \mu S$. Solving the equation gives $S(t) = \frac{\epsilon\Lambda}{\mu} + \left(S_0 - \frac{\epsilon\Lambda}{\mu}\right)e^{-\mu t}$, where S_0 is the initial information in the sensory register. It is easy to see that as $t \to \infty$, $S(t) \to \frac{\epsilon\Lambda}{\mu}$, which is the asymptotic information size. Hence, the entire information is wholly in the sensory register. Now, what happen if the information that enters into the memory is not recognized and attended or organized? Will the information-free state be achieved? Therefore, we carry out the stability analysis for the steady state. Again, we compute the Jacobian matrix of the system at IFE to obtain:

$$J_0 = \begin{pmatrix} -(\rho\theta+\psi+\mu) & 0 & 0 \\ (1-\rho)\theta & -(\delta+\delta_1+\varphi+\mu) & \sigma \\ \psi & (\delta+\varphi) & -(\sigma+\delta_2+\mu) \end{pmatrix}$$

On the roots of the characteristic equation:

$$\begin{aligned}&\lambda^3+\mu^3-\left(-\rho\theta-\psi-3\mu-\delta-2\delta_1-\varphi-\sigma\right)\lambda^2-\Big(-2\delta\mu-\delta\psi-2\delta\sigma-\delta\rho\theta-\delta\delta_1\\&\quad-3\mu^2-2\mu\psi-2\mu\sigma-2\mu\rho\theta-2\mu\varphi-4\mu\delta_1-\psi\sigma-\psi\varphi-2\psi\delta_1-\sigma\rho\theta-2\sigma\varphi\\&\quad-\sigma\delta_1-\rho\theta\varphi-2\rho\theta\delta_1-\varphi\delta_1-\delta_1^2\Big)\lambda+\delta\mu^2+\mu^2\psi+\mu^2\sigma+\mu^2\rho\theta+\mu^2\varphi+2\mu^2\delta_1\\&\quad+\mu\delta_1^2+\psi\delta_1^2+\rho\theta\delta_1^2+\delta\mu\psi+2\delta\mu\sigma+\delta\mu\rho\theta+\delta\mu\delta_1+2\delta\psi\sigma+\delta\psi\delta_1+2\delta\sigma\rho\theta\\&\quad+\delta\rho\theta\delta_1+\mu\psi\sigma+\mu\psi\varphi+2\mu\psi\delta_1+\mu\sigma\rho\theta+2\mu\sigma\varphi+\mu\sigma\delta_1+\mu\rho\theta\varphi+2\mu\rho\theta\delta_1\\&\quad+\mu\varphi\delta_1+2\psi\sigma\varphi+\psi\sigma\delta_1+\psi\varphi\delta_1+2\sigma\rho\theta\varphi+\sigma\rho\theta\delta_1+\rho\theta\varphi\delta_1\end{aligned}$$

The eigenvalues are obtained as:

$$\lambda_1 = -(\rho\theta+\psi+\mu)$$

$$\lambda_{2,3} = \frac{1}{2}\left[-(\delta+\delta_1+\varphi+\mu)\pm\sqrt{(\delta+\delta_1+\varphi+\mu)^2-4\left((\delta+\delta_1+\varphi+\mu)(\sigma+\delta_2+\mu)-\sigma(\delta+\varphi)\right)}\right]$$

$$\lambda_2 = \frac{1}{2}\left[-(\delta+\delta_1+\varphi+\mu)-\sqrt{(\delta+\delta_1+\varphi+\mu)^2-4\left((\delta+\delta_1+\varphi+\mu)(\sigma+\delta_2+\mu)-\sigma(\delta+\varphi)\right)}\right]$$

For λ_2 to be negative, this implies that

$$\left((\delta+\delta_1+\varphi+\mu)(\sigma+\delta_2+\mu)-\sigma(\delta+\varphi)\right)<0$$

$$\Rightarrow \frac{(\delta+\delta_1+\varphi+\mu)(\sigma+\delta_2+\mu)}{\sigma(\delta+\varphi)}<1$$

where

$$\frac{(\delta+\delta_1+\varphi+\mu)(\sigma+\delta_2+\mu)}{\sigma(\delta+\varphi)} = I_0 \qquad (1.6)$$

This implies that $I_0-1<0$ or $I_0<1$, I_0 is called the basic information-processing number. This number is yet to be defined in the context of memory-processing but serves as a threshold. Hence, if $I_0<1$, the information-free equilibrium (IFE) is asymptotically stable. Therefore, information dies out with time, i.e., memory decay. This goes to show that, if the rate at which people are exposed to information and the rate of elaboration, organization and retrieval in memory are not enhanced, information size becomes asymptotically stable. In other words, if the quantity representing I_0 is less than unity, the information in the memory dies out with time; thereby, providing evidence to support the theory of memory decay.

To understand system dynamics, exact model solution is determined. Note that exact solutions provide insights into the interactions and dependencies between variables within the system. They reveal how changes in one variable affect others and help uncover patterns and behaviours that might not be immediately obvious. Exact solutions enable accurate predictions of the future behaviour of the system or the state of the variables at any given time within the domain. This predictive ability is especially useful for making informed decisions and planning. Thus, the equation (4.15) are first-order linear ODEs, which can be solved via the IF technique. We obtain

$$
\begin{cases}
S(t)=\dfrac{\in\Lambda}{\rho\theta+\psi+\mu}+\left(S_0-\dfrac{\in\Lambda}{\rho\theta+\psi+\mu}\right)e^{-(\rho\theta+\psi+\mu)t}\ \ as\ \ t\to\infty,\ S(t)\to\dfrac{\in\Lambda}{\rho\theta+\psi+\mu}\\
W(t)=\left(\dfrac{\in\Lambda}{(\rho\theta+\psi+\mu)^2}e^{-(\rho\theta+\psi+\mu)t}+\left(S_0-\dfrac{\in\Lambda}{\rho\theta+\psi+\mu}\right)t\right)(1-\rho)\theta e^{-(\rho\theta+\psi+\mu)t}\\
-\sigma H(t)e^{-(\rho\theta+\psi+\mu)t}+ke^{-(\rho\theta+\psi+\mu)t}\ \ as\ \ t\to\infty,\ W(t)\to 0\\
L(t)=(\delta+\varphi)\left[\dfrac{-\in\Lambda}{a^2(2a-b)}e^{-(2a-b)t}-\dfrac{(1-\rho)\theta}{a-b}(S_0-k_1)\left(t+\dfrac{1}{a-b}\right)e^{-(a-b)t}-G(t)-\dfrac{c_1}{a-b}e^{-(a-b)t}\right]e^{-bt}\\
+\psi\left(\dfrac{k_1}{b}e^{-bt}-\dfrac{S_0-k_1}{a-b}e^{-(a-b)t}\right)+c_2\\
where\ \ a=\delta+\delta_1+\varphi+\mu; b=\sigma+\delta_2+\mu; k_1=\dfrac{\in\Lambda}{a}; H(t)=\int L(t)e^{at}dt; G(t)=\sigma\int H(t)e^{-(a-b)t}dt\\
as\ \ t\to\infty,\ L(t)\to\infty\ \ provided\ \ b>a
\end{cases}
$$

As a result, the precise solutions of the differential equations system (1.1) provide a thorough and rigorous mathematical explanation of the interactions and behaviours of the system's variables. It is an effective instrument for understanding, theoretical investigation, and prediction in a variety of scientific, engineering, and mathematical domains. In actuality, though, memories can deteriorate and fade over time for a variety of reasons, such as interference, a lack of reinforcement, and the forgetting process itself. Not all of the data kept in long-term memory will always be available or undamaged over time. Although the theoretical notion that information stored in long-term memory may eventually reach infinity as time tends to infinity is thought-provoking, the above-mentioned practical constraints and limitations imply that this claim is not directly applicable in the real world. Rather, a variety of elements that interact with one another and with the passage of time affect memory capacity and retention. In conclusion, it is critical to understand the practical constraints and complexity of human memory and cognition, even though there are theoretical scenarios in which information in long-term memory could possibly accumulate as time approaches infinite. There are several facets and practical limitations to the link between time, memory, and information.

3 The dynamic equilibrium theory

The dynamic equilibrium theory of learning as envisaged by [28] states that the total information gain (say, N) is partly constant and partly decreases with N. That is, learning is as a result of dynamic equilibrium between information acquisition and loss. Mathematically, it is written as:

$$\frac{dN(t)}{dt}=p-qN(t) \tag{2.1}$$

The equation gives the total information gain (N) at a point in time (t), where p and q are constants for a particular individual in a given learning situation. The constant p is a measure of the individual's aptitude for learning. The constant q is a measure of the probability of information being forgotten, and there is evidence that this is independent of the individual's aptitude [28]. Solving equation (2.1) using separation of variable method, we have the following analytical solution

$$N(t)=\frac{p}{q}+\left(N_0-\frac{p}{q}\right)e^{-qt} \tag{2.2}$$

Now, as $t\to 0,\ N(t)\to N_0$ and as $t\to\infty,\ N(t)\to\frac{p}{q}$.

Equation (2.2) describes information gain dynamics. It tells us that the amount of information gain approaches the equilibrium value $\frac{p}{q}$. This is an indication that, an individual is capable of learning a limited amount of information. That is, if one learned over an infinite amount of time, this individual would have approached his or her upper limit N_∞. This number is referred to as the capacity of a student to learn. However, an individual is capable of learning a very large amount of information and is difficult to determine the capacity of the person to learn [6]. Therefore, the major limitation of Hicklin's model is its disagreement with neurological findings and a set of experimental data due to forgetting function. A more realistic model is presented below. In our model, we modify equation (2.1) by proposing the probability of information being forgotten as a function of time and not a constant as envisaged by Hicklin. A study on a large set of experimental data found that long-term memory retention can be described by several mathematical functions that are in a good correlation with experimental data [7, 8]. Considering the properties of the functions and the analysis carried out in [7], the following function is chosen for the description of the forgetting process:

$$q(t) = ae^{-bt} + w(t) \tag{2.3}$$

where a is a constant associated with the quantity of information forgotten, the exponential term depicts the decaying form of forgetting in the memory as time progresses and b is the forgetting rate. This equation (2.3) shows that the effect of the acquisition tails off to (*t*) after some time, but not to zero or constant. In the present study, a modified dynamic equilibrium model of learning as envisaged by Hecklin is presented, as it takes into account one of the most important features of this theory-its exponential forgetting character. Thus, this model modification could be termed Hecklin-Obasi model of learning. The model equation due to Obasi [1] is given in (2.4) below:

$$\frac{dN(t)}{dt} = \alpha - \left(ae^{-bt} + w(t)\right)N(t) \tag{2.4}$$

where

$$w(t) = \begin{cases} \beta t + \gamma, \textit{ function of time} \\ \quad w_0, \textit{otherwise} \end{cases}$$

Solving equation (2.4) gives the integrating factor as:

$$IF = \exp\left(-\frac{a}{b}e^{-bt} + \frac{1}{2}\beta t^2 + \gamma t\right)$$

$$N(t)\,IF = \int \alpha IF dt$$

$$N(t)\exp\left(-\frac{a}{b}e^{-bt} + \frac{1}{2}\beta t^2 + \gamma t\right) = \int \alpha \exp\left(-\frac{a}{b}e^{-bt} + \frac{1}{2}\beta t^2 + \gamma t\right)dt \tag{2.5}$$

$$N(t) = \frac{\alpha t}{ae^{-bt} + \beta t + \gamma} + \left(N_0 - \frac{\alpha}{a+\gamma}\right)\exp(-1)\left(\frac{1}{2}\beta t^2 + \gamma t - \frac{a}{b}e^{-bt}\right)$$

$$as\ \ t \to \infty, N(t) \to \infty$$

All other things being equal, it appears that the solution to the model equation (2.4) validates the theory that a person can acquire a significant quantity of knowledge (taking into account the control of extraneous variables). This suggests that it becomes challenging to assess a person's learning capacity if they learnt over an endless period of time. Limits and the theoretical concept of infinite learning are related to the idea that if a person learns over an unlimited amount of time, it becomes difficult to determine their capacity to learn. This idea entails investigating what occurs when learning is prolonged indefinitely and how it affects our comprehension of human potential. The idea of a limit is used in analysis and mathematics to characterize how a function behaves as its input gets closer to a particular value, like infinity. Comparably, thinking about what would happen if someone learned over an endless period of time has intriguing implications for the field of learning. If an individual were to learn indefinitely, it suggests that they would have the opportunity to continually acquire new knowledge and skills. This implies an unlimited potential for learning and personal growth. Over an infinite time span, the person would have the chance to accumulate an immense amount of knowledge. This accumulation could cover a vast array of subjects, from basic to highly specialized topics. Thus, the notion that determining an individual's capacity to learn becomes difficult over an infinite amount of time reflects the intricate interplay between the potential for lifelong learning and the limitations posed by factors such as diminishing returns, memory, specialization, and the finite nature of human existence. While infinite learning is an interesting theoretical concept, it's important to consider the practical implications and constraints that shape human learning and knowledge acquisition.

4 Cooperative learning theory

Cooperative learning theory is an educational approach that emphasizes collaborative and interactive learning among students. It is based on the idea that students working together in small groups can achieve better learning outcomes than when working individually. The theory draws on social interdependence and the positive interplay between individual and group goals [9]. Cooperative learning theory has roots in the work of several theorists, and it has evolved over time. One of the key figures associated with the development and popularization of cooperative learning is Dr. David W. Johnson and Dr. Roger T. Johnson, a husband-and-wife team of educational psychologists [10]. They have conducted extensive research on cooperative learning and are known for their contributions to the field. Cooperative learning is widely used in classrooms across various educational levels and disciplines. Its success relies on effective implementation, careful task design, and a supportive learning environment that encourages positive interdependence and individual accountability [11].

For the field of learning research to progress, the concept of quantifying and assessing knowledge, learning, etc., is crucial. Within education, this is a current problem. According to a quote from Lord Kelvin, educational psychology and pedagogy do not have to be imprecise fields of study [3]. Mathematical models, including differential equations, can be used to describe the dynamics of learning within a cooperative group. These models may consider factors such as the rate of knowledge transfer, individual learning rates, and the impact of collaboration on learning outcomes. The combination of mathematical models with insights from educational psychology provides a more comprehensive understanding of the dynamics within cooperative learning environments. Thus, cooperative learning is seen as a system of linear differential equation which relates a knowledge function to its derivatives. This brings us to a mathematical model for cooperative learning; let us take the example of three students, each of whose knowledge G, K, and R is quantified in terms of the estimated number of connected and unlinked ideas for a given lesson. $G(t)$, $K(t)$, and $R(t)$ are assumed to be functions of time expressed in weeks. Based on previous observations and experiences, the following assumptions are made by our cooperative learning model [3]:

i. The knowledge of each learner will increase at a rate that is proportional to the knowledge of the other learners;
ii. The knowledge of each learner may decrease at a rate that is proportional to its own amount of knowledge;
iii. The rate of change of knowledge for a learner has a constant component that measures the level of amiability (i.e., trust and friendliness) of that learner toward the others.

These assumptions lead to the system of linear differential equation (3.1) below:

$$\begin{cases} \dfrac{dG(t)}{dt} = \alpha K + \delta R - \rho G + r(s,h) \\ \dfrac{dK(t)}{dt} = \beta G + \eta R - \varphi K + s(r,h) \\ \dfrac{dR(t)}{dt} = \sigma K + \tau G - \upsilon R + h(r,s) \end{cases} \tag{3.1}$$

The variables and parameters of the model are described in Table 2 below.

Table 2: Description of variables and parameters of the model

Variable	Description
$G(t)$	Knowledge of first student at a time
$K(t)$	Knowledge of second student at a time
$R(t)$	Knowledge of third student at a time
α	Openness of first student to learning from second student
δ	Openness of first student to learning from third student
ρ	Learning decay rate of first student
β	Openness of second student to learning from first student
η	Openness of second student to learning from third student
φ	Learning decay rate of second student
σ	Openness of third student to learning from second student
τ	Openness of third student to learning from first student
υ	Learning decay rate of third student
r	First student's level of cordiality towards second & third students
s	Second student's level of cordiality towards first & third students
h	Third student's level of cordiality towards first & second students

The constants $\alpha, \delta, \rho, \beta, \eta, \varphi, \sigma, \tau, and\ \upsilon$ are non-negative but the numbers $r, s\ and\ h$ have any value: positive values for attitudes of trust for the other, and negative values for antagonism and aversion. In matrix notation, the case may be written as:

$$X' = AX + B$$

$$\begin{pmatrix} G'(t) \\ K'(t) \\ R'(t) \end{pmatrix} = \begin{pmatrix} -\rho & \alpha & \delta \\ \beta & -\varphi & \eta \\ \tau & \sigma & -\upsilon \end{pmatrix} \begin{pmatrix} G \\ K \\ R \end{pmatrix} + \begin{pmatrix} r \\ s \\ h \end{pmatrix}, A = \begin{pmatrix} -\rho & \alpha & \delta \\ \beta & -\varphi & \eta \\ \tau & \sigma & -\upsilon \end{pmatrix} \tag{3.2}$$

We see that the nature of the solutions of the system will depend on the eigenvalues of the matrix A. The determinant, $det(A)$ and trace, $tr(A)$ of this matrix play a crucial role in determining the stability of the solutions. If $det(A) > 0$ and $tr(A) < 0$, the system is stable. These criteria are derived from the fact that the signs of the eigenvalues influence the stability of the system [12, 1]. Positive real parts of eigenvalues are associated with instability, while negative real parts suggest stability. The determinant and trace help in identifying these signs and making predictions about the system's behaviour. On the roots of the characteristic equation:

$$\begin{aligned} &\lambda^3 + (\upsilon + \varphi + \rho)\lambda^2 + \left((\rho + \upsilon)\varphi - \sigma\eta + \upsilon\rho - \tau\delta - \beta\alpha\right)\lambda \\ &+ (\rho\upsilon - \delta\tau)\varphi - \eta\rho\sigma - \alpha\eta\tau - \alpha\beta\upsilon - \beta\delta\sigma = 0 \end{aligned} \tag{3.3}$$

with the determinant and trace of matrix A obtained as:

$$\begin{cases} \det(A) = \sigma(\beta\delta + \eta\rho) + (\alpha\beta - \rho\varphi)\upsilon + \tau(\alpha\eta + \delta\varphi) \\ tr(A) = -(\upsilon + \varphi + \rho) \end{cases} \tag{3.4}$$

It can easily be seen from (3.4) that $tr(A) < 0$, but if $\alpha\beta - \rho\varphi > 0$ then $det(A) > 0$, which implies stability. That is, if both learners mildly like each other ($r > 0, s > 0$ and $h > 0$) and if the product of their knowledge decay indices is less than the product of their receptivity to learning from each other, there will be joint increased learning. However, if $\alpha\beta - \rho\varphi < 0$, and ($r < 0, s < 0$ and $h < 0$) there will be retrogression. That is, first student and second student do not like each other and if the product of their knowledge decay indices is greater than the product of the indices of their receptivity to each other's learning, it will lead to retrogression. Their unpracticed or unutilized knowledge will decay.

In the context of cooperative learning, stability often refers to the ability of the system to reach and maintain a desirable state, such as a state where all individuals in the group have acquired and retained knowledge or skills. Consider a simple cooperative learning scenario where individuals positively influence each other's learning, and the matrix A has eigenvalues with negative real parts. In this case, the cooperative learning system is stable, and the group is likely to converge to a state where all individuals have acquired the desired knowledge or skills. Note that stability suggests that the learning trajectories of individuals within the group are predictable and do not exhibit erratic behaviour. Predictability is desirable in educational settings to ensure a steady and effective learning process [3]. Cooperative learning often involves individual accountability and positive interdependence, where the success of one individual benefits the entire group. Systems that promote positive interdependence and individual accountability are more likely to be stable. Stability implies a balance in the interactions between individuals in the cooperative learning group. If the interactions are too competitive or too cooperative, it may lead to instability. It should be noted that stable systems tend to converge to an equilibrium state where learning is balanced and individuals have acquired the desired knowledge or skills. To further understanding system dynamics, exact model solution is determined. Thus, the system of equations (3.1) are first-order linear ODEs, which can be solved via the matrix method. We have that

$$\begin{cases} X = e^{At}X(0),\ X(0) = \begin{pmatrix} G_0 \\ K_0 \\ R_0 \end{pmatrix} \\ \dfrac{dX(t)}{dt} = AX(t) = \begin{pmatrix} -\rho & \alpha & \delta \\ \beta & -\varphi & \eta \\ \tau & \sigma & -\upsilon \end{pmatrix}\begin{pmatrix} G(t) \\ K(t) \\ R(t) \end{pmatrix},\ X(t) = e^{At}\begin{pmatrix} G_0 \\ K_0 \\ R_0 \end{pmatrix},\ but \\ e^{At} = 1 + At + \dfrac{A^2}{2!}t^2 + \dfrac{A^3}{3!}t^3 + \ldots \end{cases}$$

$$\therefore\ e^{At} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -\rho & \alpha & \delta \\ \beta & -\varphi & \eta \\ \tau & \sigma & -\upsilon \end{pmatrix} t + \frac{1}{2!}\begin{pmatrix} -\rho & \alpha & \delta \\ \beta & -\varphi & \eta \\ \tau & \sigma & -\upsilon \end{pmatrix}^2 t^2 + \frac{1}{3!}\begin{pmatrix} -\rho & \alpha & \delta \\ \beta & -\varphi & \eta \\ \tau & \sigma & -\upsilon \end{pmatrix}^3 t^3 + \ldots$$

$$= \begin{pmatrix} (1-\rho)t + \frac{1}{2}(\alpha\beta + \delta\tau + \rho^2)t^2 & \alpha t - \frac{1}{2}(\alpha\rho + \alpha\varphi + \delta\alpha)t^2 & \delta t + \frac{1}{2}(\alpha\eta - \delta\rho + \delta\upsilon)t^2 \\ \beta t + \frac{1}{2}(\eta\tau - \beta\rho - \beta\varphi)t^2 & (1-\varphi)t + \frac{1}{2}(\alpha\beta + \eta\sigma + \varphi^2)t^2 & \eta t + \frac{1}{2}(\beta\delta - \eta\upsilon - \eta\varphi)t^2 \\ \tau t + \frac{1}{2}(\beta\sigma - \rho\tau - \tau\upsilon)t^2 & \sigma t + \frac{1}{2}(\alpha\tau - \sigma\upsilon - \sigma\varphi)t^2 & (1-\upsilon)t + \frac{1}{2}(\delta\eta + \eta\sigma + \upsilon^2)t^2 \end{pmatrix}$$

$$\begin{cases} X = \begin{pmatrix} (1-\rho)t+\frac{1}{2}(\alpha\beta+\delta\tau+\rho^2)t^2 & \alpha t-\frac{1}{2}(\alpha\rho+\alpha\varphi+\delta\alpha)t^2 & \delta t+\frac{1}{2}(\alpha\eta-\delta\rho+\delta\upsilon)t^2 \\ \beta t+\frac{1}{2}(\eta\tau-\beta\rho-\beta\varphi)t^2 & (1-\varphi)t+\frac{1}{2}(\alpha\beta+\eta\sigma+\varphi^2)t^2 & \eta t+\frac{1}{2}(\beta\delta-\eta\upsilon-\eta\varphi)t^2 \\ \tau t+\frac{1}{2}(\beta\sigma-\rho\tau-\tau\upsilon)t^2 & \sigma t+\frac{1}{2}(\alpha\tau-\sigma\upsilon-\sigma\varphi)t^2 & (1-\upsilon)t+\frac{1}{2}(\delta\eta+\eta\sigma+\upsilon^2)t^2 \end{pmatrix}\begin{pmatrix} G_0 \\ K_0 \\ R_0 \end{pmatrix} \\ X(t)\equiv(G(t),K(t),R(t)) = \begin{pmatrix} G_0(1-\rho)t+\frac{1}{2}G_0(\alpha\beta+\delta\tau+\rho^2)t^2+K_0\alpha t-\frac{1}{2}K_0(\alpha\rho+\alpha\varphi+\delta\alpha)t^2+R_0\delta t+\frac{1}{2}R_0(\alpha\eta-\delta\rho+\delta\upsilon)t^2 \\ G_0\beta t+\frac{1}{2}G_0(\eta\tau-\beta\rho-\beta\varphi)t^2+K_0(1-\varphi)t+\frac{1}{2}K_0(\alpha\beta+\eta\sigma+\varphi^2)t^2+R_0\eta t+\frac{1}{2}R_0(\beta\delta-\eta\upsilon-\eta\varphi)t^2 \\ G_0\tau t+\frac{1}{2}G_0(\beta\sigma-\rho\tau-\tau\upsilon)t^2+K_0\sigma t+\frac{1}{2}K_0(\alpha\tau-\sigma\upsilon-\sigma\varphi)t^2+R_0(1-\upsilon)t+\frac{1}{2}R_0(\delta\eta+\eta\sigma+\upsilon^2)t^2 \end{pmatrix} \end{cases}$$

Therefore, the solution of the system of equations (1) is obtained as:

$$\begin{cases} G(t)=G_0(1-\rho)t+\frac{1}{2}G_0(\alpha\beta+\delta\tau+\rho^2)t^2+K_0\alpha t-\frac{1}{2}K_0(\alpha\rho+\alpha\varphi+\delta\alpha)t^2+R_0\delta t+\frac{1}{2}R_0(\alpha\eta-\delta\rho+\delta\upsilon)t^2+\mathrm{O}(t^3) \\ K(t)=G_0\beta t+\frac{1}{2}G_0(\eta\tau-\beta\rho-\beta\varphi)t^2+K_0(1-\varphi)t+\frac{1}{2}K_0(\alpha\beta+\eta\sigma+\varphi^2)t^2+R_0\eta t+\frac{1}{2}R_0(\beta\delta-\eta\upsilon-\eta\varphi)t^2+\mathrm{O}(t^3) \\ R(t)=G_0\tau t+\frac{1}{2}G_0(\beta\sigma-\rho\tau-\tau\upsilon)t^2+K_0\sigma t+\frac{1}{2}K_0(\alpha\tau-\sigma\upsilon-\sigma\varphi)t^2+R_0(1-\upsilon)t+\frac{1}{2}R_0(\delta\eta+\eta\sigma+\upsilon^2)t^2+\mathrm{O}(t^3) \end{cases}$$

$as\ t\to\infty, G(t)\to\infty, K(t)\to\infty, R(t)\to\infty$

The exact solutions above show the idea that knowledge of students in cooperative learning could theoretically approach infinity as time tends to infinity. This indicates that there is a situation or condition described as "unbounded in time" which leads to a consequence of "unleashed" increased learning for the learners. This means a growth or enhancement in the process of acquiring knowledge or skills. Thus, the result of a situation being "unbounded in time" is that it allows for a significant and unrestrained increase in learning for the individuals involved. This might imply that without time constraints or limitations, learners are able to explore, absorb, and acquire knowledge in a more effective or accelerated manner. The use of "unleashed" suggests a kind of freedom or release that enables a fuller and more extensive learning experience.

Therefore, the mathematical formulation provides a theoretical framework for exploring the cooperative theory of learning in a controlled and abstract manner. The study indicates that the mathematical model proposed for the cooperative theory of learning demonstrates a high degree of accuracy in predicting learning outcomes within cooperative learning environments. The model has successfully captured the complex interactions and dynamics involved in collaborative learning settings. The mathematical formulations have allowed for a deeper understanding of the dynamics between individual and group learning. The study highlights how individual contributions impact the overall group learning process and vice versa, providing a nuanced perspective on the interplay between individual and collective knowledge acquisition. The mathematical model and findings from this paper have direct implications for educational practices. Educators can use the insights to design and implement cooperative learning activities that are not only pedagogically sound but also backed by quantitative evidence supporting their effectiveness.

5 Abraham Maslow's motivation theory

Abraham Maslow's motivation theory, often represented as Maslow's Hierarchy of Needs, outlines a hierarchical model of human needs, with each level building upon the previous one. Abraham Maslow proposed a hierarchical model of human needs, arranged in a pyramid. According to [13], individuals are motivated to fulfill basic physiological needs (such as food and shelter) before progressing to higher-level needs like safety, belongingness, esteem, and self-actualization. This theory, introduced by Maslow in the mid-20th century, is not specifically focused on learning but provides a framework for understanding human motivation, including aspects related to education and learning [14]. Maslow's Hierarchy of Needs has since become a foundational concept in psychology and education, influencing discussions on motivation, human development, and the design of effective learning environments [15]. While Maslow did not specifically focus on education in his original works, educators and psychologists have applied and adapted his theory to better understand and address the motivational aspects of the learning process [16]. We can attempt to create a simplified mathematical model inspired by the theory. Let us consider a system of differential equations that represent the dynamics of the hierarchy of needs:

$$\begin{cases} \frac{dP(t)}{dt} = -\alpha P(t) \\ \frac{dS(t)}{dt} = \beta P(t) - \delta S(t) \\ \frac{dB(t)}{dt} = \phi S(t) - \rho B(t) \\ \frac{dE(t)}{dt} = \varphi B(t) - \eta E(t) \\ \frac{dA(t)}{dt} = \sigma E(t) \end{cases} \tag{4.1}$$

The first equation represents the rate of change of physiological needs (P), indicating a natural decrease over time without external inputs. The second equation represents the dynamics of safety needs. Safety needs (S) increase when physiological needs (P) are satisfied but decrease naturally over time without further inputs. The third equation describes Belongingness and love needs (B) which increase when safety needs (S) are satisfied but decrease naturally over time without further inputs. In the fourth equation, esteem needs (E) increase when belongingness and love needs (B) are satisfied but decrease naturally over time without further inputs. The fifth equation represents the dynamics of Self-actualization (A) which increases when esteem needs (E) are satisfied. In these equations: $P(t), S(t), B(t), E(t), and\ A(t)$ represent the levels of physiological needs, safety needs, belongingness and love needs, esteem needs, and self-actualization, respectively. The values of $\alpha, \beta, \delta, \phi, \rho, \varphi, \eta, \sigma$ are positive constants representing the impact of one need level on the rate of change of the next. These equations attempt to capture the idea that the satisfaction of one level of need contributes positively to the next level while recognizing that needs naturally decrease over time if not continuously satisfied. In matrix notation, the case may be written as:

$$X'(t) = AX(t)$$

$$\begin{pmatrix} P'(t) \\ S'(t) \\ B'(t) \\ E'(t) \\ A'(t) \end{pmatrix} = \begin{pmatrix} -\alpha & 0 & 0 & 0 & 0 \\ \beta & -\delta & 0 & 0 & 0 \\ 0 & \phi & -\rho & 0 & 0 \\ 0 & 0 & \varphi & -\eta & 0 \\ 0 & 0 & 0 & \sigma & 0 \end{pmatrix} \begin{pmatrix} P(t) \\ S(t) \\ B(t) \\ E(t) \\ A(t) \end{pmatrix}, A = \begin{pmatrix} -\alpha & 0 & 0 & 0 & 0 \\ \beta & -\delta & 0 & 0 & 0 \\ 0 & \phi & -\rho & 0 & 0 \\ 0 & 0 & \varphi & -\eta & 0 \\ 0 & 0 & 0 & \sigma & 0 \end{pmatrix} \tag{4.2}$$

We see that the nature of the solutions of the system will depend on the eigenvalues of the matrix A. On the roots of the characteristic equation:

$$\lambda^5+(\alpha+\delta+\rho+\eta)\lambda^4+(\alpha\delta+\alpha\eta+\alpha\rho+\delta\eta+\delta\rho+\eta\rho)\lambda^3 \\ +(\alpha\delta\eta+\alpha\delta\rho+\alpha\eta\rho+\delta\eta\rho)\lambda^2+\eta\rho\delta\alpha\lambda=0 \tag{4.3}$$

with eigenvalues obtained as:

$$\lambda_{i,i=1,2,3,4,5}=\begin{pmatrix}0\\-\rho\\-\eta\\-\delta\\-\alpha\end{pmatrix} \tag{4.4}$$

It can easily be seen from (4.4) that all the eigenvalues are negative except one which is zero. The zero eigenvalue can lead to different stability scenarios depending on the nature of the accompanying eigenvalues. If all other eigenvalues have negative real parts, the equilibrium is a stable center [1]. The negative eigenvalues indicate that small perturbations from the equilibrium will decay over time, and the system will return to the balanced state. In the context of the hierarchy of needs, negative eigenvalues would imply that, when the physiological, safety, and love and belonging needs are at an equilibrium point, any deviations from this point are dampened, and the individual tends to return to a balanced state in terms of these needs. This suggests a stable and adaptive psychological state where the person's needs are generally met and maintained over time. In practical terms, this means that individuals are inclined to return to a balanced state where their physiological, safety, and social needs are met.

Stability in the system implies that when an individual's needs are satisfied at a certain level, they are more likely to maintain a state of well-being. The negative eigenvalues reflect a tendency for individuals to maintain a balanced and satisfied state in terms of their needs, fostering psychological well-being. Negative eigenvalues suggest that the system is adaptive and resilient to perturbations. In the face of external or internal challenges, individuals tend to return to a stable state. This aligns with Maslow's idea that individuals have an inherent drive for self-actualization and growth, and negative eigenvalues indicate a capacity to adapt and recover from disruptions. The hierarchy of needs is characterized by the transition from lower-level needs to higher-level needs as lower-level needs are satisfied. Individuals, having satisfied lower-level needs, are more likely to progress to addressing higher-level needs, such as esteem and self-actualization, in a stable manner. To further understand the system dynamics, exact model solution is determined. Thus, the system of equations (4.1) are first-order linear ODEs, which can be solved via the integrating factor method. Therefore, the exact solutions of the system of equations (4.1) is obtained as:

$$
\left\{
\begin{array}{l}
P(t)=P_0e^{-\alpha t},\ as\, t\to\infty,\ P(t)\to 0 \\
S(t)=\dfrac{P_0\beta}{\delta-\alpha}e^{(\delta-\alpha)t}+\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)e^{-\delta t}, as\, t\to\infty,\ \boldsymbol{S}(t)\to 0, \delta<\alpha \\
B(t)=\phi e^{-\beta t}\left[\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}e^{(\delta-\alpha+\beta)t}+\dfrac{1}{\beta-\delta}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)e^{(\beta-\delta)t}\right]+\left[B_0-\phi\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\dfrac{1}{\beta-\delta}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)\right)\right]e^{-\beta t}, as\, t\to\infty,\ \boldsymbol{B}(t)\to 0 \\
E(t)=\varphi\phi\left[\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)}e^{(\delta-\alpha+\eta)t}-\dfrac{1}{(\beta-\delta)(\delta-\eta)}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)e^{(\delta-\eta)t}-\dfrac{B_0}{\beta-\eta}e^{-(\beta-\eta)t}-\dfrac{\phi}{2\beta-\eta}\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\dfrac{1}{\beta-\delta}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)\right)e^{-(2\beta-\eta)t}\right] \\
+\left[E_0-\varphi\phi\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)}-\dfrac{1}{(\beta-\delta)(\delta-\eta)}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)-\dfrac{B_0}{\beta-\eta}-\dfrac{\phi}{2\beta-\eta}\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\dfrac{1}{\beta-\delta}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)\right)\right)\right] \\
as\, t\to\infty,\ \boldsymbol{E}(t)\to E^*, provided\ \delta-\alpha+\eta<0, \delta-\eta<0, \beta-\eta>0, 2\beta-\eta>0, \\
where \\
E^*=E_0-\varphi\phi\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)}-\dfrac{1}{(\beta-\delta)(\delta-\eta)}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)-\dfrac{B_0}{\beta-\eta}-\dfrac{\phi}{2\beta-\eta}\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\dfrac{1}{\beta-\delta}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)\right)\right) \\
A(t)=\sigma\varphi\phi\left[\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)^2}e^{(\delta-\alpha+\eta)t}-\dfrac{1}{(\beta-\delta)(\delta-\eta)^2}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)e^{(\delta-\eta)t}+\dfrac{B_0}{(\beta-\eta)^2}e^{-(\beta-\eta)t}+\dfrac{\phi}{(2\beta-\eta)^2}\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\dfrac{1}{\beta-\delta}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)\right)e^{-(2\beta-\eta)t}\right] \\
+\left[E_0-\varphi\phi\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)}-\dfrac{1}{(\beta-\delta)(\delta-\eta)}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)-\dfrac{B_0}{\beta-\eta}-\dfrac{\phi}{2\beta-\eta}\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\dfrac{1}{\beta-\delta}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)\right)\right)\right]t \\
+A_0-\sigma\varphi\phi\left[\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)^2}-\dfrac{1}{(\beta-\delta)(\delta-\eta)^2}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)+\dfrac{B_0}{(\beta-\eta)^2}+\dfrac{\phi}{(2\beta-\eta)^2}\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\dfrac{1}{\beta-\delta}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)\right)\right] \\
as\, t\to\infty,\ \boldsymbol{A}(t)\to A^*, provided\ \delta-\alpha+\eta<0, \delta-\eta<0, \beta-\eta>0, 2\beta-\eta>0, \\
where \\
A^*=A_0-\sigma\varphi\phi\left[\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)(\delta-\alpha+\eta)^2}-\dfrac{1}{(\beta-\delta)(\delta-\eta)^2}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)+\dfrac{B_0}{(\beta-\eta)^2}+\dfrac{\phi}{(2\beta-\eta)^2}\left(\dfrac{P_0\beta}{(\delta-\alpha)(\delta-\alpha+\beta)}+\dfrac{1}{\beta-\delta}\left(S_0+\dfrac{P_0\beta}{\delta-\alpha}\right)\right)\right]
\end{array}
\right.
$$

From the exact solutions, the tendency of physiological, safety, belongingness and love, esteem, and self-actualization needs to approach zero or equilibrium values indicates a long-term stability in the system. Individuals in the model reach a balanced state where their various needs, spanning from basic survival to higher-level self-fulfillment, are either completely satisfied or in a state of sustained equilibrium. The result suggests that, over time, individuals progress through Maslow's hierarchy of needs and eventually reach a state where the pursuit of these needs becomes less dominant. Basic physiological and safety needs tend to zero, indicating that individuals achieve a stable state of physical well-being and security. Meanwhile, higher-level needs like belongingness, esteem, and self-actualization tend to equilibrium values, suggesting a sustained and balanced pursuit of social connection, recognition, and personal growth.

The findings imply that individuals in the model adapt to their environments and find satisfaction in the fulfillment of their diverse needs. As basic needs are met, the focus shifts to higher-level needs, leading to a dynamic and evolving psychological state. The tendency of self-actualization needs to approach equilibrium values indicates that individuals, even in a state of equilibrium, continue to experience personal growth and pursue self-fulfillment. This aligns with Maslow's concept of self-actualization as an ongoing process of realizing one's potential. This goes to show that individuals have a natural inclination to maintain a stable and satisfied state, adapting to changing circumstances while progressing through the hierarchy of needs. This stability aligns with the notion that individuals strive for fulfillment and self-actualization in a resilient and balanced manner. The result suggests a long-term stability and adaptation in the modeled hierarchy of needs. Individuals reach a balanced state where basic needs are satisfied, and higher-level needs are pursued in a sustainable and evolving manner. The exact solutions highlight the dynamic and continuous nature of human motivation and well-being as individuals progress through the hierarchy over an extended period.

6 Lewin's Field Theory

Lewin's Field Theory, developed by the psychologist Kurt Lewin [17], is a psychological framework that emphasizes the importance of considering the totality of influences on an individual's behaviour. Lewin's Field Theory is particularly relevant to social psychology and understanding the dynamics of social environments. This theory was influential in shaping the way psychologists and researchers approach the study of human behaviour [18]. Key concepts and principles of Lewin's Field Theory include: life space, psychological field, force field analysis, equilibrium and change. Lewin introduced the concept of "life space," which refers to the internal psychological environment of an individual. It encompasses all the psychological influences and experiences that shape a person's thoughts, feelings, and behaviours. The psychological field is the totality of influences acting on an individual at a given time. It includes both the external environment and the individual's internal psychological state. According to [17], behaviour is a function of the psychological field.

$$B = f\left(P, E\right) \qquad (5.1)$$

One of the fundamental equations associated with Lewin's Field Theory is equation (5.1), where B represents behaviour, P represents the person's psychological state, and E represents the external environment. This equation highlights that behaviour is a function of both the person and the environment. Lewin introduced the concept of force field analysis to describe the driving and restraining forces that influence behaviour. Driving forces propel an individual towards a particular behaviour, while restraining forces act as barriers or obstacles. Behaviour is seen as the result of the dynamic interplay between these opposing forces [19]. Lewin argued that individuals seek a state of equilibrium, where driving and restraining forces are balanced. Changes in behaviour occur when there is an imbalance in the force field, leading to a shift in the equilibrium. Lewin referred to the process of achieving change as "unfreezing" the existing equilibrium, making the desired change, and then "freezing" the new equilibrium [20]. Lewin's ideas influenced the development of action research, an approach that integrates research and practical action to address social issues. Action research involves collaboration between researchers and participants to understand and change social systems.

Lewin's Field Theory has had a lasting impact on the field of social psychology and organizational development. It provides a framework for understanding the complex interactions between individuals and their social environments, emphasizing the need to consider both internal and external factors when studying behaviour and implementing interventions [21]. The researcher can create a simplified mathematical representation inspired by the key concepts of Lewin's Field Theory. Let: $B(t)$ be behaviour at time t, $P(t)$ is psychological state at time t, and $E(t)$ is external environment at time t. The dynamics of Lewin's Field Theory are represented using a simple differential equation in (5.2) below:

$$\begin{cases} \dfrac{dP(t)}{dt} = \alpha\left(E(t) - P(t)\right) \\ \dfrac{dB(t)}{dt} = \beta P(t) \\ \dfrac{dE(t)}{dt} = 0 \end{cases} \qquad (5.2)$$

Here, the first equation describes how the psychological state changes over time. The rate of change is proportional to the difference between the external environment and the current psychological state, α is a parameter that captures the influence of the external environment on the psychological state. $(E - P)$ represents the difference between the external environment and the psychological state, indicating the driving or restraining forces. This equation suggests that the rate of change in an individual's psychological state is proportional to the difference between the external environment and the current psychological state. It reflects Lewin's idea that behaviour is influenced by both internal and external factors, and the psychological state seeks equilibrium with the environment [22]. The second equation models how behaviour changes over time. The rate of change is proportional to the psychological state. This suggests that behaviour is influenced by the current psychological state, with β determining the strength of this influence. The third equation assumes a constant external environment, implying that it does not change over time in this simple model. In matrix notation, the case may be written as:

$$X'(t) = AX(t)$$

$$\begin{pmatrix} P'(t) \\ B'(t) \\ E'(t) \end{pmatrix} = \begin{pmatrix} -\alpha & 0 & \alpha \\ \beta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} P(t) \\ B(t) \\ E(t) \end{pmatrix} \tag{5.3}$$

On the roots of the characteristic equation:

$$\lambda^3 + \alpha\lambda^2 = 0 \Rightarrow \lambda = 0\,(twice),\, \lambda = -\alpha \tag{5.4}$$

Obtaining zero and negative eigenvalues in the system of equations describing Lewin's Field Theory carries significant implications for the stability and dynamics of the psychological system. Zero eigenvalues typically indicate that there are dimensions or directions in the psychological field where there is no change or movement. In the context of Lewin's Field Theory, this might suggest the presence of stable or unchanging aspects within the psychological system. These could represent elements that remain constant or have a long-term influence, contributing to a sense of stability or continuity in certain aspects of behaviour or mental processes. The negative eigenvalues would suggest that the psychological system is inherently stable. This stability could represent a tendency for individuals or groups to return to a certain psychological equilibrium over time, even when perturbed by external or internal factors.

To further understand the system dynamics, exact model solution is determined. An exact solution means that you can precisely determine the values of the variables over time or in a given state. It provides a comprehensive understanding of how the psychological field evolves and how individuals or groups respond to different influences [21]. They reveal how changes in one variable affect others and help uncover patterns and behaviours that might not be immediately obvious. An exact solution in the mathematical model of Lewin's Field Theory implies a clear and precise understanding of the system dynamics. Thus, the system of equations (5.2) are first-order linear ODEs, which can be solved via the integrating factor method. The exact solutions of the equation are given as:

$$\begin{cases} P(t) = k + (P_0 - k)e^{-\alpha t};\, as\, t \to \infty,\, P(t) \to k \\ B(t) = \beta\left(kt - \frac{1}{\alpha}(P_0 - k)e^{-\alpha t}\right) + B_0 + \frac{\beta}{\alpha}(P_0 - k);\, as\, t \to \infty,\, \boldsymbol{B}(t) \to \infty \\ E(t) = k,\, k\ is\ cons\tan t;\, as\, t \to \infty,\, \boldsymbol{E}(t) \to k \end{cases} \tag{5.5}$$

So, the implication of an exact solution is that it provides a solid foundation for understanding and manipulating the psychological dynamics described by Lewin's Field Theory, offering a more precise and actionable framework for studying and influencing human behaviour. The exact solution reflects the idea that behaviour and the external environment tend to reach a stable or equilibrium state over time, while the psychological state may not necessarily converge to a fixed value and can potentially become more complex or diverse as time progresses. The result that behaviour and external environment reaching a certain value suggests that, with time, individuals or groups tend to settle into specific patterns of behaviour and adapt to their external surroundings. It aligns with the notion of psychological equilibrium or stability, where certain behavioural patterns become established and predictable over time [22]. On the other hand, the psychological state not converging to a certain value implies that the internal psychological dynamics may not settle into a stable equilibrium. Instead, the psychological state might continue to evolve, change, or diversify over time. This could be due to the complexity of internal factors, emotions, and cognitive processes that may not have a fixed endpoint. In essence, this observation underscores the dynamic and evolving nature of the psychological state in Lewin's Field Theory. It suggests that while external behaviours and environmental interactions may stabilize, the internal psychological landscape remains open-ended and may continuously unfold in various ways. This dynamic perspective aligns with Lewin's emphasis on understanding behaviour within a changing and interactive psychological field, highlighting the ongoing nature of personal and group development [23]. Thus, the mathematical results from Lewin's Field Theory suggest a system characterized by a balance of stability and dynamism, while stability is observed in behaviour and the external environment, the internal psychological landscape remains open-ended, continually evolving and possibly influenced by a variety of factors. Therefore, researchers and practitioners in psychology should consider the dynamic and evolving nature of the psychological state. Interventions and strategies may need to account for the potential for ongoing change and adaptation.

7 Social learning theory

Social learning theory (SLT), developed by psychologist Albert Bandura [24], is a comprehensive theory that emphasizes the importance of social interactions in learning. It proposes that individuals learn not only through direct experiences but also by observing others and modeling their behaviours, attitudes, and emotional reactions. Bandura's work expanded traditional behaviourist theories by integrating cognitive and social aspects into the learning process. The key concepts of social learning theory include: observational learning, modeling, reinforcement, attention, retention, reproduction, motivation, and self-efficacy [25]. Observational learning also known as modeling or vicarious learning occurs when individuals observe the actions and outcomes of others and incorporate this information into their own behaviours. Bandura's famous Bobo doll experiment demonstrated how children learned aggressive behaviours by observing adults. Modeling involves imitating the behaviours of role models or significant others. Individuals are more likely to emulate behaviours they perceive as rewarding or socially acceptable. For example, children may imitate the behaviours of parents, teachers, or peers. While traditional behaviourist theories emphasize external reinforcement, SLT suggests that reinforcement can be vicarious [26]. Individuals are motivated to imitate behaviours that result in positive outcomes for others. Conversely, observing negative consequences for certain behaviours may inhibit imitation. Bandura proposed a four-step process involved in observational learning. Individuals must pay attention to the model's behaviour and its consequences. They must remember the observed behaviour. Individuals must have the ability to reproduce the observed behaviour. Finally, individuals are more likely to imitate behaviours if they are motivated to do so, either through intrinsic or extrinsic factors. Central to social learning theory is the concept of self-efficacy, which refers to an individual's belief in their own ability to succeed in specific situations or accomplish tasks [27]. Bandura argued that self-efficacy influences motivation, behaviour, and resilience in the face of challenges.

Representing social learning theory using mathematics offers several advantages. Mathematical models provide a systematic framework for quantitatively analyzing the dynamics of social learning. By expressing SLT concepts in mathematical terms, researchers can conduct rigorous analyses, perform simulations, and make predictions about learning outcomes. Mathematics enables clear and precise formulation of SLT concepts, making them easier to understand and communicate.

Mathematical models can formalize complex relationships between variables, parameters, and processes involved in social learning. Mathematical modeling allows researchers to dissect and understand the underlying mechanisms driving social learning. By representing SLT in mathematical terms, researchers can identify key factors influencing learning outcomes and explore how they interact. Well-constructed mathematical models of SLT can yield predictive insights into learning behaviours and outcomes. Researchers can use simulations to explore hypothetical scenarios and assess the potential impact of interventions or changes in social contexts. In the words of [3], mathematics serves as a common language across disciplines, facilitating interdisciplinary integration of SLT with fields such as psychology, sociology, education, and computer science. Mathematical modeling encourages collaboration and exchange of ideas among researchers from diverse backgrounds. Mathematical models of SLT can inform the design of educational interventions and instructional strategies. By understanding the underlying mechanisms of social learning, educators can develop more effective approaches to teaching and fostering collaborative learning environments.

By leveraging mathematical tools and techniques, learning theorists can develop precise models, formulate hypotheses, conduct empirical studies, and derive insights into the mechanisms underlying learning processes and behaviour. Mathematics provides a rigorous framework for studying and understanding the complexities of learning theory, facilitating advancements in educational research and practice [3]. Therefore, representing social learning theory using mathematics enhances our understanding of the complex interplay between social interactions, cognitive processes, and learning outcomes. Mathematical modeling provides a powerful tool for exploring, analyzing, and advancing our knowledge of social learning phenomena. To develop a system of differential equations describing social learning theory, we need to consider the key components of this theory, which emphasize learning through observation, modeling, and social interaction. The model variables and parameters are defined in Table 3.

Table 3: Model variable and parameter descriptions

Variables	Descriptions
$L(t)$	Level of learning or knowledge at time
$I(t)$	Intensity of exposure to social interactions or observational learning at time.
Parameters	
α	Rate at which learning occurs through observation.
β	Rate at which learning occurs through direct experience or instruction.
γ	Rate at which forgetting occurs.
δ	Influence of reinforcement or feedback.

Based on these variables and parameters, we can formulate a system of differential equations describing the social learning theory as developed Albert Bandura:

$$\frac{dL(t)}{dt} = \alpha I(t) - \beta L(t) - \gamma L(t) \tag{6.1}$$

This equation describes how the level of learning ($L(t)$) changes over time. Learning increases based on the intensity of exposure to social interactions ($I(t)$) with a rate proportional to α. Learning also decreases over time due to forgetting ($\gamma L(t)$) and through direct experience or instruction ($\beta L(t)$). Additionally, we can introduce a differential equation to model the intensity of exposure to social interactions ($I(t)$):

$$\frac{dI(t)}{dt} = \delta\left(L(t) - I(t)\right) \tag{6.2}$$

This equation describes how the intensity of exposure to social interactions changes over time. It depends on the difference between the level of learning ($L(t)$) and the current intensity of exposure to social interactions ($I(t)$), with a rate proportional to δ. Thus, putting the two equations together we have

$$\begin{cases} \dfrac{dL(t)}{dt} = \alpha I(t) - \beta L(t) - \gamma L(t) \\ \dfrac{dI(t)}{dt} = \delta\left(L(t) - I(t)\right) \end{cases} \tag{6.3}$$

This system of differential equations captures the essence of social learning theory by modeling how learning evolves over time through observation, direct experience, forgetting, and feedback from social interactions. Adjustments and refinements can be made to this model to account for specific contexts and factors influencing social learning. In matrix notation, the case may be written as:

$$X'(t) = AX(t)$$

$$\begin{pmatrix} L'(t) \\ I'(t) \end{pmatrix} = \begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix} \begin{pmatrix} L(t) \\ I(t) \end{pmatrix} \tag{6.4}$$

We see that the nature of the solutions of the system will depend on the eigenvalues of the matrix A. The determinant, $det(A)$ and trace, $tr(A)$ of this matrix play a crucial role in determining the stability of the solutions. If $det(A) > 0$ and $tr(A) < 0$, the system is stable. These criteria are derived from the fact that the signs of the eigenvalues influence the stability of the system [12, 1]. Positive real parts of eigenvalues are associated with instability, while negative real parts suggest stability. The determinant and trace help in identifying these signs and making predictions about the system's behaviour. On the roots of the characteristic equation:

$$\lambda^2 + (\delta+\beta+\gamma)\lambda - \delta\alpha + \delta\beta + \delta\gamma = 0 \tag{6.5}$$

with the determinant and trace of matrix A obtained as:

$$\begin{cases} \det(A) = -\delta\alpha + \delta\beta + \delta\gamma \\ tr(A) = -(\delta+\beta+\gamma) \end{cases} \tag{6.6}$$

It can easily be seen from (6.6) that $tr(A) < 0$, but if $\beta + \gamma > \alpha$ then $det(A) > 0$, which implies stability. That is, if the sum of rate at which learning occurs through direct experience or instruction and the rate at which forgetting occurs is greater than the rate at which learning occurs through observation, there will be increased learning. In the context of social learning theory, stability in the system of differential equations implies that the behaviours or patterns modeled by the equations tend to settle down or remain consistent over time. When the system is stable, it suggests that the dynamics of learning and behaviour tend to reach an equilibrium or a steady state. In other words, the behaviours modeled by the equations do not wildly fluctuate or diverge from each other over time. Instead, they converge towards certain patterns or norms, reflecting the stability of the social learning process within the theoretical framework being studied. This finding suggests that the effectiveness of learning can be enhanced when the combined rate of learning from direct experience or instruction, along with the rate of forgetting, surpasses the rate of learning through observation. This underscores the importance of active learning approaches, such as hands-on practice, interactive instruction, and spaced repetition, in promoting deep understanding and long-term retention of knowledge and skills. By balancing direct experience with effective instructional techniques and reinforcement strategies, educators and learners can optimize the learning process and enhance educational outcomes.

The stability of social learning theory in the context of learning has several important implications. Stable social learning processes imply that individuals are likely to consistently adopt and exhibit behaviours that are learned from observing others in their social environment. This consistency can lead to the establishment and maintenance of social norms and cultural practices within a group or society. When social learning processes are stable, it becomes easier to predict how individuals will respond to different social cues or stimuli. This predictability can be valuable for educators, policymakers, and social scientists who seek to understand and influence behaviour within various social contexts. It facilitates the transmission of knowledge, skills, and cultural traditions across generations. When individuals reliably learn from their social environment, valuable information and practices can be passed down from one generation to the next, contributing to the continuity and evolution of society. More so, it can contribute to social cohesion by promoting shared values, beliefs, and behaviours within a group or community.

When individuals consistently learn from and imitate each other, it fosters a sense of belonging and solidarity, which can strengthen social bonds and cooperation. While stability implies consistency, it does not necessarily mean rigidity. Social learning processes can also be adaptive, allowing individuals and societies to respond to changing circumstances and environments. Stable social learning frameworks provide a foundation upon which new behaviours and innovations can be integrated and disseminated. A stable social learning system can enhance the resilience of individuals and communities by providing a reliable mechanism for acquiring knowledge and coping strategies in the face of challenges or adversity. Consistent learning from others can help individuals navigate uncertain or unfamiliar situations more effectively. Thus, the stability of social learning theory in learning underscores its significance as a fundamental mechanism through which individuals acquire knowledge, skills, and social behaviours within their cultural and social contexts. To further understand the system dynamics, exact model solution is determined. The system of equations (6.3) are first-order linear ODEs, which can be solved via the matrix method. We have that

$$X = e^{At}X(0),\; X(0) = \begin{pmatrix} L_0 \\ I_0 \end{pmatrix}$$

$$\begin{cases} \dfrac{dX(t)}{dt} = AX(t) = \begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix}\begin{pmatrix} L(t) \\ I(t) \end{pmatrix},\; X(t) = e^{At}\begin{pmatrix} L_0 \\ I_0 \end{pmatrix},\; but \\ e^{At} = 1 + At + \dfrac{A^2}{2!}t^2 + \dfrac{A^3}{3!}t^3 + \ldots \\ \therefore\; e^{At} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix}t + \dfrac{1}{2!}\begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix}^2 t^2 + \dfrac{1}{3!}\begin{pmatrix} -(\beta+\gamma) & \alpha \\ \delta & -\delta \end{pmatrix}^3 t^3 + \ldots \\ = \begin{pmatrix} 1-(\beta+\lambda)t + \dfrac{1}{2}\left((\beta+\gamma)^2 + \delta\alpha\right)t^2 & \alpha t - \dfrac{1}{2}(\beta+\gamma-\delta)\alpha t^2 \\ \delta t - \dfrac{1}{2}(\beta+\gamma-\delta)\delta t^2 & 1-\delta t + \dfrac{1}{2}(\alpha+\delta)\delta t^2 \end{pmatrix} \end{cases}$$

$$\begin{cases} X = \begin{pmatrix} 1-(\beta+\lambda)t+\frac{1}{2}\left((\beta+\gamma)^2+\delta\alpha\right)t^2 & \alpha t-\frac{1}{2}(\beta+\gamma-\delta)\alpha t^2 \\ \delta t-\frac{1}{2}(\beta+\gamma-\delta)\delta t^2 & 1-\delta t+\frac{1}{2}(\alpha+\delta)\delta t^2 \end{pmatrix} \begin{pmatrix} L_0 \\ I_0 \end{pmatrix} \\ X(t) \equiv (L(t), I(t)) = \begin{pmatrix} L_0-(\beta+\lambda)L_0 t+\frac{1}{2}L_0\left((\beta+\gamma)^2+\delta\alpha\right)t^2+I_0\alpha t-\frac{1}{2}I_0(\beta+\gamma-\delta)\alpha t^2 \\ L_0\delta t-\frac{1}{2}L_0(\beta+\gamma-\delta)\delta t^2+I_0(1-\delta t)+\frac{1}{2}I_0(\alpha+\delta)\delta t^2 \end{pmatrix} \end{cases}$$

Therefore, the solution of the system of equations (6.3) is obtained as:

$$\begin{cases} L(t) = L_0-(\beta+\lambda)L_0 t+\frac{1}{2}L_0\left((\beta+\gamma)^2+\delta\alpha\right)t^2+I_0\alpha t-\frac{1}{2}I_0(\beta+\gamma-\delta)\alpha t^2+\mathrm{O}(t^3) \\ I(t) = L_0\delta t-\frac{1}{2}L_0(\beta+\gamma-\delta)\delta t^2+I_0(1-\delta t)+\frac{1}{2}I_0(\alpha+\delta)\delta t^2+\mathrm{O}(t^3) \end{cases} \qquad (6.7)$$

$as\ t\to\infty, \boldsymbol{L}(t)\to\infty, \boldsymbol{I}(t)\to\infty$

The exact solutions (6.7) above show the idea that level of learning or knowledge and the intensity of exposure to social interactions or observational learning could theoretically approach infinity as time tends to infinity. This suggests that, theoretically, there is no upper limit to the level of learning or knowledge that individuals can attain, nor to the intensity of exposure to social interactions or observational learning, given an infinite amount of time. Level of learning or knowledge refers to the depth and breadth of understanding that individuals can achieve in various domains, such as academic subjects, skills, or personal development. The finding suggests that as individuals engage in learning activities over an extended period, their level of knowledge can continue to grow indefinitely. This implies that there is always more to learn, discover, and understand, and that the process of learning is potentially limitless. It goes to show that social interactions and observational learning play crucial roles in the learning process. Through interactions with others, individuals can exchange ideas, perspectives, and information, which can enrich their understanding and stimulate further learning. Observational learning, where individuals learn by observing and imitating others, also contributes significantly to the acquisition of new skills and behaviours. The finding suggests that as individuals engage in more social interactions and observational learning experiences over time, the potential for learning increases without bounds.

Therefore, given an infinite amount of time, there is no limit to how much individuals can learn or how intensively they can engage in social interactions or observational learning. In other words, as time goes on indefinitely, the potential for learning and exposure to learning opportunities continues to grow without constraint. However, it is essential to note that while this finding provides an intriguing theoretical perspective on the potential for lifelong learning and continuous personal growth, it is practically impossible for individuals to have infinite time or resources for learning. In reality, people have finite lifespans, limited resources, and various constraints that affect their learning opportunities. Nonetheless, the idea that learning and exposure to learning experiences can continue to expand without limit underscores the importance of fostering a lifelong learning mindset and seeking out opportunities for growth and development throughout one's life.

8 Concluding remarks

This paper delves into the intersection of mathematical modeling and educational theory, exploring how mathematical frameworks have been instrumental in advancing our understanding of educational processes and pedagogical practices. Advancing educational theories through mathematical modeling offers several significant benefits and opportunities. This is because mathematical models provide a precise and systematic framework for representing complex educational phenomena, allowing researchers to formalize theoretical concepts, relationships, and mechanisms in a clear and rigorous manner. This paper focused on such educational theories as information memory-processing theory, dynamic equilibrium theory, cooperative learning theory, Maslow motivation theory, Lewin's field theory and social learning theory. By employing mathematical models, researchers have been able to dissect complex educational phenomena, elucidate underlying mechanisms, and predict outcomes with a level of precision that traditional qualitative approaches often struggle to achieve.

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