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Mathematical model of terrorism population dynamics: a case of a community under the attack of two terrorists groups

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Abstract

We formulated in this work, a mathematical model which is made up of system of non-linear ordinary differential equations that is analyzed so to study the dynamics of terrorism, a case of a community under the attack of two terrorists groups. The basic terrorism multiplication number (M_0) was computed after which we investigated the existence of Terrorism Free Equilibrium point (TFE) which was investigated to be locally and globally aymptotically stable whenever $(M_0) < 1$ and $(M_0) > 1$ respectively. A unique equilibrium point known as, Terrorism Persistent Equilibrium point (TPE) was also investigated to exist whenever$(M_0) > 1$. Sensitivity analysis of the basic terrorism multiplication number (M_0) was then carried out to see the impacts of some parameters to the growth of terrorism in a population. Results show that, the rate at which individuals are exposed to terrorists group two and the rate at which terrorists in group one are recruited into terrorists group two represented by (ω) and (θ_1) respectively are the most sensitive parameters that increase terrorism in a population, as an increase in the parameter values reduces the value of the basic terrorism multiplication number (M_0) to a value less than unity. The obtained mathematical result implies that, the terrorists group two (T_2) has a mighty negative effect on the growth of terrorism in a population. Finally, numerical simulation was performed which results were in conformity with our analytical results.

Keywords and phrases: terrorism; community; terrorists; groups; population; stability.

1 Introduction

Terrorism simply refers to the application of criminal violence to cause a state of fear or terror, majorly with the aim to achieve a political or religious motives. Terrorism in this context primarily refers to purposeful violence during peacetime or during war against civilians and non-combatants in most cases [1, 2]. Bruce defind terrorism in [1] as a deliberate killing of innocent individuals at random, to generate fear through a whole population and thereby forcing the hands of its political leaders. The term is mostly used with implication of something that is wrong morally. Non governmental and governmental societies make use of terrorism to abuse opposing groups [3-6].

Terrorism originated during the French revolution of the late 18th century but became globally used and got a universal attention in the 1970s during the fights in the Northern Ireland [3, 7, 8, 9]. Terrorism is grouped into six categories namely; civil disorder terrorism, political disorder terrorism, Non-political terrorism and limited political terrorism [8]. Groups choose terrorism as a tactic because it can; act as a form of asymmetric warfare so as to directly force a government to agree to demands, intimidate a group of people into capitulating to the demands in order to avoid future injury, get attention and thus political support for a cause, directly inspire more people to the cause such as revolutionary acts [10-12].

Nigeria as a country has been under a serious siege created by terrorists in the past 10 years [13]. Currently in Nigeria, the activities of a group known as Boko Haram over the years has caused alot

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of damages to lifes and properties. Research reveals that Boko Haram group is responsible for more than 80% of terrorism activities in the country with more than 27000 deaths as at 2018 as provided in [13]. The negative impacts of terrorism in Nigeria and the world at large over the years can not be over-emphasized as their activities has tempered economic growth and developments, to this end, the entire world requires urgently an effective counter terrorism strategies so as to bring the activities of these deadly groups to an end.

This paper provides both the theoretical and mathematical understanding so as to enable us gain insight into terrorism dynamics; a case of a community or a population under the attack of two terrorists groups. Mathematical model is regarded as a vital tool used in solving real life problems like that of terrorism which currently affects almost every part of the entire world. Many real life problems like disease outbreaks have been studied extensively via mathematical modeling in [14-18].

Udoh and Oladejo presented in [19] a simple mathematical model of terrorist evolutional dynamics represented by sets of ordinary differential equations. Their model provides a long-term solution of allocating resources towards eradicating terrorism in a population. In 2016, Lopes et al presented in [20], a terrorism global stability dynamical model to study the concepts of terrorism for the period of 1970-2014. Several mathematical tools and techniques were used like the Least Square Methods, Fractal dimensions for their model analysis. Udwadia et al presented also in [21], a dynamical mathematical model of terrorism, where they considered a long term evolutional operations of terrorists. In [13] similarly, Okoye et al studied terrorism dynamics using mathematical approach. They investigated the impact of counter-terrorism measures in tackling terrorism.

After searching literatures, we discovered that most terrorism mathematical models developed over the years did not cover a special case where a community or population is under an attack of two different terrorists groups. In this present work, we consider and formulate a mathematical model of terrorism population dynamics that incorporates a case of a community under the attack of two terrorists groups in the presence of a sustained military intelligence operations to eliminate and take over the camp of terrorists.

2 Model Formulation and Procedures

We consider a population size of $N(t)$ which is divided into five sub-classes namely; Susceptible Individuals (S), Exposed Individuals (E), Terrorists Group One (T_1), Terrorists Group Two (T_2) and Removed Individuals (R). Where; $N(t) = S(t) + E(t) + T_1(t) + T_2(t) + R(t)$.

Individuals are recruited into the susceptible compartment at a rate (Λ) and the susceptible individuals are exposed to the two terrorists groups (T_1) and (T_2) at the rates of (β_1) and (β_2) respectively. The exposed individuals (E) become full terrorists and therefore moved to either terrorist group (T_1) or terrorist group (T_2) at the rates of (α) or (ω).

We equally assume that those in terrorists group one (T_1) can join terrorists group two (T_2) and terrorist group two (T_2) joining terrorist group one (T_1) at the rates of (θ_1) and (θ_2) respectively. Both terrorists groups can be eliminated by a sustained military intelligence operations in a bid to take ove terrorists camps at rates (k_1) and (k_2) respectively. The two terrorists groups due to the risk nature of their operations can also die in a bid to attack a community at terrorism induced death rates of (μ_1) and (μ_2) respectively. The two groups due to terrorism induced deaths and continuous elimination of their members by effective military operations can renounce memberships of both groups at rates of (γ_1) and (γ_2) respectively, they repent and become removed fully from the population.

We assume that the sustained military operation to eliminate terrorists camps is 100% effective so as the renounced terrorists do not go back to terrorism again. Finally, individuals in all the five classes can die naturally at rate (μ). Based on these assumptions, the model flow diagram is shown in Fig.1 below;

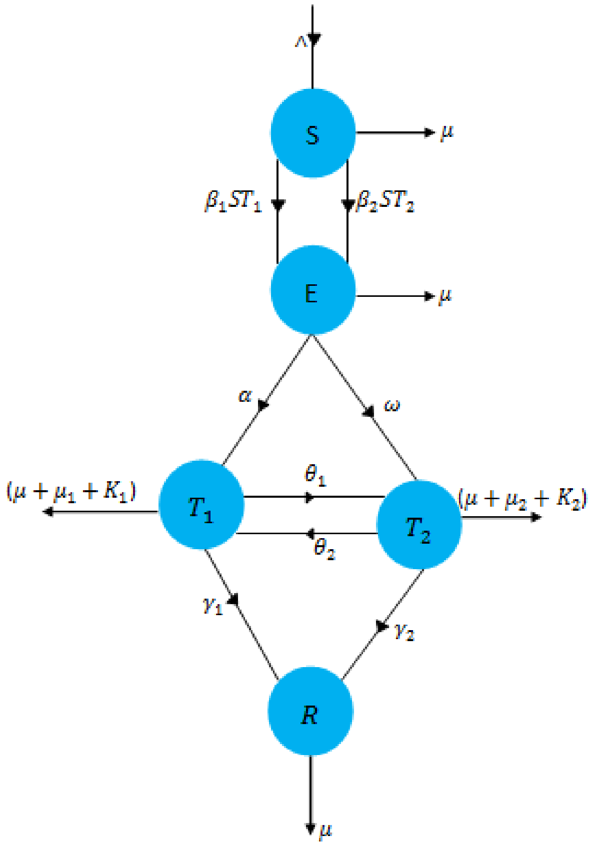

Fig. 1 Model flow diagram for terrorism dynamics.

The associated equations corresponding to the model flow diagram in Fig. 1 is given by;

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - (\beta_1 T_1 + \beta_2 T_2)S - \mu S \\ \dfrac{dE}{dt} = (\beta_1 T_1 + \beta_2 T_2)S - (\alpha + \omega + \mu)E \\ \dfrac{dT_1}{dt} = \alpha E + \theta_2 T_2 - (\theta_1 + \gamma_1 + \mu + \mu_1 + k_1)T_1 \\ \dfrac{dT_2}{dt} = \omega E + \theta_1 T_1 - (\theta_2 + \gamma_2 + \mu + \mu_2 + k_2)T_2 \\ \dfrac{dR}{dt} = \gamma_1 T_1 + \gamma_2 T_2 - \mu R. \end{cases} \tag{1}$$

The meaning of variables(parameters) used in model (1) with their various units is shown below.
Table 1. Descriptions of variables and parameters used in model (1) with their various units.

Now without of loss of generality, from model (1), we let; $m_0 = (\alpha + \omega + \mu)$, $m_1 = (\theta_1 + \gamma_1 + \mu + \mu_1 + k_1)$ and $m_2 = (\theta_2 + \gamma_2 + \mu + \mu_2 + k_2)$.

Variables/Parameters	Descriptions	Units
S	Susceptible Individuals	Individuals km^-2 months$^-1$
E	Exposed Individuals	Individuals km^-2 months$^-1$
T_1	Terrorists Group One	Individuals km^-2 months$^-1$
T_2	Terrorists Group Two	Individuals km^-2 months$^-1$
R	Removed Individuals	Individuals km^-2 months$^-1$
Λ	Recruitment rate	Months$^-1$
β_1	Exposure rate of (S) to (T_1)	Months$^-1$
β_2	Exposure rate of (S) to (T_2)	Months$^-1$
α	Rate at which (E) moves to (T_1)	Months$^-1$
ω	Rate at which (E) moves to (T_2)	Months$^-1$
θ_1	Rate at which (T_1) moves to (T_2)	Months$^-1$
θ_2	Rate at which (T_2) moves to (T_1)	Months$^-1$
γ_1	Rate at which (T_1) moves to (R)	Months$^-1$
γ_2	Rate at which(T_2) moves to (R)	Months$^-1$
μ_1	Terrorist induced death rate for (T_1)	Months$^-1$
μ_2	Terrorist induced death rate for (T_2)	Months$^-1$
k_1	Rate at which military eliminates (T_1)	Months$^-1$
k_2	Rate at which military eliminates (T_2)	Months$^-1$
μ	Natural death rate for all classes	Months$^-1$

Model (1) can therefore be written as:

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - (\beta_1 T_1 + \beta_2 T_2)S - \mu S \\ \dfrac{dE}{dt} = (\beta_1 T_1 + \beta_2 T_2)S - m_0 E \\ \dfrac{dT_1}{dt} = \alpha E + \theta_2 T_2 - m_1 T_1 \\ \dfrac{dT_2}{dt} = \omega E + \theta_1 T_1 - m_2 T_2 \\ \dfrac{dR}{dt} = \gamma_1 T_1 + \gamma_2 T_2 - \mu R \end{cases} \tag{2}$$

Taking a critical look at equation (2.2), the equation involving removed individuals (R) is independent of other equations since (R) does not appear in the remaining equations, we therefore decoupled (R) from the rest and consider the model presented in equation (2.3) below as a submodel.

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - (\beta_1 T_1 + \beta_2 T_2)S - \mu S \\ \dfrac{dE}{dt} = (\beta_1 T_1 + \beta_2 T_2)S - m_0 E \\ \dfrac{dT_1}{dt} = \alpha E + \theta_2 T_2 - m_1 T_1 \\ \dfrac{dT_2}{dt} = \omega E + \theta_1 T_1 - m_2 T_2 \end{cases} \tag{3}$$

2.1 Positivity of Solutions

For us to show that our model is mathematically meaningful, we therefore show that all the variables of model (2.2) are positive for all time $t > 0$.

Lemma 2.1 We let the initial data be :

$$S(0) \geq 0, E(0) \geq 0, T_1(0) \geq 0, T_2(0) \geq 0, R(0) \geq 0,$$

The solutions $S(t), E(t), T_1(t), T_2(t), R(t)$ of the model (2.2) are non- negative for all $t > 0$.
Proof
If $S(0) \geq 0, E(0) \geq 0, T_1(0) \geq 0, T_2(0) \geq 0, R(0) \geq 0$, then from the first equation of model (2.2), we have that;

$$\frac{dS(t)}{dt} = \Lambda - (\beta_1 T_1 + \beta_2 T_2)S - \mu S \tag{4}$$

which can be written as;

$$\frac{dS(t)}{dt} exp[\int_0^t (\beta_1 T_1(k) + \beta_2 T_2(k))dk + \mu t] + S(t)(\beta_1 T_1(k) + \beta_2 T_2(k) + \mu) exp[\int_0^t (\beta_1 T_1(k) + \beta_2 T_2(k) dk + \mu t]$$
$$= \Lambda exp[\int_0^t (\beta_1 T_1(k) + \beta_2 T_2(k) + \mu t]$$

Therefore;

$$\frac{d}{dt}(S(t) exp[\int_0^t (\beta_1 T_1(k) + \beta_2 T_2(k))dk + \mu t]) = \Lambda exp[\int_0^t (\beta_1 T_1(k) + \beta_2 T_2(k))dk + \mu t]$$

We therefore have that;

$$S(0) = exp[-\int_0^t (\beta_1 T_1(k) + \beta_2 T_2(k) - \mu t] + exp[-\int_0^t (\beta_1 T_1(k) + \beta_2 T_2(k))dk - \mu t][\int_0^t \Lambda exp[\int_0^t (\beta_1 T_1(n)$$
$$+ \beta_2 T_2(n))dn + \mu k]dk$$

In the same way, it can also be shown that $E(t) > 0, T_1(t) > 0, T_2(t) > 0$ and $R(t) > 0$. We therefore conclude that the solutions of $S(t), E(t), T_1(t), T_2(t)$ and $R(t)$ of model (2.2) are non-negative for all $t > 0$.

2.2 Invariant Region

For our model to be relevant mathematically, we need to show that all model variables and parameters are positive for all time $t \geq 0$ in a feasible region (Ω).
Lemma 2.2
The model feasible region (Ω) is defind by;

$$\Omega = [(S, E, T_1, T_2, R)] \in R_+^5 : S + E + T_1 + T_2 + R \leq \frac{\Lambda}{\mu}$$

with initial condition $S(0) \geq 0, E(0) \geq 0, T_1(0) \geq 0, T_2(0) \geq 0, R(0) \geq 0$ is positively invariant for model (2.2).
Proof
From equation (2.2); we have that;

$$\frac{dN}{dt} = \Lambda - \mu N - m_1 T_1 - m_2 T_2 \leq \Lambda - \mu N \tag{5}$$

therefore, $0 \leq N(t) \leq \frac{\Lambda}{\mu} + N(0)\ell^{-\mu t}$, where $N(0)$ represents the initial values of the total population. Therefore $0 \leq N(t) \leq \frac{\Lambda}{\mu}$ as t tends to infinity.
We can then say that the rejion;
$\Omega = [(S, E, T_1, T_2, R)] \in R_+^5 : S + E + T_1 + T_2 + R \leq \frac{\Lambda}{\mu}$ is positively invariant set for model (2.2).
To make our model analysis simple, we shall therefore consider the dynamics of the submodels (2.3).

3 Terrorism Model Analysis

3.1 Terrorism Free Equilibrium (TFE)

This is the equilibrium point where terrorism does not exit in the population. The state is represented as;

$$E_0 = (\frac{\Lambda}{\mu}, 0, 0, 0) \tag{6}$$

3.2 Basic Terrorism Multiplication Number(M_0)

The Basic terrorism multiplication number (M_0) is the same as the basic reproduction number (R_0) in epidemiological models which is also obtained using similar approach, that is the well known next generation matrix method [15, 18, 22, 23].

The basic terrorism multiplication number is therefore defined as the average number of individuals a terrorist can recruit into terrorism throughout his/her life time in the absence of counter-terrorism measure [13].

Now, at the terrorism free equilibrium point, we obtain the basic terrorism multiplication number (M_0) using the next generation matrix.

Let $y = (E, T_1, T_2, S)^T$, then our model (2.3) can be expressed as;

$$\frac{dy}{dt} = F(y) - V(y) \tag{7}$$

where;

$$F(y) = \begin{bmatrix} (\beta_1 T_1 + \beta_2 T_2)S \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

and

$$V(y) = \begin{bmatrix} m_0 E \\ m_1 T_1 - \alpha E - \theta_2 T_2 \\ m_2 T_2 - \omega E - \theta_1 T_1 \\ \mu S + (\beta_1 T_1 + \beta_2 T_2)S - \Lambda \end{bmatrix}$$

where;

$$F = \begin{bmatrix} 0 & \frac{\beta_1 \Lambda}{\mu} & \frac{\beta_2 \Lambda}{\mu} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{9}$$

$$V = \begin{bmatrix} m_0 & 0 & 0 \\ -\alpha & m_1 & -\theta_2 \\ -\omega & -\theta_1 & m_2 \end{bmatrix}$$

But $M_0 = FV^{-1}$ and is obtained as;

$$M_0 = \frac{\Lambda \beta_1 (m_2 \alpha + \theta_2 \omega) + \beta_2 (\theta_1 \alpha + m_1 \omega)}{\mu m_0 (m_1 m_2 - \theta_1 \theta_2)} \tag{10}$$

Following the theorem 3 adopted in [18], we have the following theorem.

Theorem 3.2

The terrorism free equilibrium point (E_0) is locally asymptotically stable for $M_0 < 1$ and unstable for $M_0 > 1$.

3.3 Global Stability of the Terrorism Free Equilibrium

Theorem 3.3
The model (2.3) terrorism free equilibrium point is globally asymptotically stable provided $M_0 > 1$.
Proof
Considering the Lyapunov function;

$$P = M_0 E + \frac{\Lambda(\beta_1 m_2 + \beta_2\theta_1)T_1}{\mu(m_1 m_2 - \theta_1\theta_2)} + \frac{\Lambda(\beta_2 m_1 + \beta_1\theta_2)T_2}{\mu(m_1 m_2 - \theta_1\theta_2)}$$

which its derivative along the solutions to our model (2.3) is;

$$P^1 = M_0(\beta_1 T_1 + \beta_2 T_2)S - (\beta_1 T_1 + \beta_2 T_2)\frac{\Lambda}{\mu} \leq M_0(\beta_1 T_1 + \beta_2 T_2)\frac{\Lambda}{\mu} - (\beta_1 T_1 + \beta_2 T_2)\frac{\Lambda}{\mu}$$
$$= (\beta_1 T_1 + \beta_2 T_2)\frac{\Lambda}{\mu}(M_0 - 1) \leq 0$$

Then $P^1 = 0$ only if $E = T_1 = T_2 = 0$ or if $M_0 = 1$. Then the maximum invariant set in $[(S, E, T_1, T_2) : P^1 = 0]$ is the singleton (E_0). The global stability of (E_0) is now obtained when $M_0 < 1$ by Lasalle's Invariance principle.

3.4 Terrorism Persistent Equilibrium (TPE)

At this stage, terrorism is present in the population. From our model (2.3), we have that;

$$S = \frac{\Lambda - M_0 E}{\mu}, T_1 = \frac{(m_2\alpha + \theta_2\omega)E}{m_1 m_2 - \theta_1\theta_2}, T_2 = \frac{(\theta_1\alpha + m_1\omega)E}{m_1 m_2 - \theta_1\theta_2} \tag{11}$$

where $m_0 = (\alpha + \omega + \mu)$, $m_1 = (\theta_1 + \gamma_1 + \mu + \mu_1 + k_1)$ and $m_2 = (\theta_2 + \gamma_2 + \mu + \mu_2 + k_2)$
Now by substituting equation (3.6) into $(\beta_1 T_1 + \beta_2 T_2)S - m_0 E = 0$, the Exposed class (E) satisfies the following equation;

$$\frac{(\beta_1 m_2 + \theta_2\omega)}{(m_1 m_2 - \theta_1\theta_2)} + \frac{(\beta_2\theta_1\alpha + m_1\omega)(\Lambda - m_0 E)}{(m_1 m_2 - \theta_1\theta_2)\mu} - m_0 = 0$$

when E is not equal to zero, we therefore have that;

$$Q(E) = \frac{(\beta_1 m_2 + \theta_2\omega)}{(m_1 m_2 - \theta_1\theta_2)} + \frac{(\beta_2\theta_1\alpha + m_1\omega)(\Lambda - m_0 E)}{(m_1 m_2 - \theta_1\theta_2)\mu} - m_0 \tag{12}$$

equation (3.7) implies that;

$$Q(E) = \frac{m_0\beta_1(m_2 + \theta_2\omega) + \beta_2(\theta_1\alpha + m_1\omega)}{(m_1 m_2 - \theta_1\theta_2)\mu} < 0 \tag{13}$$

Equation (3.8) confirms that $Q(E)$ decreases since;

$$Q(E) = \frac{\Lambda\beta_1(m_2 + \theta_2\omega) + \beta_2(\theta_1\alpha + m_1\omega)}{(m_1 m_2 - \theta_1\theta_2)\mu} - m_0 = m_0(M_0 - 1)$$

provided that the basic terrorism multiplication number $(M_0) > 1$, then $Q(0)$ is non-negative and that;

$$Q(\frac{\Lambda}{m_0}) = -m_0 < 0 \tag{14}$$

We therefore say that, by monotonicity of $Q(E)$, we have a special non-negative root of which the result is presented in theorem (3.4) below.
Theorem 3.4
If $M_0 > 1$, model (2.3) has a special endemic equilibrium point $E_1^*(S^*, E^*, T_1^*, T_2^*)$ where;

$$S^* = \frac{\Lambda - M_0 E^*}{\mu}, T_1^* = \frac{(m_2\alpha + \theta_2\omega)E^*}{m_1 m_2 - \theta_1\theta_2}, T_2^* = \frac{(\theta_1\alpha + m_1\omega)E^*}{m_1 m_2 - \theta_1\theta_2} \tag{15}$$

where E^* represents the special non-negative root of $Q(E) = 0$

4 Sensitivity Analysis

We performed sensitivity analysis of the basic terrorism multiplication number (M_0) to see the impacts of some parameters to the persistence of terrorism in a population.
We recall that, the basic terrorism multiplication number;

$$M_0 = \frac{\Lambda\beta_1(m_2\alpha + \theta_2\omega) + \beta_2(\theta_1\alpha + m_1\omega)}{\mu m_0(m_1m_2 - \theta_1\theta_2)}$$

By substituting the expressions for m_0,m_1 and m_2, we have that;

$$M_0 = \frac{\Lambda\beta_1((\theta_2 + \gamma_2 + \mu + \mu_2 + k_2)\alpha + \theta_2\omega) + \beta_2(\theta_1\alpha + (\theta_1 + \gamma_1 + \mu + \mu_1 + k_1)\omega}{\mu(\alpha + \omega + \mu)((\theta_1 + \gamma_1 + \mu + \mu_1 + k_1)(\theta_2 + \gamma_2 + \mu + \mu_2 + k_2 - \theta_1\theta_2)}$$

then;

$$\begin{aligned}\frac{\partial M_0}{\partial \alpha} &= \frac{\Lambda(\beta_1m_2 + \beta_2\theta_1)(\alpha + \omega + \mu) - (\beta_1m_2 + \beta_2\theta_1)\alpha - (\beta_1\theta_2 + \beta_2m_1)\omega}{\mu(\alpha + \omega + \mu)^2(m_1m_2 - \theta_1\theta_2)}\\ &= \frac{\Lambda(\beta_1m_2 + \beta_2\theta_1)(\omega + \mu) - (\beta_1\theta_2 + \beta_2m_1)\omega}{\mu(\alpha + \omega + \mu)^2(m_1m_2 - \theta_1\theta_2)}\\ &\geq \frac{\Lambda\omega[(\beta_1m_2 + \beta_2\theta_1) - (\beta_1\theta_2 + \beta_2m_1)]}{\mu(\alpha + \omega + \mu)^2(m_1m_2 - \theta_1\theta_2)}\end{aligned}$$

similarly,

$$\begin{aligned}\frac{\partial M_0}{\partial \omega} &= \frac{\Lambda(\beta_1\theta_2 + \beta_2m_1)(\alpha + \omega + \mu) - (\beta_1m_2 + \beta_2\theta_1)\alpha - (\beta_1\theta_2 + \beta_2m_1)\omega}{\mu(\alpha + \omega + \mu)^2(m_1m_2 - \theta_1\theta_2)}\\ &= \frac{\Lambda(\beta_1\theta_2 + \beta_2m_1)(\alpha + \mu) - (\beta_1m_2 + \beta_2\theta_1)\alpha}{\mu(\alpha + \omega + \mu)^2(m_1m_2 - \theta_1\theta_2)}\\ &\geq \frac{\Lambda\alpha[(\beta_1\theta_2 + \beta_2m_1) - (\beta_1m_2 + \beta_2\theta_1)]}{\mu(\alpha + \omega + \mu)^2(m_1m_2 - \theta_1\theta_2)}\end{aligned}$$

when $(\beta_1m_2 + \beta_2\theta_1) > (\beta_1\theta_2 + \beta_2m_1)$, we have that; $\frac{\partial M_0}{\partial \alpha} > 0$ and $\frac{\partial M_0}{\partial \omega} < 0$.
This implies that the basic terrorism multiplication number (M_0) increases when (α) increases and decreases when (ω).
Similarly

$$\frac{\partial M_0}{\partial \theta_1} = \frac{-\Lambda\beta_2(m_1\omega + \theta_1\alpha) + \beta_1(m_2\alpha + \theta_2\omega)(\mu + \mu_2 + k_2) + \beta_2(\alpha + \omega)(\theta_1\theta_2 - m_1m_2)}{\mu m_0(m_1m_2 - \theta_1\theta_2)^2}$$

also in the same way;

$$\frac{\partial M_0}{\partial \theta_2} = \frac{-\Lambda\beta_2(m_1\omega + \theta_1\alpha) + \beta_1(m_2\alpha + \theta_2\omega)(\mu + \mu_1 + k_1) + \beta_2(\alpha + \omega)(\theta_1\theta_2 - m_1m_2)}{\mu m_0(m_1m_2 - \theta_1\theta_2)^2}$$

when, $\beta_2(m_1\omega + \theta_1\alpha) + \beta_1(m_2\alpha + \theta_2\omega)(\mu + \mu_2 + k_2) + \beta_2(\alpha + \omega)(\theta_1\theta_2 - m_1m_2) > 0$,
we have that $\frac{\partial M_0}{\partial \theta_1} < 0$.
Also when, $\beta_2(m_1\omega + \theta_1\alpha) + \beta_1(m_2\alpha + \theta_2\omega)(\mu + \mu_1 + k_1) + \beta_1(\alpha + \omega)(\theta_1\theta_2 - m_1m_2) < 0$,
we have that $\frac{\partial M_0}{\partial \theta_2} > 0$.

This implies that the basic terrorism multiplication number (M_0) increases when (θ_2) increases and decreases when (θ_1) increases. The mathematical interpretations of this result is that, the terrorist group two has a mighty bad effect on the increase rate of terrorism in a population under some specific conditions. Figure (8) confirms this claim.

Variables/Parameters	Descriptions	Units
S	10000	[13]
E	5000	[Assumed]
T_1	200	[13]
T_2	100	[Assumed]
R	100	[Assumed]
Λ	0.08	[Assumed]
β_1	0.11	[13]
β_2	0.08	[Assumed]
α	0.09	[Assumed]
ω	0.87	[Assumed]
θ_1	0.92	[Assumed]
θ_2	0.069	[Assumed]
γ_1	0.2	[Assumed]
γ_2	0.3	[19]
μ_1	0.002	[21]
μ_2	0.003	[Assumed]
k_1	0.1	[Assumed]
k_2	0.15	[Assumed]
μ	0.0012	[19]

4.1 Numerical Simulation and Discussion of Results

Numerical simulation is performed to support our analytical results. Using the value of variables and parameters tabulated in Table 2 we perfomed the numerical simulation of our model.
Table 2. Value of variables and parameters used for model simulation.

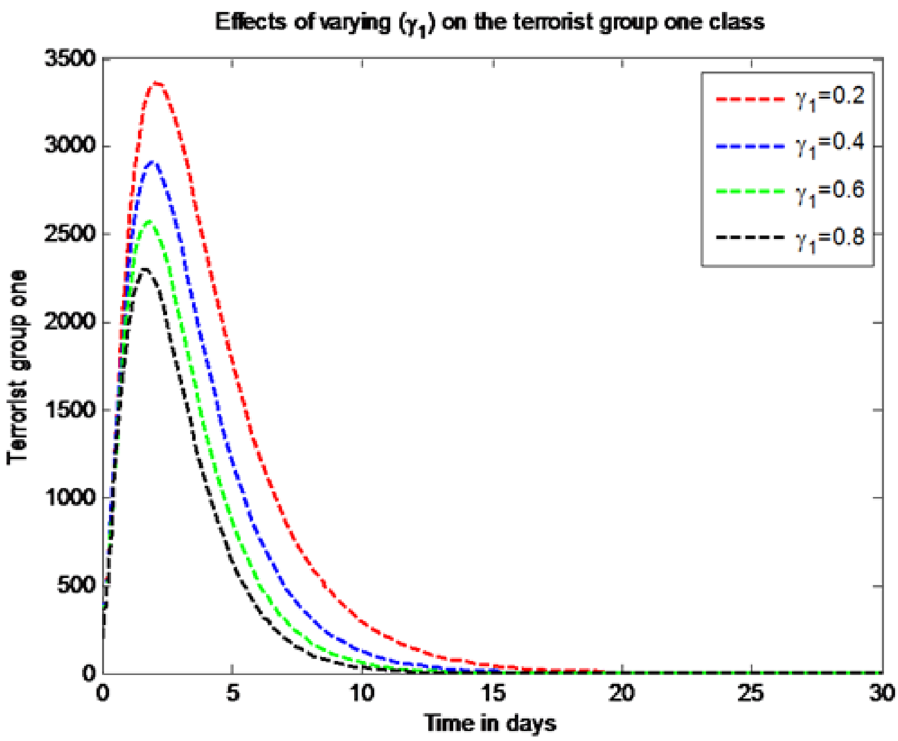

Fig. 2 Effects of varying (γ_1) on the terrorists group one population.

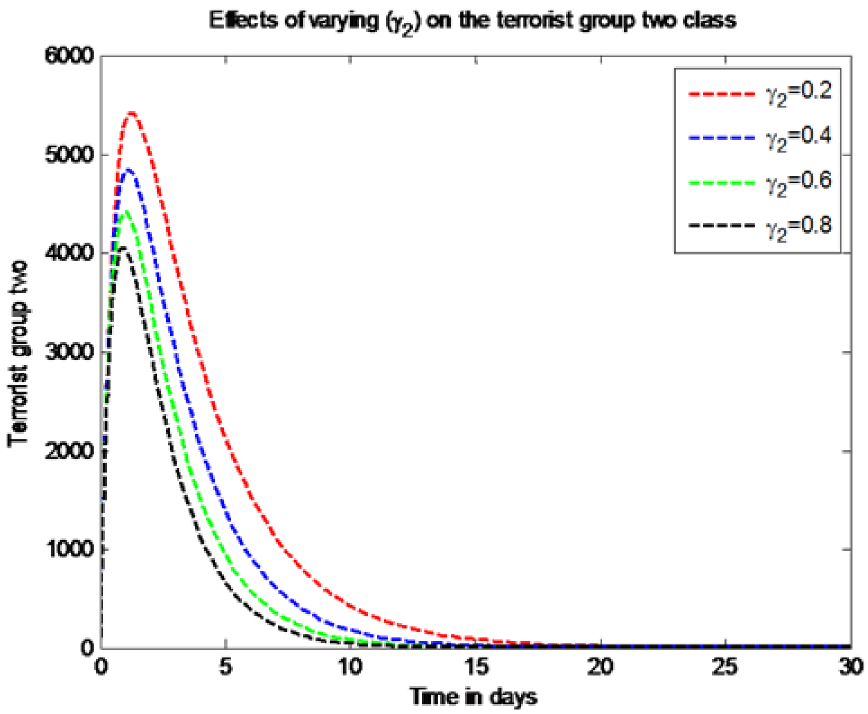

Fig. 3 Effects of varying (γ_2) on the terrorists group two population.

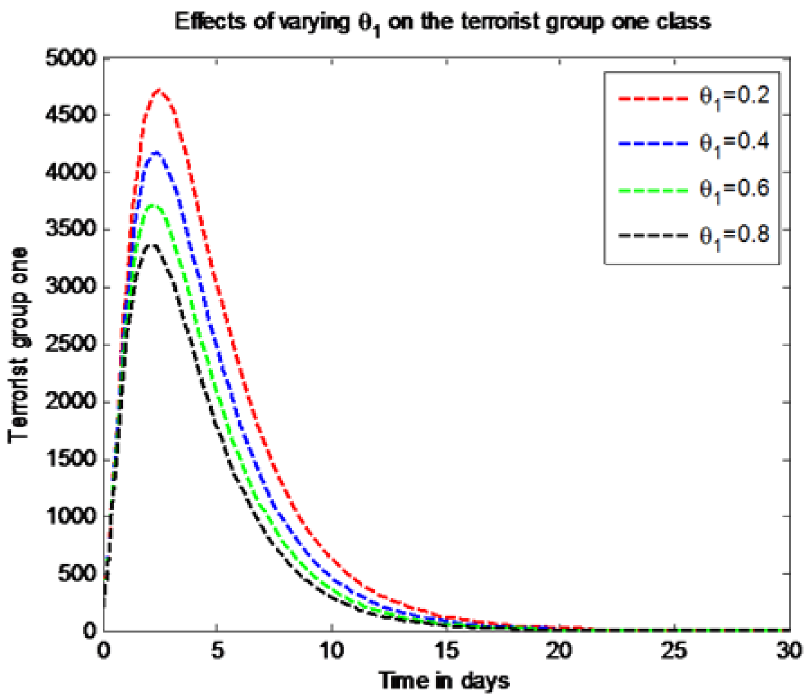

Fig. 4 Effects of varying (θ_1) on the terrorists group one population.

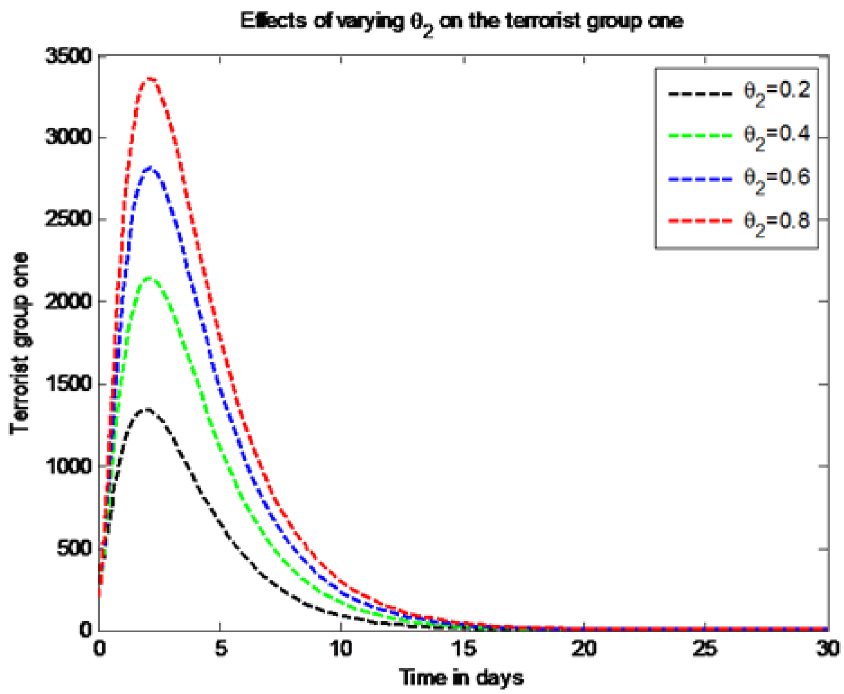

Fig. 5 Effects of varying (θ_2) on the terrorists group one population.

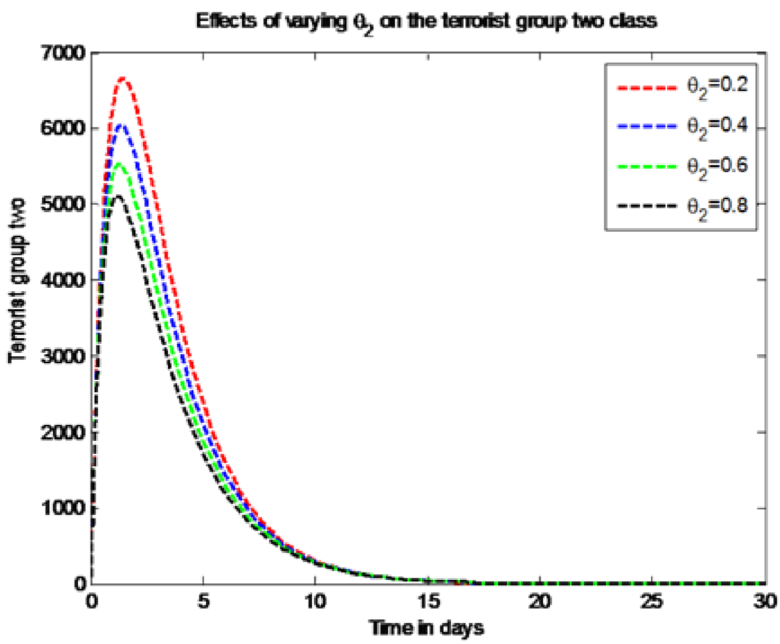

Fig. 6 Effects of varying (θ_2) on the terrorists group two population.

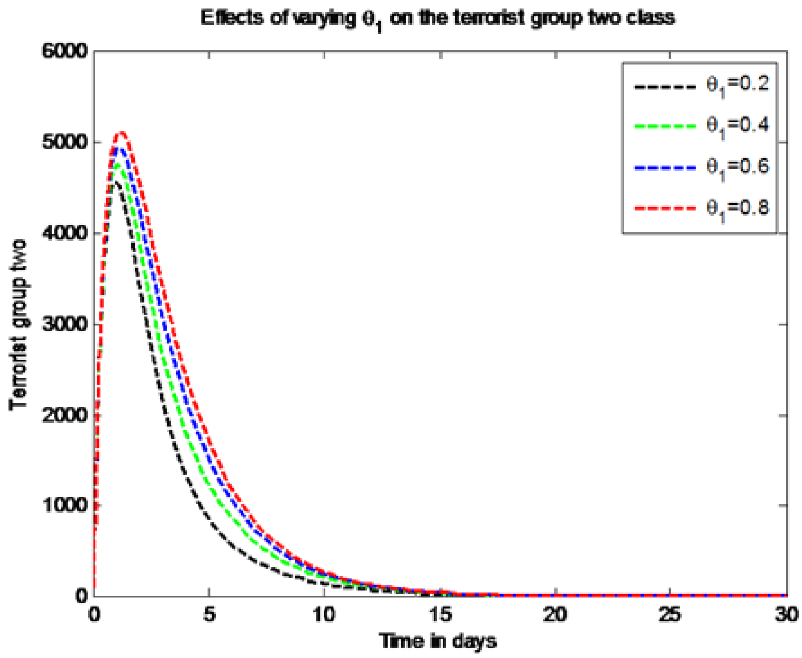

Fig. 7 Effects of varying (θ_1) on the terrorists group two population.

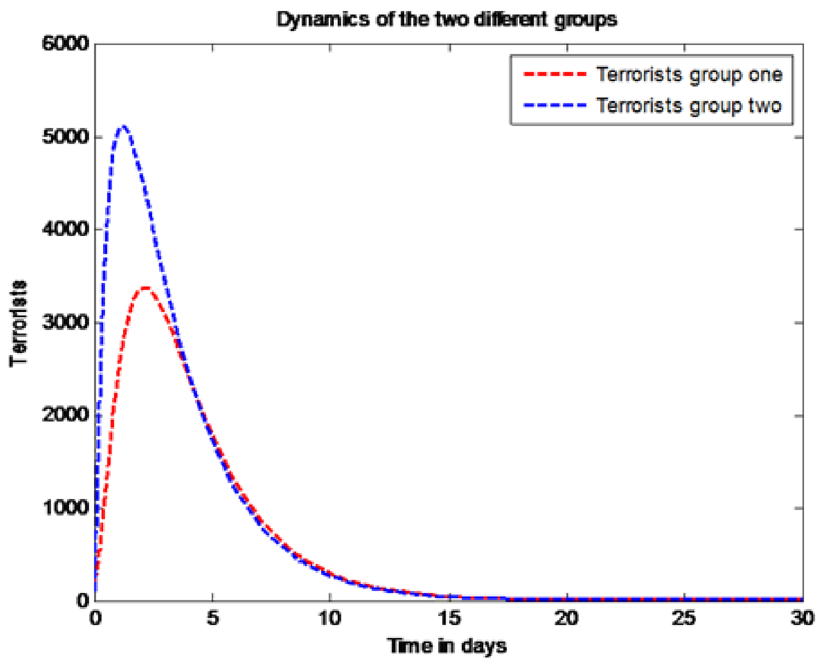

Fig. 8 Dynamics of the two different terrorism groups.

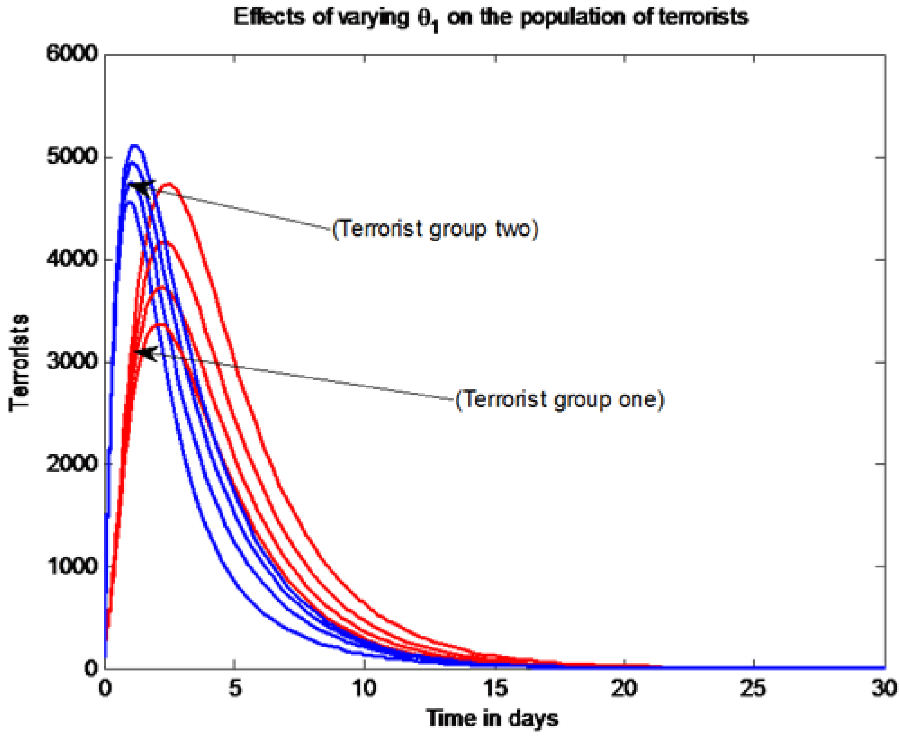

Fig. 9 Effects of varying (θ_1) on terrorists population.

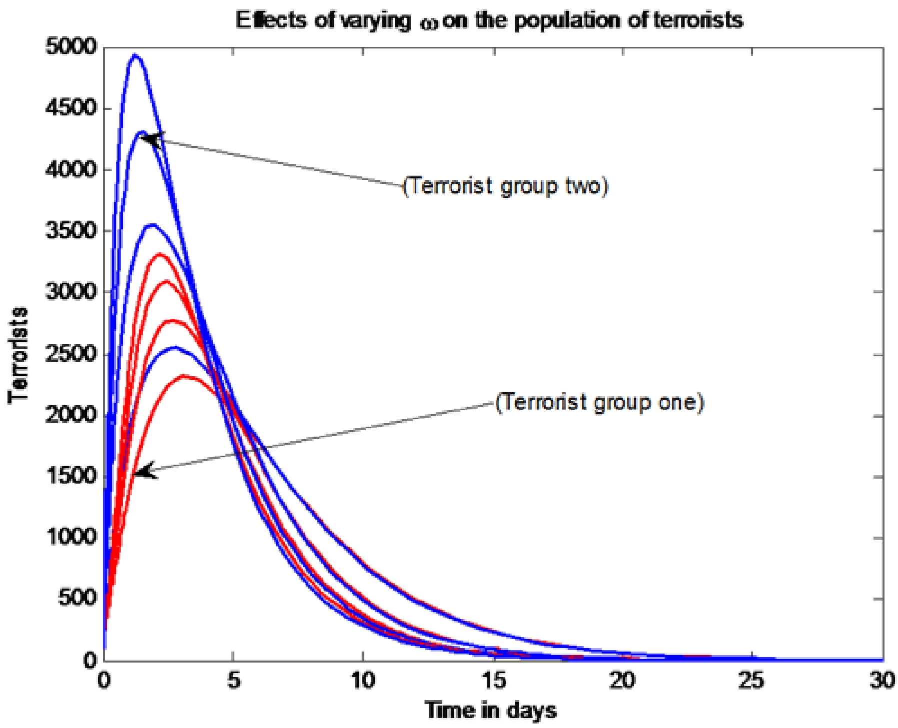

Fig. 10 Effects of varying (ω) on terrorists population.

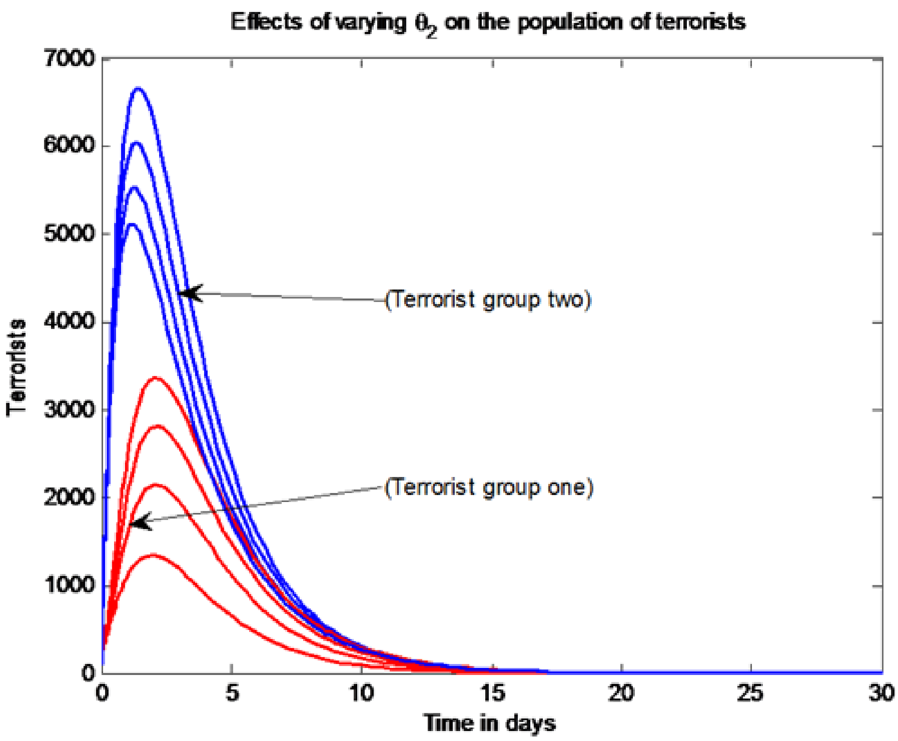

Fig. 11 Effects of varying (θ_2) on terrorists population.

Fig. 2 and Fig. 3 show the effects of varying (γ_1) on the two terrorists groups, where an increase in the value of this parameter leads to a decrease in the population on terrorists. This result implies that, as the millitary sustained intelligence operation to eliminate and take over terrorists camps increases, their population reduces drastically as many terrorists will desert their respective camps and many will equally die.

Fig.4 and Fig.5 also show the effects of varying (θ_1) and (θ_2) on the terrorists group one population dynamics, where an increase in the value of these parameters lead to a decrease in the population of terrorists in group one class. Similar dynamics was obtained in Fig. 6, where an increase in the value of (θ_2) reduces the population of terrorists in group two.

It is also evident from Fig.7, that as terrorists in group one join terrorists in group two, those in group two reinforce and dominate the population as their population increases, the side effect of this reinforcement is seen in Fig. 8 where terrorists in group become so powerfull and then dominates the terrorists population and as a result, terrorism activities increase in such population.

Fig 9-11, shows the impact of vaying (θ_1), (ω) and (θ_2) on the population of both terrorists groups. We observed that the activities of terrorists in group two dominates the activities of those in terrorist group one.

5 Conclusion

A mathematical model of terrorism population dynamics; a case of a community under attack of two terrorists groups is presented in this paper. As evident from our model analysis, the continious use of millitary intelligence operations to take over terrorists camps reduces terrorism in a population, as many terrorists will be eliminated and intending members will be discourged in joining terrorism and if this practice is sustained with time activities of terrorists will be reduced significantly.

Finally, our obtained mathematical results agree with what is obtainable in real life and as such the obtained results can be applied to reduce terrorism activities in a population.

Conflict of interest:
The Authors declare no conflict of interest in this research work.

Data Availability:
The data supporting this work are previously published articles and they have been duly cited in this paper.

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References

1. Agusto, F.B. (2009). Optimal Chemoprophylasis and Treatment Control Strategies of Tuberculosis Transmission Model, World J. Model. Simul. 5: 163-175.

2. Atokolo, W. and Mbah, G.C.E.(2020). Modeling the control of zika virus vector population using the sterile insect technology. Journal of Applied Mathematics, Pp.1-12.

3. Atokolo, W., Aja, R.O., Aniaku, S.E., Onah, S.I., Mbah, G.C.E. (2022). Approximate solution of the fractional order sterile insect technology model via the laplace-adomian decomposition method for the spread of zika virus disease. International Journal of Mathematics and Mathematical Sciences, Pp.1-24.

4. Bakker, E. (2014). Forecasting the unpredictable; a review of forecasts on terrorism. 2000-2012, International Centre for Counter-Terrorism.

5. Bennet, J. (2021). In Chaos, Palestinians Struggle for way out, The New York Times, 1,10-11.

6. Bowie, N. (2021). Terrorism databases and data sets; a new inventory. Perspectives on Terrorism. Leiden University. XV(2),ISSN 2334-3745.

7. Burleigh, M. (2009). Blood and rage; A cultural history of terrorism, Columbia University Press.

8. Burgess, M. (2003). A brief history of Terrorism, Washington, D.C; Center for Defense Information (CDI), The Rhetorical Dimensions of Terrorism, (2nd edition). Thousand oaks, C.A: Sage.

9. Bruce, H. (1988). Inside Terrorism, New York Columbia University Press.

10. Bruce, H. (2006). Inside Terrorism, New York Columbia University Press, 2nd edition.

11. Collins, O.C., Govinder, K.S. (2016). Stability and optimal vaccination of a waterborne disease model with multiple water sources. Natural Resources Modeling 29: 426-447.

12. Collins, O.C., Govinder, K.S. (2014). Incorporating heterogeneity into the transmission dynamics of a waterborne disease model. Journal of Theoretical Biology, 356:133-143.

13. Crenshaw, M. (1981). The causes of terrorism. Comparative Polities, 13(4):379-399.

14. Lasalle, J.P. (1976). The stability of dynamical systems. Regional Conference Series in Applied Mathematics, Society for Industry and Applied Mathematics.

15. Lopes, A.M., Machado, J.A.T., Marta, M.E. (2016). Analysis of global terrorism dynamics by means of entropy and state space portrait. Nonlinear Dynamics, 85(3):1547-1560.

16. Okoye, C., Collins, O.C., Mbah, G.C.E. (2020). Mathematical approach to the analysis of terrorism dynamics. Security Journal, https://doi.org/10.1057/s41284-020-00235-5.

17. Ramon, S. (2021). Understanding lone wolf terrorism: global patterns, motivations and preventions. Perspectives on Terrorism. Leiden University. XV(2),ISSN 2334-3745.

18. Simon, J.D. (1994). The Terrorists Trap. Bloomington, Indian University Press.

19. Tchuenche, J.M., Khamis, S.A., Agusto, F.B., Mpeche, S.C. (2011). Optimal control and sensitivity analysis of an influenza model with treatment and vaccination. Acta Biotheor, 59:1-28.

20. Tishkov, V. (2004). Life in a War-Torn Society. University of California Press.

21. Udoh, I.J., and Oladejo, M.O. (2019). Optimal human resources allocation in counter-terrorism (ct) operation; a mathematical deterministic model. International Journal of Advances in Scientific Research and Engineering, 5(1):96-115.

22. Udwadia, F.G., Leitmann, L.M., Lambertini, O. (2006). A dynamical model of terrorism. Discrete Dynamics in Nature and Society, https:// doi.org/10.1155/DDNS/85653.

23. van den Driessche, P., and Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Mathematical Bioscience, 180:29-48.

24. White, J.R. (2011). Terrorism and Homeland Security. Belmont, C.A. Wadsworth, 7nd edition.