International Journal of Mathematical Analysis and Modelling

(Formerly Journal of the Nigerian Society for Mathematical Biology)

Volume 6, Issue 2 (Oct.-Nov.), 2023

ISSN (Print): 2682 - 5694
ISSN (Online): 2682 - 5708

International Journal of Mathematical Analysis and Modelling
Volume 6, Issue 2, October-November 2023, pages 418 - 427

Modelling 3D stokes flows in one-sided bumpy curved channels

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Abstract

The mathematical models of 3D flows through curved channels with one-sided bumps are developed. Two channel configurations are considered, namely, the top bumpy wall-smooth wall (T-S) configuration and the smooth wall-bottom bumpy wall (S-B) configuration. The analytical solutions of the complex flow models are given. The dependence of the flows on the parameters of the problems are shown explicitly through the analytical solutions. The present work gives a fundamental insight to describing and understanding flows in channels with protrusions.

Keywords and phrases: bumps; radius of curvature; wavenumber; phase shift; flow rate

1 Introduction

For decades, flow in channels have been investigated extensively by researchers in the scientific community. Since the pioneering work of Poiseuille, several analyses of pressure driven flows have emerged with significant practical applications in science and engineering. In particular, due to the manifestations of rough channels in various aspects of science and engineering, flows in channels enclosed or bounded by rough walls have been investigated to understand the effects of both two-dimensional (2D) and three-dimensional (3D) surface roughness on the flow characteristics in conduits with uneven walls. The flow naturally depends on the morphology of the surface roughness and can be considered to be a perturbation of the primary flow, i.e., the flow in a smooth conduit devoid of uneven walls. Wang [1,2] presented a perturbation analysis for flows in straight channels with corrugations. The analysis of Stokes flow in a corrugated pipe was given by Phan-Thien [3]. Flows in annuli with longitudinal grooves were presented by Moradi and Floryan [4]. The enhancement of mass transfer through a sinusoidal-wavy walled channel was examined by Nishimara and Kojima [5], for an unsteady flow. Convective heat transfer in periodic wavy passages was studied by Wang and Vanka [6]. Poiseuille flow of a Bingham fluid in a channel with a superhydrophobic groovy wall was

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recently presented by Rahmani, and Taghari [7]. The works in Refs. [1]-[7] demonstrate that perturbed boundaries have significant impacts on the flow behaviours, and can be used to achieve flow augmentation to a certain extent depending on physical and geometric parameters associated with the flow problem. Dean's problem [8] has also been extended to include the effects of surface roughness, to examine the change in the flow characteristics in curved channels with rough boundaries. Peterson [9] studied steady flow through a curved tube with wavy walls perturbing from the Deans problem. Okechi and Asghar [10,11] investigated steady Stokes flows in corrugated curved channels perturbing from the primary flow in a smooth curved channel without corrugations.
The theoretical models of 3D flows in straight channels and tubes with 3D bumps have been developed for both pressure and non-pressure driven flows. Stokes flow in bumpy tube and channel were reported by Wang [12,13]. The problems were extended by Yu and Wang [14,15] to include the analyses of flows in bumpy channel and tube enclosing a porous medium to determine the effects of the bumps subject to the permeability of the medium. For flows generated by electroosmotic mechanism, the works of Chang et al. [16] and lei et al. [17] can be referred for bumpy channel and tube, respectively. However, the study of 3D flow in curved channel with 3D bumps has not be considered so far.

In the present work, the models of 3D inertia free flow of a Newtonian fluid in curved channels are given. The essence of this work is to expand the current literature on channel flows by developing mathematical models for flows in bumpy curved channels, which is a salient advancement in the area of bumpy channel flows. Two distinct cases of the flow geometry are considered, including, a curved channel with bumpy top wall and a smooth bottom wall as well as a curved channel with smooth top wall and a bumpy bottom wall. The analytical solutions for the Newtonian flow are derived via boundary perturbation analysis, for each case, with the relative amplitude of the bumps as the perturbation parameter. In the subsequent sections, the 3D mathematical models are given, followed by the analytical solutions and the results are expressed explicitly as functions of the inherent parameters of the problems.

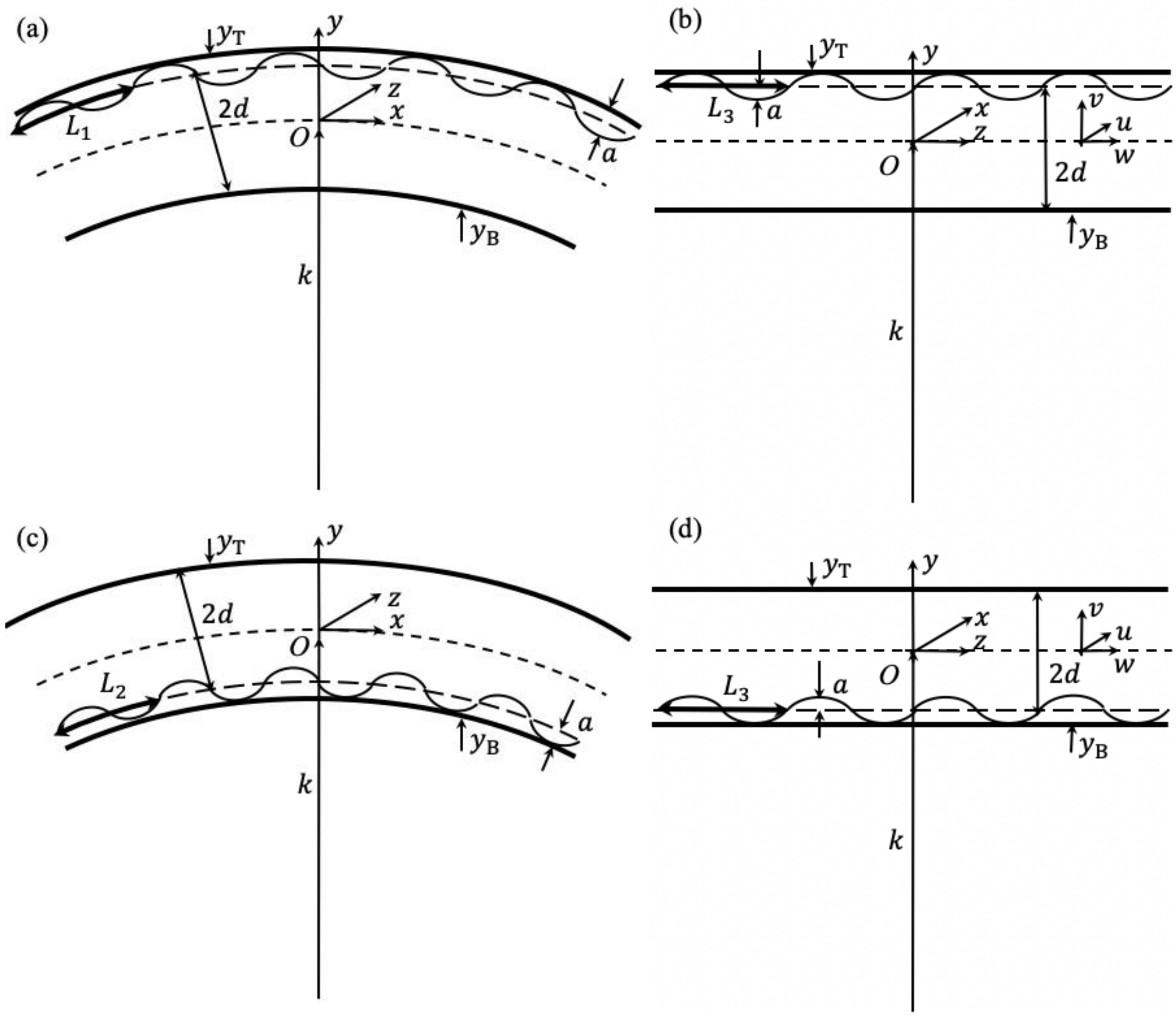

Fig. 1. Flow schematic. (a) and (b) are the axial and cross sections, respectively, of the T-S configuration, while (c) and (d) are the axial and cross sections, respectively, of the S-B configuration. The flow is pressure driven, along the curvilinear x-direction.

2 Mathematical model

Consider a flow through a mildly curved channel of radius of curvature k. Let (x, y, z) denote the curvilinear coordinates for the flow geometry. The flow is assumed to be incompressible and generated by a mean pressure gradient applied in the x-direction. Two distinct flow cases are of interest: A case where the flow is enclosed within a bumpy top wall and a smooth bottom wall and a case where the flow bounded by a smooth top wall and a bumpy bottom wall. The walls are set apart by a mean distance $2d$, the wavelength of the bumps of amplitude a in the x direction are taken to be L_1 and L_2 as shown in Fig. 1, whereas L_2 is the wavelength in the z direction.

The equations governing three-dimensional incompressible steady flow are given by

$$\nabla \cdot \mathbf{U} = 0, \tag{1}$$
$$\mu \nabla^2 \mathbf{U} - \nabla p = \mathbf{0}, \tag{2}$$

The Eq. (1) is the continuity equation, whereas Eq. (2) is the momentum equation. The vector $\mathbf{U} = (u, v, w)$ is the velocity in the curvilinear coordinates (x, y, z), μ is the dynamic viscosity of the fluid and p is the pressure. The flow is assumed to be dominated by the viscous forces, thus the Stokes approximation is used. Suppose $\mathcal{G}$ is the mean pressure gradient driving the flow, we can normalize (x, y, z), (u, v, w) and pressure gradient in Eqs. (1) and (2) by d, $\mathcal{G}d^2/\mu$, and $\mathcal{G}$, respectively. Thus, we have the following equations governing the 3D flow

$$\partial_y\big((y+k)v\big) + k\partial_x u + (y+k)\partial_z w = 0, \tag{3}$$
$$(y+k)^2\partial_y\Big((y+k)^{-1}\partial_y\big((y+k)u\big)\Big) + 2k\partial_x v + k^2\partial_{xx}u + (y+k)^2\partial_{zz}u = k(y+k)\partial_x p, \tag{4}$$
$$(y+k)^2\partial_y\Big((y+k)^{-1}\partial_y\big((y+k)v\big)\Big) - 2k\partial_x u + k^2\partial_{xx}v + (y+k)^2\partial_{zz}v = (y+k)^2\partial_y p, \tag{5}$$
$$(y+k)\partial_y\Big((y+k)\partial_y w\Big) + k^2\partial_{xx}w + (y+k)^2\partial_{zz}w = (y+k)^2\partial_z p, \tag{6}$$

where $\partial_j(\cdot)$ is the derivative with respect to j. Let the bumps be sinusoidal, then the no-slip boundary conditions for the top bumpy wall-smooth wall (T-S) configuration is given by

$$u = v = w = 0 \text{ at } y = \begin{cases} y_{\mathrm{T}} = 1 + \varepsilon\sin(\alpha x)\sin(\vartheta z) \\ y_{\mathrm{B}} = -1 \end{cases} \tag{7}$$

And that of the smooth wall- bottom bumpy wall (S-B) read

$$u = v = w = 0 \text{ at } y = \begin{cases} y_{\mathrm{T}} = 1 \\ y_{\mathrm{B}} = -1 + \varepsilon\sin(\alpha x)\sin(\vartheta z) \end{cases}. \tag{8}$$

Here, $\varepsilon = a/d$, $\alpha = 2\pi d/L_1 \approx 2\pi d/L_2$ and $\vartheta = 2\pi d/L_3$ are the relative amplitude, longitudinal wavenumber and transverse wavenumber, respectively.

3 Analytical method and results

We apply perturbation method to obtain the solutions to Eqs. (3)-(8). The velocity and the pressure can be written in the following regular perturbation expansions for small ε, given by

$$u = \sum_{\beta=0}^{\sigma} \varepsilon^\beta u_\beta, \tag{9}$$
$$v = \sum_{\beta=0}^{\sigma} \varepsilon^\beta v_\beta, \tag{10}$$

(11)

$$w = \sum_{\beta=0}^{\sigma} \varepsilon^{\beta} w_{\beta}, \qquad (12)$$
$$p = \sum_{\beta=0}^{\sigma} \varepsilon^{\beta} p_{\beta}.$$

The boundary conditions in Eqs. (7) and (8) can be expanded as

$$u|_{y=y_{\mathrm{T}}} = \sum_{\beta=0}^{\sigma} \varepsilon^{\beta} \left(\sin^{\beta}(\alpha x)\sin^{\beta}(\vartheta z)/\beta!\right)^{1-j} \partial_{y^{\beta}} u\Big|_{y=1} = 0, \qquad (13)$$
$$v|_{y=y_{\mathrm{T}}} = \sum_{\beta=0}^{\sigma} \varepsilon^{\beta} \left(\sin^{\beta}(\alpha x)\sin^{\beta}(\vartheta z)/\beta!\right)^{1-j} \partial_{y^{\beta}} v\Big|_{y=1} = 0, \qquad (14)$$
$$w|_{y=y_{\mathrm{T}}} = \sum_{\beta=0}^{\sigma} \varepsilon^{\beta} \left(\sin^{\beta}(\alpha x)\sin^{\beta}(\vartheta z)/\beta!\right)^{1-j} \partial_{y^{\beta}} w\Big|_{y=1} = 0, \qquad (15)$$
$$u|_{y=y_{\mathrm{B}}} = \sum_{\beta=0}^{\sigma} \varepsilon^{\beta} \left(\sin^{\beta}(\alpha x)\sin^{\beta}(\vartheta z)/\beta!\right)^{j} \partial_{y^{\beta}} u\Big|_{y=-1} = 0, \qquad (16)$$
$$v|_{y=y_{\mathrm{B}}} = \sum_{\beta=0}^{\sigma} \varepsilon^{\beta} \left(\sin^{\beta}(\alpha x)\sin^{\beta}(\vartheta z)/\beta!\right)^{j} \partial_{y^{\beta}} v\Big|_{y=-1} = 0, \qquad (17)$$
$$w|_{y=y_{\mathrm{B}}} = \sum_{\beta=0}^{\sigma} \varepsilon^{\beta} \left(\sin^{\beta}(\alpha x)\sin^{\beta}(\vartheta z)/\beta!\right)^{j} \partial_{y^{\beta}} w\Big|_{y=-1} = 0. \qquad (18)$$

For $j = 0$ we get the T-S configuration with corresponding boundary conditions, while for $j = 1$ we have the B-S configuration.

The notation $\partial_{j^{\beta}}(\cdot)$ is βth the derivative with respect to *j*. The differential equations at each perturbation order are solved subject to the corresponding boundary conditions. We stop at the second-order of perturbation, i.e., $\sigma = 2$, since the terms of $\sigma > 2$ are small for $\varepsilon \ll 1$.

***O*(1) problem**
At the zeroth-order, the curved channel is smooth and free of bumps, hence, the velocity vector is given by $\mathbf{U} = (u_0^j, 0, 0)$. The velocity components v_0^j and w_0^j in *y* and *z* directions, respectively, are zero when $\varepsilon = 0$. The pressure $p_0^j = -x$ generates the one-dimensional flow. Therefore, Eqs. (3)-(8) with Eqs. (9)-(12) and Eqs. (13)-(18) read

$$(y+k)^2 \partial_y \left((y+k)^{-1} \partial_y \left((y+k) u_0^j \right) \right) = k(y+k), \qquad (19)$$
$$u_0^j\big|_{y=1} = u_0^j\big|_{y=-1} = 0.$$

The solution of Eq. (19) is expressed as

$$u_0^j = a_0^j (y+k) + b_0^j (y+k)^{-1} + k(y+k)[1 - 2\ln{(y+k)}]/4, \qquad (20)$$

where

$$a_0^j = (k+1)^2 \ln(k+1)/8 - (k-1)^2 \ln(k-1)/8 - k/4, \tag{21}$$

and

$$b_0^j = (k+1)^2 (k-1)^2 \ln(k-1/k+1)/8. \tag{22}$$

***O*(*ε*) problem**

The velocity vector for the first-order equation is given by $\mathbf{U} = (u_1^j, v_1^j, w_1^j)$. At second-order, the flow is 3D and the pressure gradients along (x, y, z) are non-zero due to the bumpy walls. Eqs. (3)-(8) with Eqs. (9)-(12) and Eqs. (13)-(18) allow the solutions

$$u_1^j = U_1^j \sin(\alpha x) \sin(\vartheta z), \tag{23}$$

$$v_1^j = V_1^j \cos(\alpha x) \sin(\vartheta z), \tag{24}$$

$$w_1^j = W_1^j \cos(\alpha x) \cos(\vartheta z), \tag{25}$$

$$p_1^j = P_1^j \cos(\alpha x) \sin(\vartheta z), \tag{26}$$

using Eqs. (23)-(26), we have

$$\partial_y \left((y+k) V_1^j \right) + \alpha k U_1^j - \vartheta (y+k) W_1^j = 0, \tag{27}$$

$$(y+k)\partial_y \left((y+k)\partial_y U_1^j \right) - 2\alpha k V_1^j - (\vartheta^2 (y+k)^2 + \alpha^2 k^2 + 1) U_1^j = -\alpha k (y+k) P_1^j, \tag{28}$$

$$(y+k)\partial_y \left((y+k)\partial_y V_1^j \right) - 2\alpha k U_1^j - (\vartheta^2 (y+k)^2 + \alpha^2 k^2 + 1) V_1^j = (y+k)^2 \partial_y P_1^j, \tag{29}$$

$$(y+k)\partial_y \left((y+k)\partial_y W_1^j \right) - (\alpha^2 k^2 + \vartheta^2 (y+k)^2) W_1^j = \vartheta (y+k)^2 P_1^j, \tag{30}$$

with boundary conditions

$$U_1^j\big|_{y=1} = -(1-j)\partial_y u_0^j\big|_{y=1}, \tag{31}$$

$$U_1^j\big|_{y=-1} = -j \partial_y u_0^j\big|_{y=-1}, \tag{32}$$

$$V_1^j{}_{y=1} = V_1^j{}_{y=-1} = 0, \tag{33}$$

$$W_1^j\big|_{y=1} = W_1^j\big|_{y=-1} = 0. \tag{34}$$

The solutions of the 3D coupled system Eqs. (27)-(30) are obtained as

$$U_1^j = \frac{1}{2}\Big(a_1^j I_{\alpha k+1}\big(\vartheta(y+k)\big) + b_1^j K_{\alpha k+1}\big(\vartheta(y+k)\big) + c_1^j I_{\alpha k-1}\big(\vartheta(y+k)\big) + d_1^j K_{\alpha k-1}\big(\vartheta(y+k)\big)\Big), \tag{35}$$

$$V_1^j = \frac{1}{2}\Big(a_1^j I_{\alpha k+1}\big(\vartheta(y+k)\big) + b_1^j K_{\alpha k+1}\big(\vartheta(y+k)\big) - c_1^j I_{\alpha k-1}\big(\vartheta(y+k)\big) - d_1^j K_{\alpha k-1}\big(\vartheta(y+k)\big) + e_1^j (y+k) I_{\alpha k}\big(\vartheta(y+k)\big) + f_1^j (y+k) K_{\alpha k}\big(\vartheta(y+k)\big)\Big), \tag{36}$$

$$W_1^j = \frac{1}{2\vartheta}\Big(e_1^j\vartheta(y+k)I_{\alpha k+1}\big(\vartheta(y+k)\big) - f_1^j\vartheta(y+k)K_{\alpha k+1}\big(\vartheta(y+k)\big) \tag{37}$$

$$+\left(\vartheta\big(a_1^j - c_1^j\big) + f_1^j(\alpha k+2)\right)I_{\alpha k}\big(\vartheta(y+k)\big)$$

$$+\left(\vartheta\big(d_1^j - b_1^j\big) + e_1^j(\alpha k+2)\right)K_{\alpha k}\big(\vartheta(y+k)\big)\Big),$$

$$P_1^j = e_1^j I_{\alpha k}\big(\vartheta(y+k)\big) + f_1^j K_{\alpha k}\big(\vartheta(y+k)\big). \tag{38}$$

where I_ϖ and K_ϖ are order ϖ modified Bessel functions of the first and second kind, respectively.

The functions $a_1^j, b_1^j, c_1^j, d_1^j, e_1^j$ and f_1^j can be obtained algebraically from the boundary conditions in Eqs. (31)-(34).

$O(\varepsilon^2)$ problem

To proceed, we introduce an overbar which represents the average with respect to *z*. That is to say,

$$\bar{m} = \frac{\vartheta}{2\pi}\int_0^{2\pi/\vartheta} m\, dz. \tag{39}$$

This procedure is required as only the *z*-averaged second-order terms are needed to evaluate the flow rate [11-16]. The first-order solution is periodic in the *y*-direction, such that the net flow rate through the channel is unaffected by the terms of the first-order. Taking Eq. (39) into consideration, Eqs. (3)-(6) with Eqs. (9)-(12) become

$$\partial_y\left((y+k)\bar{v}_2^j\right) + k\partial_x\bar{u}_2^j = 0, \tag{40}$$

$$(y+k)^2\partial_y\left((y+k)^{-1}\partial_y\left((y+k)\bar{u}_2^j\right)\right) + 2k\partial_x\bar{v}_2^j + k^2\partial_{xx}\bar{u}_2^j = k(y+k)\partial_x p_2^j, \tag{41}$$

$$(y+k)^2\partial_y\left((y+k)^{-1}\partial_y\left((y+k)\bar{v}_2^j\right)\right) - 2k\partial_x\bar{u}_2^j + k^2\partial_{xx}\bar{v}_2^j = (y+k)^2\partial_y\bar{p}_2^j, \tag{42}$$

$$(y+k)\partial_y\left((y+k)\partial_y\bar{w}_2^j\right) + k^2\partial_{xx}\bar{w}_2^j = 0, \tag{43}$$

with the corresponding boundary conditions

$$\bar{u}_2^j\big|_{y=1} = -\frac{1}{4}(1-j)\big(1-\cos(2\alpha x)\big)\big(\partial_y U_1^j - \partial_{yy}u_0^j\big)\big|_{y=1}, \tag{44}$$

$$\bar{u}_2^j\big|_{y=-1} = -\frac{1}{4}j\big(1-\cos(2\alpha x)\big)\big(\partial_y U_1^j - \partial_{yy}u_0^j\big)\big|_{y=-1}, \tag{45}$$

$$\bar{v}_2^j\big|_{y=1} = -\frac{1}{4}(1-j)\sin(2\alpha x)\partial_y V_1^j\big|_{y=1}, \tag{46}$$

$$\bar{v}_2^j\big|_{y=-1} = -\frac{1}{4}j\sin(2\alpha x)\partial_y V_1^j\big|_{y=-1}, \tag{47}$$

$$w_2^j\big|_{y=1} = w_2^j\big|_{y=-1} = 0. \tag{48}$$

By Eqs. (44)-(48), the solutions of Eqs. (40)-(43) take the forms

$$u_2^j = U_2^j + U_3^j \cos(2\alpha x), \tag{49}$$

$$v_2^j = V_2^j \sin(2\alpha x), \tag{50}$$

$$w_2^j = 0, \tag{51}$$

$$p_2^j = P_2^j \sin(2\alpha x). \tag{52}$$

Therefore, we have

$$\partial_y\left((y+k)V_2^j\right) + 2\alpha k U_3^j = 0, \tag{53}$$

$$(y+k)\partial_y\left((y+k)\partial_y U_2^j\right) - U_2^j = 0, \tag{54}$$

$$(y+k)\partial_y\left((y+k)^{-1}\partial_y\left((y+k)U_3^j\right)\right) + 4\alpha k\left(V_2^j - \alpha k U_3^j\right) = -2\alpha k(y+k)P_2^j, \tag{55}$$

$$(y+k)\partial_y\left((y+k)^{-1}\partial_y\left((y+k)V_2^j\right)\right) + 4\alpha k(V_2^j - \alpha k U_3^j) = (y+k)^2\partial_y P_2^j, \tag{56}$$

and the boundary conditions

$$U_2^j\big|_{y=1} = -\frac{1}{4}(1-j)\left(\partial_y U_1^j + \partial_{yy}u_0^j\right)\big|_{y=1}, \tag{57}$$

$$U_2^j\big|_{y=-1} = -\frac{1}{4}j\left(\partial_y U_1^j + \partial_{yy}u_0^j\right)\big|_{y=-1}, \tag{58}$$

$$U_3^j\big|_{y=1} = \frac{1}{4}(1-j)\left(\partial_y U_1^j + \partial_{yy}u_0^j\right)\big|_{y=1}, \tag{59}$$

$$U_3^j\big|_{y=-1} = \frac{1}{4}j\left(\partial_y U_1^j + \partial_{yy}u_0^j\right)\big|_{y=-1}, \tag{60}$$

$$V_2^j\big|_{y=1} = -\frac{1}{4}(1-j)\partial_y V_1^j\big|_{y=1}, \tag{61}$$

$$V_2^j\big|_{y=-1} = -\frac{1}{4}j\partial_y V_1^j\big|_{y=-1}. \tag{62}$$

The solutions of second-order are found to be

$$U_2^j = a_2^j(y+k) + b_2^j/(y+k), \tag{63}$$

$$\begin{aligned} U_3^j = {} & c_2^j(y+k)^{2\alpha k-1} - d_2^j(y+k)^{-2\alpha k-1} \\ & + \frac{1}{4}\left\{e_2^j\left[(2\alpha k-1)^{-1}-1\right](y+k)^{1-2\alpha k} \right. \\ & \left. + f_2^j\left[(2\alpha k+1)^{-1}-1\right](y+k)^{1+2\alpha k}\right\}, \end{aligned} \tag{64}$$

$$\begin{aligned} V_2^j = {} & c_2^j(y+k)^{2\alpha k-1} + d_2^j(y+k)^{-2\alpha k-1} \\ & + \frac{\alpha k}{2}\left[e_2^j(2\alpha k-1)^{-1}(y+k)^{1-2\alpha k} + f_2^j(2\alpha k+1)^{-1}(y+k)^{1+2\alpha k}\right], \end{aligned} \tag{65}$$

$$P_2^{\pm} = e_2^j(y+k)^{-2\alpha k} + f_2^j(y+k)^{2\alpha k}. \tag{66}$$

The coefficients $a_2^j, b_2^j, c_2^j, d_2^j$, e_2^j and f_2^j are readily obtained algebraically using the boundary condition in Eqs. (57)-(62).

The total volumetric flow rate per unit area in the *x*-direction evaluated at $x = 0$, where the cross section is simpler [11-16] reads

$$Q^j = \frac{1}{2}\int_{-1}^{1} \bar{u}^j\big|_{x=0}\, dy = \frac{1}{2}\int_{-1}^{1} \left(\bar{u}_0^j + \varepsilon\bar{u}_2^j + \cdots\right)\big|_{x=0}\, dy. \tag{67}$$

Eq. (67) can be written in the form

$$Q^j = q^j\left(1 - \varepsilon^2\chi^j + O(\varepsilon^4)\right). \tag{68}$$

Here

$$q^j = ka_0^j + b_0^j \ln((k+1)/2(k-1)) - k((k+1)^2\ln(k+1) - (k-1)^2\ln(k-1))/8 \tag{69}$$

is the flow rate of the curved channel with smooth walls and

$$\begin{aligned}\chi^j = 2a_2^j\, k/q^j &+ b_2^j \ln((k+1)/q(k-1)) \\ &+ \left(c_2^j((k+1)^{2\alpha k} - (k-1)^{2\alpha k}) + d_2^j((k+1)^{-2\alpha k} - (k-1)^{-2\alpha k})\right)/2\alpha k q^j \\ &+ e_2^j((2\alpha k - 1)^{-1} - 1)((k+1)^{-2\alpha k+2} - (k-1)^{-2\alpha k+2})/16\alpha k q^j(1-\alpha k) \\ &+ f_2^j((2\alpha k + 1)^{-1} - 1)((k+1)^{2\alpha k+2} - (k-1)^{2\alpha k+2})/16\alpha k q^j(1+\alpha k).\end{aligned} \tag{70}$$

The function χ^j imposes effect of the bumpy curved walls on the flow rate, for both T-S and S-B situations, such that for $\chi^j > 0, Q < q^j$ and for $\chi^j < 0, Q > q^j$, whenever $0 < \varepsilon \ll 1$.

4 Conclusion

Mathematical models describing 3D viscous flows of a Newtonian fluid in one-sided bumpy curved channels is presented. Two configurations are considered including the T-S and S-B configurations. A regular perturbation solution for the 3D complex configurations is derived from perturbation analysis with the relative amplitude as the perturbation parameter. The results show that irrespective of the position of the bumpy wall, the flow rate is a function of χ^j and the amplitude of the bumps. For a fixed pressure gradient, $\varepsilon^2\chi^j$ represents the percentage decrease in the total flow rate through the curved channels due to the flow resistance imposed by the bumps. The increase in the flow resistance is determined by the function χ^j. Maximum and minimum values of χ^j would result in maximal and minimal reductions in the total flow, respectively (see Eq. (68)). The given mathematical models give the descriptions of flows in a curvilinear geometry with sinusoidal 3D surface roughness and can be further extended to include other morphologies of roughness.

References

[1] Wang, C. Y. (1976). Parallel flow between corrugated plates. J. Eng. Mech. Div. 102: 1088-1090.
[2] Wang, C. Y. (1979). On Stokes flow between corrugated plates. J. Appl. Mech. 46:462-464.
[3] Phan-Thien, N. (1980). On Stokes flow of a viscous fluid through corrugated pipes. J. Appl. Mech. 47:961-963.
[4] Moradi, H. V., Floryan, J.M. (2013). Flows in annuli with longitudinal grooves. AIAA, 716:280-315.
[5] Nishimara, T., Kojima, N. Y. (1995). Mass transfer enhancement in a sinusoidal wavy-walled channel for pulsatile flow. Int. J. Heat Mass Trans. 38:1719-1731.
[6] Wang, G., Vanka, S.P. (1995). Convective heat transferrin periodic wavy passages. Int. J. Heat Mass Trans. 38:3219-3230.
[7] Rahmani, H., Taghari, S. M. (2022). Poiseuille flow of a Bingham fluid in a channel with a superhydrophobic groovy wall. J. Fluid Mech. 948, A34.
[8] Dean, W. R. (1928). Fluid motion in a curved channel. Proc. R. Soc. London Ser. A 121:402-420.
[9] Peterson, S. D. (2010). Steady flow through a curved tube with wavy walls. Phys. Fluids 22, 023602.
[10] Okechi, N. F., Asghar, S., (2020). Oscillatory flow in a corrugated curved channel. Eur. J. Mech. B Fluids 84:81-92.
[11] Okechi N. F., Asghar, S. (2020). Stokes flow in a rough curved channel. Eur. J. Mech. B Fluids 84:15-22.
[12] Wang, C. Y. (2004). Stokes flow through a channel with three-dimensional bumpy walls. Phys. Fluids 16:2136–2139.
[13] Wang, C. Y. (2006). Stokes flow through a tube with bumpy wall. Phys. Fluids 18, 078101.
[14] Yu, L. H., Wang, C. Y. (2013). Darcy-Brinkman flow through a bumpy channel. Transp. Porous Med. 97:281-294.
[15] Wang, C. Y., Yu, L. H. (2015). Darcy flow through bumpy tubes. J. Porous Media 18:457–461.
[16] Chang, L., Jian, Y., Buren, M., Sun, Y., (2016). Electroosmotic flow through a microparallel channel with 3D wall roughness. Electrophoresis 37:482-492.
[17] Lei, C., Chang, C. C., Wang, C. Y. (2019). Electroosmotic pumping through a bumpy tube: Boundary perturbation and roughness detection. Phys. Fluids 31, 012001.