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An application of Homotopy Perturbation Method (HPM) for solving Influenza virus model in a population

N.N. Topman[*†], G.C.E. Mbah[†], O.C. Collins[†], and B.C. Agbata[†]

Abstract

This article examined application of Homotopy Perturbation Method (HPM) for solving influenza virus model. Some concepts and assumptions of homotopy perturbation method are discussed and homotopy perturbation method is used to solve differential equations with boundary conditions arising from influenza virus model. The total population is subdivided into eight epidemiological compartments in a homogenous and heterogeneous population. The obtained results as compared to those in existing literature is accurate. Analysis of the homotopy perturbation method showed that the method is flexible and very easy to understand.

Keywords and phrases: Homotopy Perturbation Method; infectious disease model; differential equation; convergent; series solution; Influenza virus model

1 Introduction

The homotopy perturbation method is a very unique tool and very effective technique for evaluating solutions of nonlinear equations without requiring process of linearization. This system was introduced by [4]. HPM is a combination of the perturbation and homotopy methods. The homotopy perturbation method yields a very rapid convergence series in most cases, usually only a few iterations leading to accurate solutions [16]. It is a universal one which can solve various kinds of linear and nonlinear equations. Benefits of HPM which distinguished it from other analytical methods include, HPM is a series expansion method that indirectly dependent on small or large physical parameters [2, 5], Therefore, it can be used to solve both varieties problems. It gives convergent series solutions and the perturbation equation is easy to construct. He used HPM to solve Schrodinger equation initial value problems [3]. Many researchers have used mathematical modelling to analyze infectious diseases and Homotopy Perturbation Method, see [1,6,8,9,10,11, 13,14, 15]. [1] developed basic reproduction numbers of infectious disease models and analysed basic reproduction numbers of many infectious diseases.

[*] Corresponding Author
[†] Department of Mathematics, University of Nigeria, Nsukka, Nigeria

2 The model equations

The model is given below:

$$
\begin{aligned}
&\frac{ds}{dt} = \Lambda - \beta_1 SI_1 - \beta_2 SI_2 - \beta_3 SI_3 - \mu s + \omega R,\\
&\frac{dI_1}{dt} = \beta_1 SI_1 - (\sigma_1 + \gamma_1 + \rho_1 + \mu) I_1,\\
&\frac{dT_1}{dt} = \sigma_1 I_1 - (\delta_1 + \varepsilon_1 + \mu) T_1,\\
&\frac{dI_2}{dt} = \beta_2 SI_2 - (\sigma_2 + \gamma_2 + \rho_2 + \mu) I_2,\\
&\frac{dT_2}{dt} = \sigma_2 I_2 - (\delta_2 + \varepsilon_2 + \mu) T_2,\\
&\frac{dI_3}{dt} = \beta_3 SI_3 - (\sigma_3 + \gamma_3 + \rho_3 + \mu) I_3,\\
&\frac{dT_3}{dt} = \sigma_3 I_3 - (\delta_3 + \varepsilon_3 + \mu) T_3,\\
&\frac{dR}{dt} = \delta_1 T_1 + \gamma_1 I_1 + \delta_2 T_2 + \gamma_2 I_2 + \delta_3 T_3 + \gamma_3 I_3 - \mu R - \omega R,
\end{aligned}
\tag{1}
$$

where

$k_1 = \sigma_1 + \gamma_1 + \rho_1 + \mu$

$k_2 = \sigma_2 + \gamma_2 + \rho_2 + \mu$

$\alpha_1 = \delta_1 + \varepsilon_1 + \mu$

$\alpha_2 = \delta_2 + \varepsilon_2 + \mu$

$\alpha_3 = \delta_3 + \varepsilon_3 + \mu$

P is an embedding parameter in [0,1]. Then we construct the Homotopy:

$$
\begin{aligned}
&(1-\rho)\frac{ds}{dt} + \rho\left[\frac{ds}{dt} - \Lambda + \beta_1 SI_1 + \beta_2 SI_2 + \beta_3 SI_3 + \mu S - \omega R\right] = 0\\
&(1-\rho)\frac{dI_1}{dt} + \rho\left[\frac{dI_1}{dt} - \beta_1 SI_1 + \left((\sigma_1 + \gamma_1 + \rho_1 + \mu) I_1\right)\right] = 0,\\
&(1-\rho)\frac{dT_1}{dt} + \rho\left[\frac{dT_1}{dt} = \sigma_1 I_1 + (\delta_2 + \varepsilon_2 + \mu) T_1\right] = 0,\\
&(1-\rho)\frac{dI_2}{dt} + \rho\left[\frac{dI_2}{dt} - \beta_2 SI_2 + (\sigma_2 + \gamma_2 + \rho_2 + \mu) T_2\right] = 0,\\
&(1-\rho)\frac{dT_1}{dt} + \rho\left[\frac{dT_2}{dt} = \sigma_2 I_2 + (\delta_2 + \varepsilon_2 + \mu) I_2\right] = 0\\
&(1-\rho)\frac{dI_2}{dt} + \rho\left[\frac{dI_3}{dt} - \beta_3 SI_3 + (\sigma_3 + \gamma_3 + \rho_3 + \mu) I_3\right] = 0\\
&(1-\rho)\frac{dI_3}{dt} + \rho\left[\frac{dT_3}{dt} - \sigma_3 I_3 + (\delta_3 + \varepsilon_3 + \mu) T_3\right] = 0\\
&\frac{dR}{dt} + \rho[-\delta_1 T_1 - \gamma_1 I_1 - \delta_2 T_2 + \gamma_2 I_2 + \delta_3 T_3 + \gamma_3 I_3 + (\mu + \omega) R] = 0.
\end{aligned}
$$

This implies

$$\left.\begin{aligned}
\frac{ds}{dt} &= \rho\left[\frac{ds}{dt} + \Lambda - \beta_1 SI_1 - \beta_2 SI_2 - \beta_3 SI_3 - \mu + \omega R\right]\\
\frac{dI_1}{dt} &= \rho\left[\frac{dI_1}{dt} + \beta_1 SI_1 - (\sigma_1 + \gamma_1 + \rho_1 + \mu)I_1 - \frac{dI_1}{dt}\right]\\
\frac{dT_1}{dt} &= \rho\left[\frac{dT_1}{dt} + \sigma_1 I_1 - (\delta_1 + \varepsilon_1 + \mu)T_1 - \frac{dT_1}{dt}\right]\\
\frac{dI_2}{dt} &= \rho\left[\frac{dI_2}{dt} + \beta_2 SI_2 - (\sigma_2 + \gamma_2 + \rho_2 + \mu)I_2 - \frac{dI_2}{dt}\right]\\
\frac{dI_3}{dt} &= \rho\left[\frac{dI_3}{dt} + \beta_3 SI_3 - (\sigma_3 + \gamma_3 + \rho_3 + \mu)I_3 - \frac{dI_3}{dt}\right]\\
\frac{dR}{dt} &= \rho\left[\frac{dR}{dt} + \delta_1 T_1 + \gamma_1 I_1 + \delta_2 T_2 + \gamma_2 I_2 + \delta_3 T_3 + \gamma_3 I_3 + (\mu + \omega)R - \frac{dR}{dt}\right]
\end{aligned}\right\} \tag{2}$$

$$\begin{aligned}
S_{(T)} &= S_O + \rho S_1 + \rho^2 S_2 + \cdots\\
I_{1(t)} &= I_{1(O)} + \rho I_{1(1)} + \rho^2 S_{1(2)} + \cdots\\
T_{1(t)} &= T_{1(O)} + \rho I_{1(1)} + \rho^2 T_{1(2)} + \cdots\\
I_{2(t)} &= I_{2(O)} + \rho I_{2(1)} + \rho^2 I_{1(2)} + \cdots\\
T_{2(t)} &= T_{2(O)} + \rho I_{2(1)} + \rho^2 T_{1(2)} + \cdots\\
I_{3(t)} &= I_{3(O)} + \rho I_{3(1)} + \rho^2 I_{3(2)} + \cdots\\
T_{3(t)} &= T_{3(O)} + \rho I_{3(1)} + \rho^2 T_{3(2)} + \cdots\\
R_{(t)} &= R_0 + PR_1 + \rho^2 R_2 + \cdots
\end{aligned}$$

The initial condition are as follows:

$$S_{(0)} = u_1, I_1(0) = u_2, T_1(0) = u_3, I_2(0) = u_4, T_2(0) = u_5,$$
$$I_2(0) = u_6, T_3(0) = u_7 \ and\ R(0) = u_8.$$

Carrying out the necessary substitutions, we have that,

$$\frac{d}{dt}(S_O + PS_1 + P^2S_2 + \cdots)$$
$$= P\begin{bmatrix} \Lambda - \beta_1(S_o + PS_1 + P^2S_2 + \cdots)\big(I_{1(0)} + PI_{1(0)} + P^2I_{1(2)}\big)\\ -\beta_2(S_o + PS_1 + P^2S_2 + \cdots)\left(I_{2(0)} + PI_{2_{(0)}} + P^2I_{2(2)+\cdots}\right)\\ -\beta_3(S_o + PS_1 + P^2S_2 + \cdots)\left(I_{3(0)} + PI_{2_{(0)}} + P^2I_{3(2)+\cdots}\right)\\ -\mu(S_O + PS_1 + P^2S_2 + \cdots) + \omega(R_0 + PR_1 + P^2R_2 + \cdots)\end{bmatrix}$$

$$\frac{d}{dt}\big(I_{1(0)} + PI_{2(1)} + P^2I_{2(2)+\cdots}\big)$$
$$= P\begin{bmatrix}\beta_1(S_O + PS_1 + P^2S_2 + \cdots)\big(I_{1(0)} + PI_{1(1)} + P^2I_{1(2)} + \cdots\big)\\ -(\sigma_1 + \gamma_1 + \rho_1 + \mu)\big(I_{1(0)} + pI_{1(1)} + P^2I_{1(2)}\big) + \cdots\end{bmatrix}$$

$$\frac{d}{dt}\left(T_{1(0)} + PT_{1(1)} + P^2T_{1(2)+\cdots}\right) = P\begin{bmatrix}\sigma_1\left(I_{1(0)} + PI_{1(1)} + P^2I_{1(2)} + \cdots\right)\\(\delta_1 + \varepsilon_1 + \mu)\left(T_{1(0)} + PT_{1(1)} + P^2T_{1(2)}\right) + \cdots\end{bmatrix}$$

$$\frac{d}{dt}\left(I_{1(0)} + PT_{2(1)} + P^2I_{2(2)+\cdots}\right) = P\begin{bmatrix}\beta_2(S_O + PS_1 + P^2S_2 + \cdots)\left(I_{2(0)} + PI_{2(1)} + P^2I_{2(2)} + \cdots\right)\\-(\sigma_2 + \gamma_1 + \rho_1 + \mu)\left(I_{2(0)} + pI_{2(1)} + P^2I_{2(2)}\right) + \cdots\end{bmatrix}$$

$$\frac{d}{dt}\left(T_{2(0)} + PT_{2(1)} + P^2T_{2(2)+\cdots}\right) = P\begin{bmatrix}\sigma_2\left(I_{2(0)} + PI_{2(1)} + P^2I_{2(2)} + \cdots\right)\\(\delta_2 + \varepsilon_2 + \mu)\left(T_{2(0)} + PT_{2(1)} + P^2T_{2(2)}\right) + \cdots\end{bmatrix}$$

$$\frac{d}{dt}\left(I_{3(0)} + PI_{3(1)} + P^2I_{3(3)+\cdots}\right) = P\begin{bmatrix}\beta_3(S_O + PS_1 + P^2S_2 + \cdots)\left(I_{2(0)} + PI_{2(1)} + P^2I_{2(2)} + \cdots\right)\\-(\sigma_3 + \gamma_3 + \rho_3 + \mu)\left(I_{3(0)} + pI_{3(1)} + P^2I_{3(1)} + P^2I_{3(2)}\right) + \cdots\end{bmatrix}$$

$$\frac{d}{dt}\left(T_{3(0)} + PT_{3(1)} + P^2T_{3(2)+\cdots}\right) = P\begin{bmatrix}\sigma_3\left(I_{3(0)} + PI_{3(1)} + P^2I_{3(2)} + \cdots\right)\\(\delta_3 + \varepsilon_3 + \mu)\left(T_{3(0)} + PT_{3(1)} + P^2T_{3(2)}\right) + \cdots\end{bmatrix}$$

$$\frac{d}{dt}\left(R_O + PR_1 + P^2R_{2+\cdots}\right) = P\begin{bmatrix}\delta_1\left(T_{1(0)} + PT_{1(1)} + P^2T_{1(2)} + \cdots\right) +\\\gamma_1\left(I_{1(0)} + PI_{1(1)} + P^2I_{1(2)} + \cdots\right) +\\\delta_1\left(T_{2(0)} + PI_{2(1)} + P^2I_{2(2)} + \cdots\right) +\\\gamma_2\left(I_{2(0)} + PI_{2(1)} + P^2I_{2(2)} + \cdots\right) +\\\delta_1\left(T_{3(0)} + PT_{3(1)} + P^2I_{3(2)} + \cdots\right) +\\\gamma_3\left(I_{3(0)} + PI_{3(1)} + P^2I_{3(2)} + \cdots\right) +\\(\mu - \omega)(R_o + PR_1 + P^2R_2 + \cdots)\end{bmatrix}$$

Comparing them according to their identical powers:

$$P^0: \quad \frac{ds}{dt} = 0$$

$$\frac{dI_1}{dt} = 0$$

$$\frac{dT_1}{dt} = 0$$

$$\frac{dI_2}{dt} = 0$$

$$\frac{dT_2}{dt} = 0$$

$$\frac{dI_3}{dt} = 0$$

$$\frac{dT_3}{dt} = 0$$

$$\frac{dR}{dt} = 0.$$

P^1: $\frac{dS_1}{dt} = \Lambda - \beta_1 S_0 I_{1(0)} - \beta_2 S_0 I_{2(0)} - \beta_3 S_0 I_{3(0)} - \mu S_0 + \omega R_0$

$$\frac{dI_1}{dt} = \beta_1 S_0 T_{1(0)} - (\sigma_1 + \gamma_1 + \rho_1 + \mu) I_{1(0)}$$

$$\frac{dT_1}{dt} = \sigma_1 I_{1(0)} - (\delta_1 + \varepsilon_1 + \mu) T_{1(0)}$$

$$\frac{dI_2}{dt} = \beta_2 S_0 I_{2(0)} - (\sigma_2 + \gamma_2 + \rho_2 + \mu) I_{2(0)}$$

$$\frac{dT_2}{dt} = \sigma_2 I_{2(0)} - (\delta_2 + \varepsilon_2 + \mu) T_{2(0)}$$

$$\frac{dI_3}{dt} = \beta_3 S_0 I_{3(0)} - (\sigma_3 + \gamma_3 + \rho_3 + \mu) I_{3(0)}$$

$$\frac{dT_3}{dt} = \sigma_3 I_{3(0)} - (\delta_3 + \varepsilon_3 + \mu) T_{3(0)}$$

$$\frac{dR}{dt} = \delta_1 T_{1(0)} + \gamma_1 I_{1(0)} + \delta_2 T_{2(0)} + \gamma_2 I_{2(0)} + \delta_3 T_{3(0)} + \gamma_3 I_{3(0)} + (\mu - \omega).$$

P^2: $\frac{dS_2}{dt} = \Lambda - \beta_1 S_0 I_{1(0)} - \beta_2 S_0 I_{2(0)} - \beta_3 S_0 I_{3(1)} - \mu S_1 + \omega R_1$

$$\frac{dI_1}{dt} = \beta_1 S_0 T_{1(0)} - (\sigma_1 + \gamma_1 + \rho_1 + \mu) I_{1(1)}$$

$$\frac{dT_1}{dt} = \sigma_1 I_{1(0)} - (\delta_1 + \varepsilon_1 + \mu) T_{1(1)}$$

$$\frac{dI_2}{dt} = \beta_2 S_1 I_{2(1)} - (\sigma_2 + \gamma_2 + \rho_2 + \mu) I_{2(1)}$$

$$\frac{dT_2}{dt} = \sigma_2 I_{2(1)} - (\delta_2 + \varepsilon_2 + \mu) T_{2(1)}$$

$$\frac{dI_3}{dt} = \beta_3 S_1 I_{3(1)} - (\sigma_3 + \gamma_3 + \rho_3 + \mu) I_{3(1)}$$

$$\frac{dT_3}{dt} = \sigma_3 I_{3(1)} - (\delta_3 + \varepsilon_3 + \mu) T_{3(1)}$$

$$\frac{dR}{dt} = \delta_1 T_{1(1)} + \gamma_1 I_{1(1)} + \delta_2 T_{2(1)} + \gamma_2 I_{2(1)} + \delta_3 T_{3(1)} + \gamma_3 I_{3(1)} + (\mu - \omega) R_1.$$

For powers of P^o, we have that

$S_0 = u_1, I_{1(0)} = u_2, T_{1(0)} = u_3, I_{2(0)} = u_4, T_{2(0)} = u_1, T_{2(0)} = u_5$
$I_{2(0)} = u_5, T_{3(0)} = u_7, \ R_0 = U_8$

For POWER s of p[1], we have

$$\frac{ds}{dt} = \Lambda - \beta_1 u_1\, u_2 - \beta_2 u_1\, u_4 - \beta_3 u_1\, u_6 - \mu u_1\, \omega u_8$$
$$\frac{dI_1}{dt} = \beta_1 u_1\, u_2 - (\sigma_1 + \gamma_1 + \rho_1 + \mu) u_2$$
$$\frac{dT_1}{dt} = \sigma_1\, u_2 - (\delta_1 + \varepsilon_1 + \mu) u_3$$
$$\frac{dI_2}{dt} = \beta_2 u_1\, u_4 - (\sigma_2 + \gamma_2 + \rho_2 + \mu) u_4$$
$$\frac{dT_2}{dt} = \sigma_2\, u_4 - (\delta_2 + \varepsilon_2 + \mu) u_5$$
$$\frac{dI_3}{dt} = \beta_3 u_1\, u_6 - (\sigma_3 + \gamma_3 + \rho_3 + \mu) u_6$$
$$\frac{dT_3}{dt} = \sigma_3\, u_6 - (\delta_3 + \varepsilon_3 + \mu) u_7$$
$$\frac{dR}{dt} = \delta_1 u_3 + \gamma_1 u_2 + \delta_2 u_5 + \gamma_2 u_4 + \delta_3 u_7 + \gamma_3 u_6 + (\mu - \omega) u_8.$$

$Now\ let\ u_1 u_2 = u_9, u_1 u_4 = u_{10}, u_1 u_6 = u_{11}. Hence,$ integrating with respect to t, we have

$$S = (\Lambda - \beta_1 u_9 - \beta_2 u_{10} - \beta_3 u_{11} - \mu u_1 - \omega u_8) t$$
$$S_{(1)} = D_1 t \ where\ D_1 = (\Lambda - \beta_3 u_9 - \beta_2 u_{10} - \mu u_1 - \omega u_8)$$
$$I_{1(1)} = [\beta_1 u_9 - (\sigma_1 + \gamma_1 + \rho_1 + \mu) u_2] t$$
$$I_{1(1)} = D_2 t \ where\ D_2 = [\beta_1 u_9 - (\sigma_1 + \gamma_1 + \rho_1 + \mu)]$$
$$T_{1(1)} = [\beta_1 u_2 - (\delta_1 + \varepsilon_1 + \mu_3)] t$$
$$I_{2(1)} = [\beta_2 u_{10} - (\sigma_2 + \gamma_2 + \rho_2 + \mu) u_4] t$$
$$T_{2(1)} = D_4 t \ where\ D_3 = [\beta_2 u_{10} - (\sigma_2 + \gamma_2 + \rho_2 + \mu)] u_4$$
$$T_{2(1)} = [\sigma_2 u_4 - (\delta_2 + \varepsilon_2 + \mu_3) u_5] t$$
$$T_{2(1)} = D_5 t \ where\ D_5 = [\sigma_2 u_4 - (\delta_2 + \varepsilon_2 + \mu) u_5]$$
$$I_{3(1)} = [\beta_3 u_{11} - (\sigma_3 + \gamma_3 + \rho_3 + \mu) u_6] t$$
$$T_{3(1)} = D_6 t \ where\ D_6 = [\beta_3 u_{11} - (\sigma_3 + \gamma_3 + \rho_3 + \mu)]$$
$$T_{3(1)} = [\sigma_3 u_6 - (\delta_3 + \varepsilon_3 + \mu) u_7] t$$
$$T_{3(1)} = D_7 t \ where\ D_7 = [\sigma_3 u_6 - (\delta_3 + \varepsilon_3 + \mu)]$$
$$R_{(1)} = [\gamma_1 u_2 + \delta_1 u_3 + \gamma_2 u_4 + \delta_2 u_5 + \gamma_3 u_6 + \delta_3 u_7 + (\mu - \omega)] t$$
$$R_{(1)} = D_8 t \ where\ D_8 = [\gamma_1 u_2 + \delta_1 u_3 + \gamma_2 u_4 + \delta_2 u_5 + \gamma_3 u_6 + \delta_3 u_7 + (\mu - \omega)] u_8.$$

For the power of P[2], integrating we have,

$$S_2 = (-\beta_1 D_1 D_2 - \beta_2 D_1 D_4 - \beta_3 D_1 D_2 - \mu D_1 + \omega D_8) \frac{t^2}{2}$$
$$I_{1(2)} = (\beta_1 D_1 D_2 - (\sigma_1 + \gamma_1 + \rho_1 + \mu) D_2) \frac{t^2}{2}$$

$$T_{1(2)} = (\sigma_1 D_1 - (\delta_1 + \varepsilon_1 + \mu)D_3)\frac{t^2}{2}$$
$$I_{2(2)} = (\beta_2 D_1 D_4 - (\sigma_2 + \gamma_2 + \rho_2 + \mu)D_2)\frac{t^2}{2}$$
$$T_{2(2)} = (\sigma_2 D_4 - (\delta_2 + \varepsilon_2 + \mu)D_5)\frac{t^2}{2}$$
$$I_{3(2)} = (\beta_3 D_1 D_6 - (\sigma_3 + \gamma_3 + \rho_3 + \mu)D_6)\frac{t^2}{2}$$
$$T_{3(2)} = (\sigma_3 D_6 - (\delta_3 + \varepsilon_3 + \mu)D_7)\frac{t^2}{2}$$
$$R_{(2)} = (\gamma_1 D_2 + \delta_1 D_3 + \gamma_2 D_4 + \delta_2 D_5 + \gamma_3 D_6 + \delta_3 D_7 + (\mu - \omega)D_8)\frac{t^2}{2}.$$

Let the $0(\rho^3)$ be very small such that by asymptotic series approximation:

$$S(t) = u_1 + \rho D_1 t + \rho^2(-\beta_1 D_1 D_2 - \beta_2 D_1 D_4 - \beta_3 D_1 D_6 - \mu D_1 + \omega D_8)\frac{t^2}{2} + o(\rho^3)$$
$$I_1(t) = u_2 + \rho D_2 t + \rho^2(-\beta_1 D_1 D_2 - (\sigma_1 + \gamma_1 + \rho_1 + \mu)D_2)\frac{t^2}{2} + o(\rho^3)$$
$$T_1(t) = u_3 + \rho D_3 t + \rho^2\big(\sigma_1 D_1 - (\delta_1 + \varepsilon_1 + \mu)\big)\frac{t^2}{2} + o(\rho^3)$$
$$I_2(t) = u_4 + \rho D_4 t + \rho^2(\beta_2 D_1 D_2 - (\sigma_2 + \gamma_2 + \rho_2 + \mu)D_2)\frac{t^2}{2} + o(\rho^3)$$
$$T_2(t) = u_5 + \rho D_5 t + \rho^2(\sigma_2 D_1 - (\delta_2 + \gamma_2 + \mu)D_3)\frac{t^2}{2} + o(\rho^3)$$
$$I_3(t) = u_6 + \rho D_6 t + \rho^2(\beta_3 D_1 D_6 - (\sigma_3 + \gamma_3 + \rho_3 + \mu)D_6)\frac{t^2}{2} + o(\rho^3)$$
$$T_3(t) = u_7 + \rho^2\,(\sigma_3 D_6 - (\delta_3 + \varepsilon_3 + \mu)D_5)\,\frac{t^2}{2}\,o(\rho^3)$$
$$R(t) = u_8 + \rho D_8 t + \rho(\gamma_1 D_2 + \delta_1 D_3 + \gamma_2 D_4 + \delta_2 D_5 + \gamma_3 D_6 + \delta_3 D_7 + (\mu - \omega)D_8)\frac{t^2}{2}.$$

We have

$$S(t) = u_1 + \rho D_1 t + \rho^2(-\beta_1 D_1 D_2 - \beta_2 D_1 D_4 - \beta_3 D_1 D_6 - \mu D_1 + \omega D_8)\frac{t^2}{2} < o(\rho^3),$$
$$I_1(t) = u_2 + \rho D_2 t + \rho^2(\beta_1 D_1 D_2 - (\sigma_1 + \gamma_1 + \rho_1 + \mu)D_2)\frac{t^2}{2} < o(\rho^3),$$
$$T_1(t) = u_3 + \rho D_3 t + \rho^2(\sigma_1 D_1 - (\delta_1 + \varepsilon_1 + \mu)D_3)\frac{t^2}{2} < o(\rho^3)$$
$$I_2(t) = u_4 + \rho D_4 t + \rho^2(\beta_2 D_1 D_2 - (\sigma_2 + \gamma_2 + \rho_2 + \mu)D_2)\frac{t^2}{2} < o(\rho^3)$$
$$T_2(t) = u_5 + \rho D_5 t + \rho^2(\sigma_2 D_1 - (\delta_2 + \gamma_2 + \mu)D_3)\frac{t^2}{2} < o(\rho^3)$$

$I_3(t) = u_6 + \rho D_6 t + \rho^2(\beta_3 D_1 D_6 - (\sigma_3 + \gamma_3 + \rho_3 + \mu)D_6)\frac{t^2}{2} < o(\rho^3)$
$T_3(t) = u_7 + \rho^2 D_7 - \rho^2(\sigma_3 D_6 - (\delta_3 + \varepsilon_3 + \mu)D_5)\frac{t^2}{2} < o(\rho^3)$
$R(t) = u_8 + \rho D_8 t + \rho^2(\gamma_1 D_2 + \delta_1 D_3 + \gamma_2 D_4 + \delta_2 D_5 + \gamma_3 D_6 + \delta_3 D_7 + (\mu - \omega)D_8) < 0(\rho^3).$

Thus,

$S(t) \approx u_1 + \rho D_1 t + p^2(-\beta_1 D_1 D_2 - \beta_2 D_1 D_4 - \beta_3 D_1 D_6 - \mu D_1 + \omega D_8)\frac{t^2}{2}$
$I_1(t) \approx u_2 + \rho D_2 t + \rho^2(-\beta_1 D_1 D_2 - (\sigma_1 + \gamma_1 + \rho_1 + \mu)D_2)\frac{t^2}{2}$
$T_1(t) \approx u_3 + \rho D_3 t + \rho^2\big(\sigma_1 D_1 - (\delta_1 + \varepsilon_1 + \mu)\big)\frac{t^2}{2}$
$I_2(t) \approx u_4 + \rho D_4 t + \rho^2(\beta_2 D_1 D_2 - (\sigma_2 + \gamma_2 + \rho_2 + \mu)D_2)\frac{t^2}{2}$
$T_2(t) \approx u_5 + \rho D_5 t + \rho^2(\sigma_2 D_1 - (\delta_2 + \gamma_2 + \mu)D_3)\frac{t^2}{2}$
$I_3(t) \approx u_6 + \rho D_6 t + \rho^2(\beta_3 D_1 D_6 - (\sigma_3 + \gamma_3 + \rho_3 + \mu)D_6)\frac{t^2}{2}$
$T_3(t) \approx u_7 + \rho^2 (\sigma_3 D_6 - (\delta_3 + \varepsilon_3 + \mu)D_5)\frac{t^2}{2}$
$R(t) \approx u_8 + \rho D_8 t + \rho(\gamma_1 D_2 + \delta_1 D_3 + \gamma_2 D_4 + \delta_2 D_5 + \gamma_3 D_6 + \delta_3 D_7 + (\mu - \omega)D_8).$

3 Conclusion

In this article, application of Homotopy Perturbation Method (HPM) for solving an influenza virus model was investigated, with some concepts of homotopy perturbation method and steps involved in HPM presented. It is observed that homotopy perturbation method gives a series solution that is mostly convergent for both linear and nonlinear differential equations [10]. From analysis of the model using HPM, it is discovered that the method is a sound analytical technique which is easy to apply in solving physical problems

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