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Numerical analysis on broken home and its causes

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Abstract

Non-linear $I_H B_H$ mathematical model with XYZ sub-compartment was used to study the dynamics of broken home and marriage repair. The existence and stability of the broken home free and endemic equilibria were discussed. The broken home free equilibrium was locally asymptotically stable with $R_0 < 1$ and unstable with $R_0 > 1$. Numerical simulations were conducted to confirm the analytic results. The findings were that, reducing the contact rate between the intact marriage and broken home, increases the number of marriage that go into separation and educating separators to refrain from divorce can be useful in combating the broken home. The study also shows that from the broken home free equilibrium when all parameters are fixed and 50% variation of beta and k, the intact marriage increases monotonically while separated and divorce are monotonically decreased. Again, from the Endemic equilibrium when all parameters are fixed and 50% variation of beta and k, the intact marriage fluctuates at the initial stage and later increases, the separated and divorce monotonically decreased. In further studies there should be a development of various related topics that will help in analyzing other aspects that are closely related to broken home in order to get further understanding and avoid the problem of broken home that happen to us and to give us information about the effects of broken home.

Keywords and phrases: mathematical model; intact home; broken home; separated; divorce; causes of broken home

1 Introduction

Broken home connotes the division or the separation of parents who have been together and suddenly divorce or terminated their relationship. The institution of marriage is as old as the creation of man. It is the coming together of two persons in love with consent of parents, guardians and witnesses for the purpose of procreation and companionship. It is an institution that play very important-role in the upbringing and the development of human beings. Marriage is the joining together of two matured people who agree to spend their lifetime together. Thus taking joint care or responsibility over their children who are the fruits of the union. [1,16, 24].

Marriage is expected to be an affair of intimacy and compromise, where spouses complement each other. It is also expected to be a life time companionship or unionism, but the current trend in the marriage dyad departs from these traditional assertions, as the marital union is today mere associations where rancor, conflict, savage mutual attacks are endemic. Today the couples tend to dwell together only when things are rosy and prefer to leave or separate when misunderstandings

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occur or when a conflictual situation arises. Despite the threats of the disintegration in marriages, there are still model marriages whose dispositions give hope for badly managed ones. Conflict is said to be good, especially when the tension so generated leads to change which may be thought aggressive, yet ultimately ends in convincingly peaceful resolution. Thus, though conflict is disruptive, it helps to create awareness among different people, groups and society with respect to the existence of boundaries of common interest.

Marital conflict breeds unhealthy relationship in the couples themselves, the children and dependants. For revitalisation of convivial living, counselling must come into play. Some tips on couple's interaction must be introduced as a combing device for couples in conflict [9; 10] Couples require some skills to aid them in the management of crisis point at any likely breaking point caused by pressure generated by conflict as they cohabit.

Globally many marriages are crashing because of faulty foundations. Most broken homes occur as a result of illegal marriages, improper way of paying dowries, in most cases when the bride price is not duly paid, the marital vows are not well maintained this leads to broken home. The incessant rate of the broken home in our society has become very alarming, and this call for an urgent attention from our parents and the government to rehabilitate the collapsed area in most homes. This may have future implication on our children in all spheres of their lives.

Marriage relationships are said to have become generally strained in recent past, especially among literate couples, as indicated by scholars Efforts have, however, been made by scholars, churches, professional organisations, etc. to curb this menace. Yet, the incidence of marital conflict and disharmony appears endemic as the stabilising of personality role of marriage and the family wanes at alarming rates. Scholars, teachers, churches, etc. are at a loss on the options available for identifying the causes of this situation in their attempt to proffer solutions.

This study is therefore hinged on the incessant occurrence of marital conflict among the couples. The couples are by virtue of their education expected to detect conflict signals and hence employ conflicts managing tactics to effectively control the situation without disrupting the marital dyad and cohesion. The existence of this phenomenon (marital conflict) amongst the couples points to poor marital conflict management skills.

The purpose of this research work is geared at achieving the following:

1. To discover the level of marital conflict to broken home amongst couples.
2. To explore the causes of broken home amongst the couples.

2 Review of related literature

The Concept of Marriage and Family

The terms "marriage" and "family", though difficult to separate, are not exactly the same. [15] posits that it is necessary to dichotomise the two words for the purpose of simplicity and understanding. [9] argues that there could be confusion in literature among the authorities due to the snail-shell relationship which exists between marriage and family. He further contends that the arrival of a child in a marriage makes the marriage and family two social systems operating under the roof at the same time as a subsystem of marriage and marriage as a subsystem of family. Though the two terms (concepts) seem inseparable and are often used interchangeably, they possess features that mark them out as different. This is because the features of marriage as a heterosexual union of consenting adults cannot be said to be the same with family, which is basically a social grouping that has people who are either consequently or conjugally related or both ways related. [18] contends that the concept of marriage was instituted in the Garden of Eden by God.

The work in [8] defines the term "marriage" as a legal union of a man and a woman as husband and wife. This, which represents the English definition of marriage, has been mirrored outside culture and custom, hence the definition is purely a linguistic one devoid of ethical connotations. The concept of marriage and family attracts many experts from all walks of life. [2]

views marriage as a cohabitational union of man and woman. It appears he views marriage as a social system, which operates under one roof. In the same vein, [21] sees marriage as a statutory expectation of a young man and woman who are supposed to pledge their love for each other to live as husband and wife under marriage ordinance. Marriage is something deeper than the mere union of a man and a woman under the same roof. [15] perceives marriage as a sacred and a permanent contract which is assumed to be enacted when two people decide on their own accord and in the presence of at least two witnesses to exchange the formal consent to live a life of vocation of love and sharing for the purpose of promoting their mutual growth and welfare as persons in their journey together through life.

In [6], contends that in a marriage contract, a certificate is issued to the man and woman who are involved in this contract of marriage union. [6] must be referring to the Church and Court marriages alone, ignoring the traditional customary marriage where certificates are not issued before the contract is made between the spouses among the elders and families of the spouses. In the African context, marriage is one of the most important occasions in one's life. The other occasions are one's birthday and one's day of demise. Of course, a new-born baby does not know what is happening nor is the dead person aware of anything. The most exciting day in African context is the very day one marries. Marriage, therefore, is not a private affair. It involves the families, the kith and kin of the intending couples, including the community at large. [17] points out that marriage is so exciting that the road cannot be all roses just as a coin has two phases. In the garden of love, as one adage describes the situation of the newly married couples, the other side of the coin is not envisaged. [10] posit that the marriage of young people may not be strictly accepted as marriage; rather it could be co-habiting since certain rites and or ceremonies that legitimize marriage are absent. This is to say that not all unions are marriages. In the study area, such "marriages" are not uncommon and are often caused by pregnancy as young people get themselves involved in love relationships. This situation has serious implications for the adolescent parents whose education is often truncated, especially the girl who would have to remain at home to give birth and take care of the child. Some marriages are therefore not properly constituted and may not be considered marriage in its strictest sense. Some marriages procreate while others do not; hence while some unions end up only as marriages, others become families [23:151] also contends that "Marriage, after all, continues well beyond one hundred game trials; for it is a relationship; one of the very few in our culture that is supposed to last". Taking marriages as a game of which only a few are meant to last is contrary to the religious and cultural views of marriage in terms of permanency. Marriage is supposed to be in existence till death separates the spouses and not a game of trial or temporary situations, especially in the African and Christian traditions.

The work in [7] opines that marriage is a compromising union of man and woman which poses developmental life cycle tasks that require individuals to address issues of intimacy and reciprocity. One may share the idea of compromising. The married people have to compromise as they continue to reciprocate each other in the cohabitation.

Family could be defined as two or more people who are related by birth, marriage, or adoption, and residing in the same housing unit. Although sociologists define as a group related to one another by bonds of blood, marriage, or adoption and who live together to form a unit.

Causes of Broken Homes

Family is said to be the first institution where one starts to equip oneself to grow. But differences have been found to each family. Having healthy and happy family is what everyone's dream. But many could not succeed it. Due to different problems, the healthy families have been broken up as well. When unwanted things are coming up, many families have been broken, and the members are separated. This is a tragic happening out of unrestricted reasons as well as unfortunate reasons.

When we say broken family, it is not distinct and strange thing, rather common and experiencing problem in and around us. Family life is in a crisis when some problems aroused and which causes separation of the members in the families, which is simply called broken family. It is very controversial that whether to claim every splitting up is broken family, while they still run the family well. There are many families without father, mother, and other members but still conditionally and systematically running. They may not like to call them broken, of course they are not broken, rather just some members left away.

Death

It is obvious that death reduces the family strength. More adversely death sometime leads to broken family. Like other reason all death in the family do not necessarily bring any broken, but by the death of mother or father children become mother/father less. When children realize that the parent will never return, they get emotionally disturbed. Moreover if the left partner live a drastic life in search of happiness after losing his/her partner, the results in the family could not be hale and hearty for the family life, then it starts to be broken.

In early life, loss of mother is more damaging than loss of the father. The mother plays a very important role in nurturing the child during the formative years of life. But as the children grow older, loss of the father is often more serious than loss of the mother, especially for boys. Sometimes mother may lack the time and energy to give children the care they need as she has double burden of homemaking and bread-earning. But if children loss both parents, the effects are doubly serious.

Misconception between family members

Apart from divorce parents and death, there can be a family problem caused by misconception between the members in the home. The reason may be differ from each other. However the most common reasons are drugs, abusing, drinking alcohol, misbehavior manner and doing other intoxicant things by the father, sons or any other members. Then there comes misconception, ruining and anger, which in turn caused breaking the relationship of himself and herself from the family.

Unconditional administration

Failure in administration in the family is one cause for the breaking up of family. If the father or mother or any other heads fail to administer, there will be problems among the members of the family. They will be free to do whatever and however they like in the home and outside. Then the unsystematic and improper living will affect the remaining relationship in the family. If the relationship has been worsening, there will definitely be losing of caring which can lead to broken in the family.

Friends influence

Another reason why a family has broken up is because of parental or friends influence. When a third party has involved itself in the matters inside the home, bad-mouthing will just likely to happen and misunderstanding will only grow worse. Everyone has friends; we used to share our problems with them. But there can be negative impact. To whom we thought helps will come, another unexpected advice may come. Many a time, broken family, particularly splitting up of parents took place due to the influences made by our friends.

In-Law's role in marriage

In-laws in marriage relationship refers to the relatives one acquires through marriage. These in-laws are relatives of either the husband or wife. Those who constitute in-laws in this context are parents and siblings of the couples. In a large group, members of extended family, for example, nieces, nephews, cousins, and grandparents are also regarded as in-laws.

The desire to have a home makes a young man and a young woman to come together as husband and wife. Evans-Pritchard (1973) notes that among the Nvers of the Northern Sudan, young man talks of a home with reference to his family of procreation. The moment one leaves home to marry, something happens to the bond that holds parents and child, because a child remains emotionally attached to his or her parents for life. [3; 20] point out that parents often get involved in arranging their children's marriages to help the children overcome this attachment. The fact that parents are involved in the marriage negotiation and the subsequent innate desire to help their children settle down quickly, lead to intrusion into the privacy of the couple, which could occasionally cause misunderstanding. [22] opines that: although parents' intrusion may cause strife in the home, these in-laws are often unaware that their presence is causing someone pain, some may claim objectivity in their intrusion in the couple's marital life feeling that it is their legitimate rights to interact with their children [20]. As the in-laws interact with the young couples, problems such as jealousy, bad socio-cultural practices and negative attitudes to infertility and all-female children could emanate or be aggravated.

Jealousy

It is always very rare for a mother in-law and daughter in-law to have a cordial relationship. [19] feels that the cause of the problem is accusations and counter-accusations between the two women. The mother in-law often sees her daughter in-law as a rival. She picks on everything that the daughter in-law does, for instance, good family meals, children's clothing and the wife's dressing apparels are condemned outright as acts of extravagance by the mother in-law. The young wife is accused of increasing the poverty level in the family. If the husband listens to his wife and neglects his parents' wish, the wife is accused of using love potion on their son. Once the daughter-in-law becomes aware of the negative attitude towards her, she also develops resentment for her in-laws. She sees the mother-in-law as being jealous, authoritative and attempting to take over her husband and the running of her home. The sister-in-laws of marriageable age are more desperate in their attacks since they feel that the sister-in-law is depriving them of their rights. With the consent of their mother, they keep their brothers' wife uncomfortable in her home. This ill feeling often leads to character assassination and calumny which breeds' tension, conflicts and contempt that results in exchanging of abusive language and continual physical assaults. The sour mother-in-law - daughter-in-law relationship has often ended in the break-up of marriages since the man is placed in a precarious position of not knowing whom to support. Parents could do their children well if they show love and affection to those who are married.

Infertility

When a man gets married, the anticipation by family members is having children in the nearest possible time. As the marriage progresses if there is no sign of pregnancy, worries set in for the couples; the period of waiting for pregnancy is a time of anxiety and unsolicited advice and suggestions from relatives and friends. The woman who is unable to become pregnant is the focus of discussion by the in-laws. The in-laws never care to know who is the cause of the barrenness but would simply accuse the woman either of infidelity, teenage abortion or witchcraft. The woman is called names and the man would be asked to look for another wife for he was "marrying a man". Society appreciates fecundity; anyone who falls short of achieving this aim encounters enormous difficulties in marriage. The problem of barrenness is especially excruciating in polygamous homes. Hannah the wife of Elkana in the Bible suffered most humiliating experience annually at Shiloh [18]. [11] is of the view that a childless woman is not only just tormented by co-wife/wives but also by the extended family members, especially the in-laws.

All-female children

This is a related problem to barrenness as it concerns in-laws. This is simply the inability of a wife to give birth to male children. Even literate families still blame lack of male children on the wife. The female children and their mother are often castigated for their sex. Mother-in-laws create opportunities to nag the daughter-in-law and develop disaffection for the children. Daughters are made to understand that they are not liked and are of less value than boys. In extreme cases the husband is put under pressure to keep concubines in a bid to raise male children who would sustain the family name. The desire to please parents often leads to misunderstanding. The love existing in the couples wanes when negative attributes of the woman are magnified at the expense of virtues.

3 Mathematical model formulation

Broken home in this study is considered as a spreading disease. Hence the broken home model is an epidemic model, which is liken to be a predator-prey model. Thus the predator-prey model is the building block of the bio and ecosystem as biomasses are grown out of their resource masses [12; 13; 14]. A mathematical model was formulated and the population was divided into two major compartments: such as Intact Home (I_H) i.e $X(t)$ and the Broken Home (B_H). Under the broken, we have two sub-compartments which are Separated $Y(t)$ and Divorce $Z(t)$. The marriage dynamics between the intact home and the broken home (separated and divorce) are shown in the figure below.

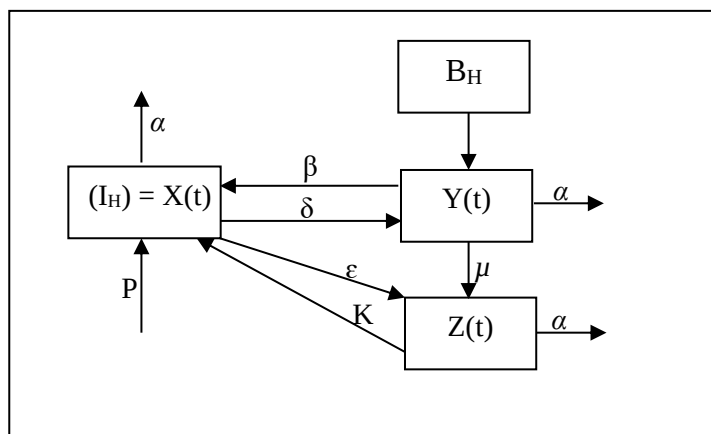


Figure 1: A schematic diagram showing intact and broken home model

From the above schematic diagram, the assumptions were: i). broken home occurs in a closed environment; ii). Factors such as sex, religion, race social and economic do not affect the occurrence of broken home; iii). There is homogeneous mixture of members. With the initial conditions $X(0) \geq 0$, $Z(0) \geq 0$ and $Y(0) \geq 0$, where $X=1$, $Y=1$ and $Z=1$.

Thus, the model for this study is given as:

$$\begin{aligned}\frac{dX}{dt} &= P - aX - \delta XY - \varepsilon XZ + \beta Y + KZ, \\ \frac{dY}{dt} &= \delta XY - \beta Y - \mu Y - \alpha Y, \\ \frac{dZ}{dt} &= \mu Y + \varepsilon XZ - KZ - \alpha Z.\end{aligned}\tag{3.1}$$

4 Numerical results

In this section, we use numerical simulations to show the dynamical behaviour of our results, by assuming that our total population (X, Y, Z) is 100% and X = 0.55, Z = 0.30 and Y = 0.15. The other parameters that would be used in this section are displayed in Table 1 and Table 2 respectively.

Table 4.1: Model parameters at broken home free equilibrium (adopted from [4])

Parameter	$\mathbf{p}$	$\mathbf{\varepsilon}$	K	$\mathbf{\alpha}$	$\mathbf{\beta}$	$\mathbf{\delta}$	$\mathbf{\mu}$
Value	0.4	0.5	0.4	0.25	0.09	0.4	0.2

Table 4.1.1: Broken home free equilibrium when all parameters are fixed.

Variation of parameter values for Broken home fr

equilibrium for $\beta = 0.09, k = 0.4$		
intact married	separated	divorced
100.0000	10.0000	10.0000
0.583902	37.34403	47.29659
0.590813	27.52676	36.18501
0.597033	20.34375	27.43093
0.603165	15.07138	20.67858
0.609364	11.19300	15.54092
0.616910	8.335290	11.66648
0.625493	6.227279	8.762274
0.636285	4.670479	6.593532
0.649441	3.519618	4.977930
0.665523	2.667771	3.775773
0.684693	2.036316	2.881469
0.707074	1.567303	2.215697
0.732801	1.217968	1.719154
0.761559	0.956892	1.347844
0.792909	0.760941	1.069157
0.826571	0.613004	0.858895
0.861677	0.500667	0.699398
0.897441	0.414784	0.577617
0.933351	0.348579	0.483916

Table 4.1.1 shows that while the intact marriage is monotonically increasing, the divorce and separated are decreasing monotonically. Intact married ranging from 0.583902 to 0.933351, Separated from 37.34403 to 0.348579 and Divorce ranging from 47.29659 to 0.483916.

Table 4.1.2: Broken home free equilibrium had $R_0 < 1$ when all parameters are fixed.

$p=0.4, \epsilon=0.5, k=0.4, \alpha=0.25, \beta=0.09, \delta=0.4, \mu=0.2$

50% variation of beta and k on broken home free equilibrium

Variation of parameter values for Broken home free equilibrium for $\beta = 0.0045, k=0.2$

intact married	separated	divorced
100	10	10
0.305518	35.00337	50.52793
0.317347	24.16905	40.4812
0.327993	16.76335	31.62099
0.338231	11.67504	24.25778
0.348734	8.164877	18.36593
0.360619	5.735388	13.77242
0.374239	4.049228	10.25798
0.39066	2.875951	7.606568
0.410697	2.057508	5.627506
0.434955	1.485099	4.162342
0.46446	1.083472	3.084192

0.499256	0.80068	2.294391
0.539482	0.600645	1.717475
0.584733	0.458363	1.29668
0.63411	0.356503	0.989801
0.686624	0.282992	0.765718
0.740959	0.229476	0.601729
0.795242	0.190214	0.481307
0.848756	0.161071	0.392536

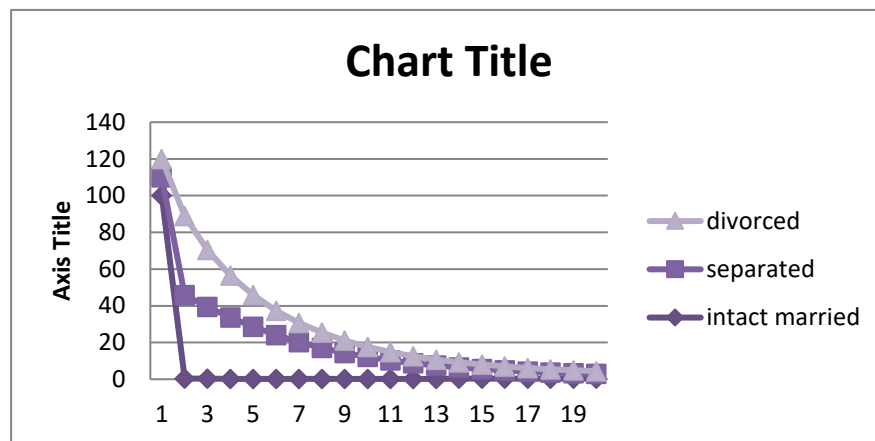


Table 4.1.2 shows that while the intact marriage is monotonically increasing, separated and divorce are decreasing monotonically. Intact married ranging from 0.305518 to 0.848756, separated is decreasing from 35.00337 to 0.161071 and divorce from 50.52793 to 0.392536

150% variation of beta and k on broken home free equilibrium.

Variation of parameter values for Broken home free equilibrium for $\beta = 0.135, k = 0.6$

Table 4.1.3

Intact marriage	Separated	Divorced
100	10	10
0.847075	39.7166	44.76104
0.840412	31.00387	33.60959
0.837469	24.15817	25.61454
0.837081	18.81323	19.70075
0.838764	14.65487	15.24149
0.842132	11.42604	11.84031
0.846472	8.921769	9.228954
0.851962	6.980373	7.216047
0.859068	5.475341	5.660489
0.867734	4.308399	4.456437
0.878295	3.403076	3.523111
0.89046	2.700307	2.798882

0.904698	2.154025	2.235908
0.92098	1.728717	1.797481
0.939134	1.396946	1.455314
0.958824	1.13755	1.187653
0.980298	0.933943	0.977342
1.003107	0.773556	0.811465
1.026733	0.646721	0.680154

Table 4.1.3 shows that while the intact marriage decreases at the initial stage and later increases while separated and divorce are decreasing monotonically. Intact married ranging from 0.847075 to 1.026733, separated is decreasing from 39.7166 to 0.646721 and divorce from 44.76104 to 0.680154.

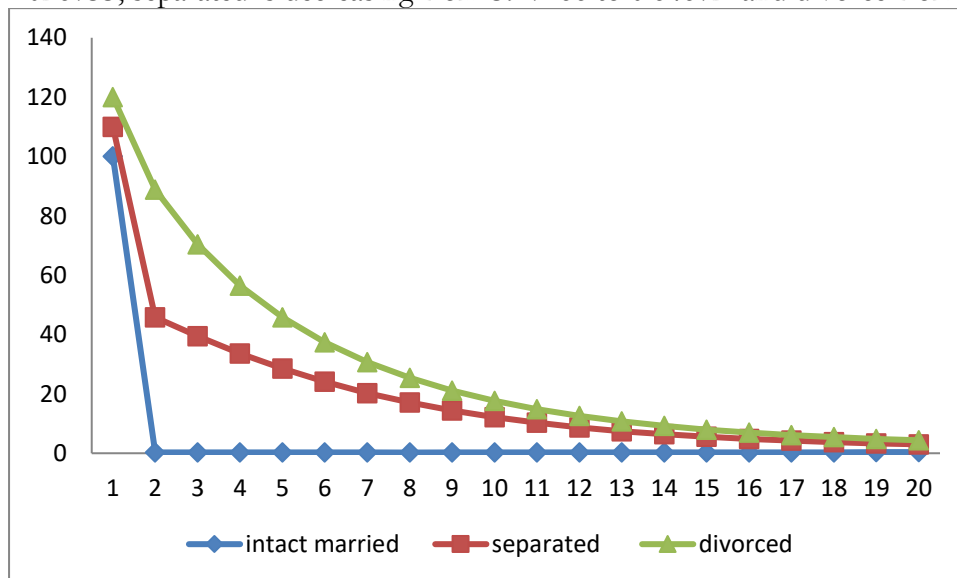


Table 4.2: Model parameters at endemic equilibrium (adopted from Gambrah & Adzadu 2018)

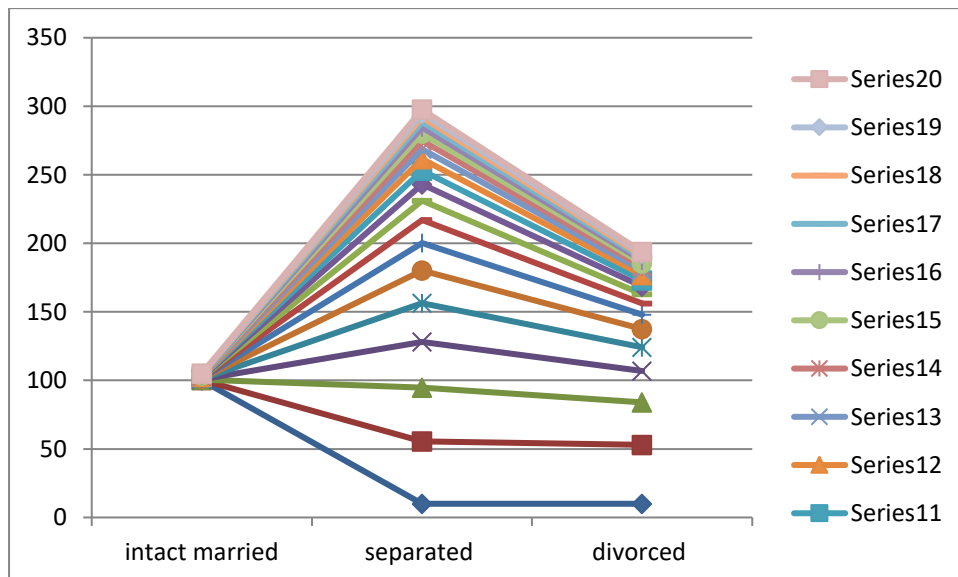
Parameter	p	ϵ	K	α	β	δ	μ
Value	0.4	0.6	0.2	0.25	0.01	0.5	0.025

Endemic equilibrium when all parameters are fixed

intact married	separated	divorced
100	10	10
0.19994	43.72628	44.71117
0.193056	36.27589	33.0635
0.187854	30.00386	24.75885
0.184272	24.76185	18.7672
0.182605	20.40814	14.39543
0.182442	16.81196	11.17208
0.184037	13.85503	8.771953
0.187631	11.43286	6.968237

0.192961	9.454973	5.601202	Variation of parameter values for endemic equilibrium $K=0.2, \beta=0.01$ Table 4.2.1
0.200031	7.843754	4.556744	
0.20904	6.533393	3.752412	
0.2198	5.46892	3.128368	
0.232261	4.604553	2.640539	
0.246329	3.902514	2.256277	
0.261624	3.331962	1.951405	
0.27807	2.867575	1.707585	
0.295438	2.488865	1.511059	
0.31338	2.17933	1.351488	
0.331645	1.925638	1.220951	

Table 4.2.1 shows that while value of the intact marriage fluctuates at the initial stage and later increases, the separated and divorce are decreasing monotonically. Intact marriage ranging from 0.19994 to 0.331645, separated is decreasing from 43.72628 to 1.925638 and divorce from 44.71117 to 1.220951.



A graph showing endemic equilibrium, $p=0.4, \epsilon=0.6, k=0.2, \alpha=0.25, \beta=0.01, \delta=0.5, \mu=0.025$

50% variation of beta and k on endemic equilibrium

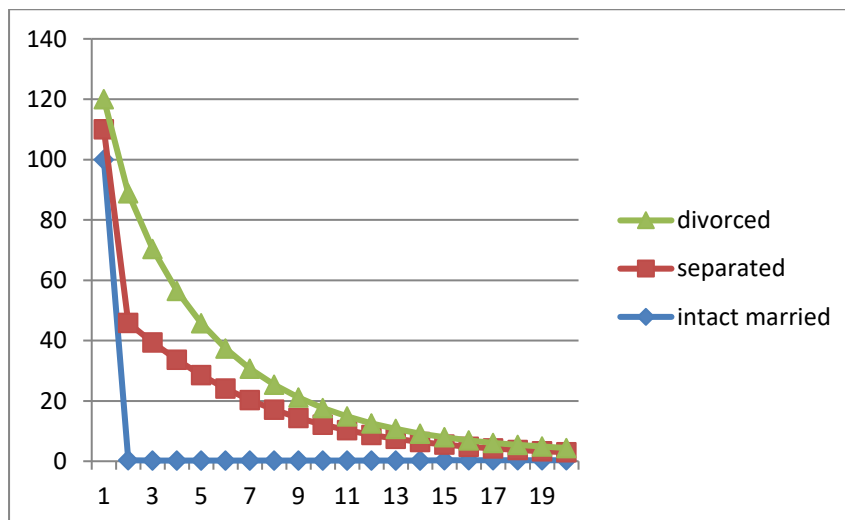
Variation of parameter values for endemic equilibrium $K=0.1, \beta=0.005$

Table 4.2.2

intact married	separated	divorced
100	10	10
0.107206	41.91423	46.62013
0.107732	33.42582	35.66491
0.108926	26.66816	27.3891

0.110991	21.29392	21.12127
0.114014	17.02398	16.3625
0.118094	13.63441	12.74088
0.123491	10.9457	9.978236
0.130457	8.814208	7.865928
0.139014	7.125261	6.247147
0.149339	5.787273	5.00354
0.161630	4.72722	4.045576
0.175879	3.88705	3.30548
0.192101	3.220595	2.731745
0.210027	2.691333	2.285329
0.22963	2.270215	1.936351
0.250425	1.934469	1.662275
0.272152	1.666034	1.445807
0.294438	1.450744	1.273823
0.316916	1.277472	1.136324

Table 4.2.2 shows that while the intact marriage decreases at the initial stage and later increases while separated and divorce are decreasing monotonically. Intact married ranging from 0.107206 to 0.316916, separated is decreasing from 41.91423 to 1.277472 and divorce from 46.62013 to 1.136324.



150% variation of beta and k on endemic equilibrium.

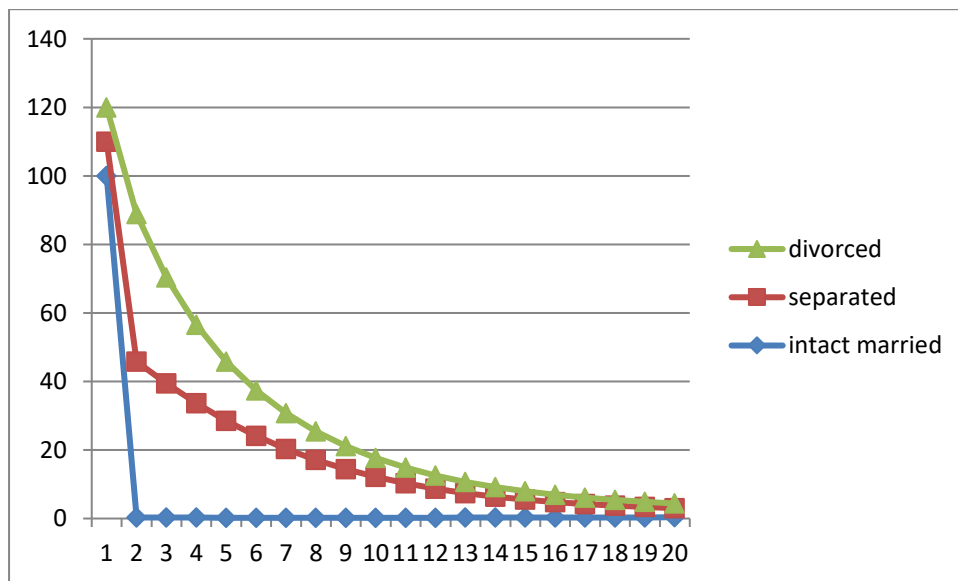
Variation of parameter values for endemic equilibrium $K=0.3$, $\beta=0.015$

Table 4.2.3

intact married	separated	divorced
100	10	10
0.286725	45.54859	42.9998
0.268082	39.14688	30.97636
0.25365	33.36716	22.87582
0.243229	28.26395	17.27472

0.235949	23.83644	13.30807
0.231901	20.04665	10.43677
0.230461	16.83651	8.31746
0.231508	14.13884	6.725685
0.234452	11.88561	5.511912
0.239556	10.01172	4.573544
0.24635	8.458418	3.839599
0.25487	7.173508	3.259355
0.264826	6.112024	2.796378
0.27613	5.235518	2.423776
0.288798	4.511521	2.121417
0.302252	3.913254	1.874546
0.316634	3.418097	1.671414
0.331689	3.007597	1.503231
0.347109	2.666628	1.363231

Table 4.2.3 shows that while value of the intact marriage fluctuates at the initial stage and later increases, the separated and divorce are decreasing monotonically. Intact married ranging from 0.286725 to 0.347109, separated is decreasing from 45.54859 to 2.666628 and divorce from 42.9998 to 1.363231.



4.1 Sensitivity analysis of the basic reproductive numbers

We investigate the nature of the model by conducting sensitivity analysis of the reproductive number (R_0).

(a) At the broken home -free equilibrium $\varepsilon = 0.5, K = 0.4$ and $\alpha = 0.25, R_0 = 0.76923 < 1$. If the value of ε is increased to any figure greater than 0.65 and the values of K and α are maintained $R_0 > 1$.

(b) At the endemic equilibrium, $\varepsilon = 0.6$, $K = 0.2$ and $\alpha = 0.25$, $R_0 = 1.333 > 1$. If ε is reduced to 0.44 and α , K are maintained the same, $R_0 < 1$.

We use XYZ model to study the dynamics of broken home as an epidemic. We discussed the existence and stability of divorce free and endemic equilibrium and performed the sensitivity analysis of the reproductive numbers. Based on the data in Table 1, the basic reproductive number of the divorce free equilibrium is estimated to be $R_0 = 0.76923 < 1$. This implies that only marriage population is present and the separated and broken home populations reduces to zero ($Y = 0$, $Z = 0$). This means that the model is asymptotically stable at $R_0 < 1$ and therefore satisfies Theorem 1. This has been verified numerically in Fig. 2. In the stability analysis of the broken home free equilibrium, the eigenvalues are $\lambda_1 = -0.25$, $\lambda_2 = -0.05$ and $\lambda_3 = -0.15$. This also indicates that the broken home free equilibrium is asymptotically stable.

Considering the situation when $R_0 > 1$, the reproductive number of the endemic equilibrium is estimated to be $R_0 = 1.333 > 1$ using the data in Table 2. This shows the situation in which the marriage, separated and broken home coexist in the population $(X^*, Y^*, Z^*) = (0.75, 0.204, 0.85)$

This indicates the existence of divorce problem in the population. People with divorce problem will continue to transform more Marriages into divorced and the broken home free equilibrium becomes unstable at R_0 . This is in line with our analytical results and has also been verified numerically.

5 Discussion

The analytical solution of the system of inferential equations shows that at the first initial condition, $X = 0$, $Y = 0$, $Z = 0$; $X(t)$ is equal to the rate of entering into marriage is the same or equal to the rate of leaving the marriage which could be by any means (death). This also means that at onset there was no marriage, no separation nor divorce. This depicts that there was no marriage between the two persons meaning that there was no concrete consent of either family involved.

Based on the zero initial boundary condition, $Y(t)$ and $Z(t)$ analytical solution show that there was no marriage existing between the two persons, this means that they do not have any objective reality even though they were cohabiting.

However, the second initial boundary conditions at $X = Y = Z = 1$; the analytical results show that where there is marriage, then separation and divorce is likely to occur if not well managed. These solutions shows that both the intact home ($X(t)$), separated ($Y(t)$) and divorced ($Z(t)$) exist and grows in an exponential rate with respect to time (t). The result showcase that for separation and divorce not to occur, then marriage should be avoided completely.

The analytical result shows that for broken home to be avoided there must be a good mutual understanding in the intact home and that will lead it to grow exponentially with love bond and without cracks or breakages.

6 Conclusion

By being human beings we all have limitations and shortcomings, due to these we used to make problems for other for some time. Family filled with mutual love and care is the ideal for all of us. But because of such human incapacity, we cannot succeed it. Then conflicts aroused which caused broken family. There can be many reasons.

Family filled with mutual love and care is the ideal for all of us. But because of such human incapacity, we cannot succeed it. Then conflicts aroused which caused broken family. There can be many reasons. Broken family also can be resolved at the same time, it could be prevented. The model shows that, broken home epidemic cannot only be controlled by reducing the contact rate

between the married and the divorced but also increasing the number of marriages that go into separation and educating separated to refrain from divorce can be useful in combating the epidemic.
Recommendations

1. In further studies there should be a development of various related topics that will help in analyzing other aspects that are closely related to broken home in order to get further understanding and avoid the problem of broken home that happen to us and to give us information about the effects of broken home.
2. The Ministry of Education and Vocational Training (MEVT) in collaboration with Universities and religious organizations should develop training programme on love, marriage and care for parents in order to educate the society on the importance of love and family care for healthy families, communities and society in general.

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