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Solution of stochastic delay differential equation on asset value changes for capital market price

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Abstract

An asset values in trading business creates opportunity where returns or profits would be used at every periods. As a result, a stochastic model of asset value function is considered only in periodic events. Close form analytical solutions were obtained which gave precise measures and conditions of generating asset values through additive effects series. The effects of time delay for all periods were critically examined. Normality tests were conducted to show the impact analysis of initial stock prices and asset values that followed some probability distributions. To this end, the Tables, graphs and other stock quantities were discussed for the purpose of investment plans as it affects asset value functions.

Keywords: asset value; normality test; additive effects; SDE and SDDE

1 Introduction

Mathematical models are produced from equations which stock price models cannot be omitted as a result of multiple application in our day-to -day business transactions. These applications consist of banking and finance, accountancy, quantitative finance, etc. In several areas in science and engineering, the design, correct evaluation, and analysis of system exposed to practical or lifelike condition should consider the prospect of "white noise" random forces that must influence the system or inaccurate assessments or analysis in the system. Uncertainly is integral to the mathematical formulation of several circumstances or situations like stock market fluctuations, noise in population systems, irregular fluctuations or communication networks in observed signals. The economic crunch in our nations are resultant effect of stock price fluctuations, as a result of randomness pertaining to stock market or exchange, investors government, policy makers, etc, are unable to give a precise prediction of the future.

Stock price fluctuations create fear and as a result, people resort to criminality so as to meet individual needs, and snowballs to buying out of anxiety in fact, financial analysis who invest in financial market are typically clueless of the behaviour of stock market; hence they go through this stock trading problem, and also faces the challenge of not knowing the type of stocks to be bought and sold for profit maximization. Hence relevant information on regular basis are required or needed by both financial analysis and potential investors for the prediction of stock price behaviour.

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The unstable characteristic and other significant influences like liquidity on stock returns, because the abrupt changes in share prices is erratic and happens regularly. So, in order to help investors and owners of corporations take decisions on the level of their investment in stock market [1], researchers are curious and fascinated in studying the behaviour of the unstable market variables.

Nonetheless, the price evolution of ab risky assets are generally modelled as a path or track of a risky assets that are generally of a diffusion process defined on several basic probability space, with the Geometric Browner motion, the main tool used as the established reference model, [2] Many researcher have modelled stock market prices with several ways obtained results. For example,[3] studied the unstable feature of stock market forces, making use of suggested differential equation model. In the research of [4], stability analysis of stochastic model of price change at the floor of a stock market was considered and precise conditions were obtained which determined the equilibrium price and growth rate of asset shares. Stochastic analysis of the behaviour of stock prices was studied by [1], and results showed that the proposed model was efficient for predicting stock prices. Similarly, [5] considered the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in this research, the drift and volatility coefficients for the stochastic differential equations were obtained and the Euler-Maruyama method for system of SDEs was utilized to invigorate the stock prices [6], developed the geometric Brownian motion and looked at the exactness of the model with comprehensive analysis of assumed data.

The work in [7] studied the stochastic modelling of stock prices applying a method of Brownian motion model to explain the stock price time series. Result from the study revealed that as long as a model based upon the white noise is fitted to the market values, the two interpretations will give different estimates of the parameters, but identical values pertaining to the predicted stock prices. Nevertheless [2] studied stochastic model of the fluctuation of stock market price. Conditions for finding out the equilibrium price, adequate conditions for robust stability and convergence to equilibrium of the growth rate of the value function of shares. However, [8] looked at a stochastic model of price changes at the floor of stock market. In the work of [9], the equilibrium price and the market growth rate of shares we determined. See [10-14] for other articles on stock prices.

Past researches for example, [5] examined a stochastic model of several selected stocks in the Nigerian Stock Exchange (NSE) where the Euler-Maruyam method for system of (SDE) was utilized to invigorate the stock prices and result revealed that stock1 yields the best returns on investment compared to stock2, stock3 and stock4. The merit of this research over [5] is that the present research models the effect of growth-rates on stock market price estimation or forecast regarding to volatility and the drift.

Aforementioned studies have consequently considered similar problems of SDE. For instance, [22],[20] and [23] have only focused on Growth rate of assets, stability of stock prices, asset value and its return rates In all, none has provided any asset values with time dependent delay, with variations of initial closing stock prices, to guarantee the consistency of such solutions and goodness of test to show some levels of probability distributions.

In this study, we considered SDE with time delay parameter in the model which gave three different solutions with aid of initial conditions The effects of time delay for all asset values were critically examined. Normality tests were conducted to show the impact analysis of initial stock prices and asset values that followed some probability distributions. Furthermore, the Tables, graphs and other stock quantities were discussed.

To the best of our understanding this is the first study that has one model equation with three initial conditions (initial closed stock prices) to produce three investment analytical solutions to realistically assess the dynamics of asset values as a result of delay parameter.

The organization of this paper is set as follows: Section 2.1 presents the preamble, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2 Preamble

For us to adequately understand stochastic processes, we present few intricate definitions on probability spaces rudiment of this dynamic area of study:

Definition 1: Probability space is a triple $(\Omega, \mathrm{F}, \tilde{A})$ where Ω represents a set of sample space, F represents a collection of subsets of Ω, while $\wp$ is the probability measure defined on each event $A \in \mathrm{F}$. The collection F is a σ -algebra or σ -field such as $\Omega \in \mathrm{F}$ and F is closed under the arbitrary unions and finite intersections. Hence it is called probability measure when the following condition holds.

(i) $P(A) \geq 0$ *for all* $A \subset \Omega$

(ii) $P(\Omega) = 1$

(iii) $A, B \subset \Omega,\ A \cap B = \phi$ *then* $P(A \cup B) = P(A) + P(B)$

Definition 2: Normal Distribution: A normal distribution function is a peculiar distribution in probability theory and is usually used for modeling asset returns. A normal distribution is used in the Black-Scholes Partial differential equation to value European options. A normal distribution depends on two parameters, Westergren and Rade (2003).

Mean, $\mu \in$, is the expectation of a random variable normal distribution.

Variance , $\sigma^2 > 0$, deals with the magnitude of the spread from the mean.

The cumulative distribution, usually denoted as $\phi(\mathrm{X})$, is the probability that X will be equal to or less than X ,expressed as $F_x(x) = P(X \leq x)$. A standard normal cumulative distribution function is defined as.

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt \ .$$

A normal distribution is a symmetric distribution, which means that it touches around a vertical axis of symmetry. Obviously, there is a connection between any given points with same distance to the vertical axis.

Definition 3 : A σ -algebra is a set F of subsets of Ω with the following axioms:

(i) $\phi, \Omega \in \mathrm{F}$

(ii) If $A \in \mathrm{F}$, *then* $A^c \in \mathrm{F}$

(iii) If $A_1, A_2, \ldots, \in \mathrm{F}$, *then* $\bigcup_{k=1}^{\infty} A_k, \bigcap_{k=1}^{\infty} A_k \in \mathrm{F}$

Clearly $A^c := \Omega - A$ is the complement of A.

Definition 4: If $\mathcal{F}$ is a σ -algebra in Ω, then Ω is called a measurable space and the members of $\mathcal{F}$ are called the measurable sets in Ω [16].

Definition 5: Let $(\Omega, \mathcal{M})$ be a measurable space .A map $\mu : \mathcal{M} \to \square = [0, \infty) \cup \{\infty\}$ is called a measure provided that

(i) $\mu(\phi) = 0$

(ii) $\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n)$

Definition 6: Stochastic process: A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e, for each t in the index set $T, X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t . In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

Definition 7: A stochastic process whose finite dimensional probability distributions are all Gaussian.(Normal distribution).

Definition 8: Random Walk: There are different methods to which we can state a stochastic process. Then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from a point $x = a$ to a point $x = b$. A random walk is a stochastic sequence $\{S_n\}$ with $S_0 = 0$, defined by

$$S_n = \sum_{k-1}^{n} X_k \tag{1}$$

where X_k are independent and identically distributed random variables.

Definition 9: (Differential Equation): is an equation which has functions and their derivatives. In reality the functions is associated to real quantities whereas the derivatives denotes rate of change. Example of differential equation is follows

$$\frac{dS(t)}{dt} = \mu S(t) \tag{2}$$

$$S(0) = S_0 \tag{3}$$

where $S(t)$ represent asset price, μ rate of return, $\frac{dS(t)}{dt}$ is the rate of change of asset price and S_0 is the initial stock price; (2) and (3) can be obtained using variable separable which gives:

$$S(t) = S_0 e^{\mu t}. \tag{4}$$

Therefore μ is not known completely which is subject to environmental effects. Therefore (2) can be written as

$$dS(t) = \mu S(t)\,\mathrm{dt} + \sigma S(t)\,\mathrm{dZ(t)} \tag{5}$$

where σ is the volatility, dZ is the Brownian motion or Wierners process which is random term, the stochastic term added to (2) gives (5) which makes it stochastic differential equation.

Definition 10: A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that $(\Omega, \mathrm{F}, \wp)$ is a probability space with filteration $\{f_t\}_t \geq 0$ and $W(t) = (W_1(t), W_2(t), ..., W_m(t))^T, t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g *as* follows

$$dX(t) = f(t, X(t))dt + g(t, X(t))dZ(t),\ 0 \leq t \leq T, \tag{6}$$
$$X(0) = x_0, \tag{7}$$

where $T > 0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f:[0,\mathrm{T}]\times \mathbb{R}^n$ *and* $g:[0,T]\times \mathbb{R}^n \to \times \mathbb{R}^{n\times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S))dS + \int_0^t g(S, X(S))dZ(S) \tag{8}$$

where dX, dZ are terms known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$ satisfying (6).

Theorem 1.1: let $T > 0$, be a given final time and assume that the coefficient functions $f:[0,\mathrm{T}]\times \mathbb{R}^n \to \mathbb{R}^n$ *and* $g:[0,T]\times \mathbb{R}^n \to \times \mathbb{R}^{n\times n}$ are continuous. Moreover, $\exists$ finite constant numbers λ *and* β such that $\forall t \in [0,\mathrm{T}]$ and for all $x, y \in \mathbb{R}^n$, the drift and diffusion term satisfy

$$\| f(t,x) - f(t,y) \| + \| \mathrm{g}(t,x) - g(t,y) \| \leq \lambda \| x - y \| , \tag{9}$$
$$\| f(t,x) \| + \| \mathrm{g}(t,x) \| - g(t,\mathrm{x}) \| \leq \beta(1 + \| x \|). \tag{10}$$

Suppose also that x_0 is any $\mathbb{R}^n$ -valued random variable such that $E(\| x_0 \|^2) < \infty.$ then the above SDE has a unique solution X in the interval $[0,T]$. Moreover, it satisfies $E\left(\sup_{0\leq t\leq T} \| X(t) \|^2\right) < \infty$. the proof of the theorem 1.1 is seen in [17].

Theorem 3.2:(Ito's lemma). Let $f(S,t)$ be a twice continuous differential function on $[0,\infty) \times \mathrm{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0 \ .$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (dS_t)^2 + \text{higer order terms } (h.\mathrm{o}, \mathrm{t}) \ .$$

So, ignoring h.o.t and substituting for dS_t we obtain

$$dF_t = \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}\left(a_t dt + bdz(t)\right)^2 \tag{11}$$

$$= \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2 dt, \tag{12}$$

$$= \left(\frac{\partial F}{\partial S_t}a_t + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2\right)dt + \frac{\partial F}{\partial S_t}b_t dz(t). \tag{13}$$

More so, given the variable $S(\mathrm{t})$ denotes stock price, then following GBM implies (5) and hence, the function $F\left(S,t\right)$,Ito's lemma gives:

$$dF = \left(\mu S\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 F}{\partial S^2}\right)dt + \sigma S\frac{\partial F}{\partial S}dz(t) \tag{14}$$

3 Mathematical formulation of the problem

The stochastic analysis on the value valuation of asset value with time delay and additive effects as stock rate of returns is considered. The positive influence of time delay on the value of asset is equally considered. The volatility dynamics of stock prices was taken to be constant throughout the trading days. The initial stock price which is assumed to follow different trend series was categorized. The entire origin of stock dynamics is found in a complete probability space$\left(\Omega, \mathrm{F}, \wp\right)$ with a finite time investment horizon$T > 0$. Therefore, we have the following stochastic delay differential equation below;

$$dX_i\left(t\right) = \left(t-\tau\right)X_i(t)dt + \sigma X_1(t)dZ(t) \tag{15}$$

whereμ is an expected rate of returns on stock,σ is the volatility of the stock ,dt is the relative change in the price during the period of time and Z is a Wiener process .Following the method of [18-19] on rate of returns gives as:

$$\begin{aligned} &R_t := \left(\lambda_1 + \lambda_2\right)^2, .., \\ &\text{where } t = 1, 2.. \end{aligned} \tag{16}$$

Using (15) and (16) gives the following system of delay stochastic differential equations :

$$dX_1(t) = \left(t-\tau\right)\left(\theta_1 + \theta_2\right)^2 \mathrm{X}_1(t)dt + \sigma X_1(t)dZ^{(1)}(\mathrm{t}), \tag{17}$$

where $\mathrm{X}_1(\mathrm{t}), \mathrm{X}_2(\mathrm{t}), S_3(\mathrm{t})$ *and* $S_4(\mathrm{t})$ are underlying stocks with the following initial conditions:

$$\mathrm{X}_1(0) = X_0,\ t > 0 \tag{18}$$

$$X_2\left(0\right) = e^t\ ,t > 0 \tag{19}$$

$$X_3\left(0\right)=e^{-t}\ ,t>0 \tag{20}$$

where, $X_1(t), X_2(t)$ and $X_3\left(t\right)$ are asset prices, The expression dZ , which contains the randomness that is certainly a characteristic of asset prices is called a Wiener process or Brownian motion. λ_1 *and* λ_2 represents rate of returns of first and second investments respectively, $\lambda_1+\lambda_2$ is additive effects

3.1 Method of solution

The model (17)-(20) is made up of a system of stochastic delay differential equations whose solutions are not trivial. We implement the methods of Ito's lemma in solving for $X_1(t), X_2(t)$ *and* $X_3(t)$ To grab this problem we note that we can forecast the future worth of the asset with sureness.

From (17) Let $f\left(X_1,t\right)=\ln X_1$ so differentiating partially gives

$$\frac{\partial f}{\partial X_1}=\frac{1}{X_1},\ \frac{\partial^2 f}{\partial X_1^2}=-\frac{-1}{X_1^2},\ \frac{\partial f}{\partial t}=0. \tag{21}$$

According to Ito's gives

$$df\left(X_1,t\right)=\sigma X_1\frac{\partial f}{\partial X_1}dZ(t)+\left((\theta_1+\theta_2)^2X_1(t)\frac{\partial f}{\partial X_1}+\frac{1}{2}\sigma^2X_1^2\frac{\partial^2 f}{\partial X_1^2}+\frac{\partial f}{\partial t}\right)dt. \tag{22}$$

Substituting (17) and (21) into (22) gives

$$\begin{aligned}&=\sigma \cancel{X_1}\frac{1}{\cancel{X_1}}dZ(t)+\left(\left(t-\tau\right)(\theta_1+\theta_2)^2\cancel{X_1}(t)\frac{1}{\cancel{X_1}}+\frac{1}{2}\sigma^2\cancel{X_1}^2(-\frac{1}{\cancel{X_1}^2})+0\right)dt\\&=\sigma dZ(t)+\left(\left(t-\tau\right)(\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)dt.\end{aligned}$$

Integrating both sides , talking upper and lower limits gives

$$\begin{aligned}&\int_0^t d\ln X_1=\int_0^t df\left(X_u,u\right)=\left(t-\tau\right)\int\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)du+\int_0^t\sigma dZ(t)\\&\ln X_1-\ln X_0=\left(t-\tau\right)\left((\theta_1+\theta_2)^2u-\frac{1}{2}\sigma^2u\right)\Big|_0^t+\left(\sigma Zu\right)\Big|_0^t\\&\ln\left(\frac{X_1}{X_0}\right)=\left(t-\tau\right)\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)t+\sigma Z(t)\end{aligned} \tag{23}$$

Taking the ln of the both sides

$$X_1(t) = X_0 \exp\left(t-\tau\right)\left((\theta_1+\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma Z(t)\,. \tag{24}$$

Applying other initial stock in (19)- (20) yields

$$X^*{}_2(t) = e^t\, \mathrm{e}\left(t-\tau\right)\left((\theta_1+\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma Z(t) \tag{25}$$

$$X^{*****}{}_3(t) = \frac{1}{e^t}\, \mathrm{e}\left(t-\tau\right)\left((\theta_1+\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma Z(t) \tag{26}$$

3.2 Results and discussion

This Section presents results whose solutions are in (24)-(26), Hence we have the following:

Table 1: The impact of time delay in the assessment of additive Asset values through the solution below: $X_1(t) = X_0 \exp\left\{\left(t-\tau\right)\left(\left(\theta_1+\theta_2\right)^2 - \frac{1}{2}\sigma^2\right)t + \sigma dz(t)\right\}$ t = 2, $dz = 1$

τ	S_0	σ	$(\theta_1+\theta_2)$	$X_1(t)$	$(\theta_1+\theta_2)$	$X_1(t)$
0.1	1.000	0.2	1.0000	50.6025	2.0000	4519898.067
0.2	1.000	0.2	1.0000	41.5958	2.0000	2039061.07
0.3	1.000	0.2	1.0000	34.1923	2.0000	919881.3745
0.4	2.000	0.2	1.0000	56.2130	2.0000	829971.9471
0.5	2.000	0.2	1.0000	46.2077	2.0000	374425.1444
0.6	2.000	0.2	1.0000	37.9833	2.0000	168914.3702
0.7	3.000	0.2	1.0000	46.8341	2.0000	114303.4792
0.8	3.000	0.2	1.0000	38.4982	2.0000	51565.7148
0.9	3.000	0.2	1.0000	31.6460	2.0000	23262.8347
1.0	4.000	0.2	1.0000	34.6846	2.0000	13992.7464
1.1	4.000	0.2	1.0000	28.5111	2.0000	6312.5460
1.2	4.000	0.2	1.0000	23.4365	2.0000	2847.7781

Table 2: The impact of time delay in the assessment of additive Asset values through the solution below: $X^*{}_2(t) = e^t \exp\left\{(t-\tau)\left((\theta_1+\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma dz(t)\right\}$ t = 2, $dz = 1$

τ	S_0	σ	$(\theta_1+\theta_2)$	$X_1(t)$	$(\theta_1+\theta_2)$	$X_1(t)$
0.1	7.38	0.2	1.0000	373.9066	2.0000	33397978.81
0.2	7.38	0.2	1.0000	307.3558	2.0000	15066826.15
0.3	7.38	0.2	1.0000	252.6502	2.0000	6797095.465
0.4	7.38	0.2	1.0000	207.6816	2.0000	3066372.857
0.5	7.38	0.2	1.0000	170.7168	2.0000	1383332.417
0.6	7.38	0.2	1.0000	140.3313	2.0000	624062.5866
0.7	7.38	0.2	1.0000	115.3540	2.0000	281533.2794
0.8	7.38	0.2	1.0000	94.8224	2.0000	127008.0744
0.9	7.38	0.2	1.0000	77.9452	2.0000	57297.1373
1.0	7.38	0.2	1.0000	64.0719	2.0000	25848.4506
1.1	7.38	0.2	1.0000	52.6679	2.0000	11661.0084
1.2	7.38	0.2	1.0000	43.2937	2.0000	5260.6293

Undoubtedly in Tables 1 and 2 show a little increase in time delay significantly reduces asset values through additive effects. This is physically consistent because as time passes some certain asset values continue to depreciates over time; example of such assets are cars, machines etc. This remark is an eye opener to an investor on how to manage such depreciating assets.

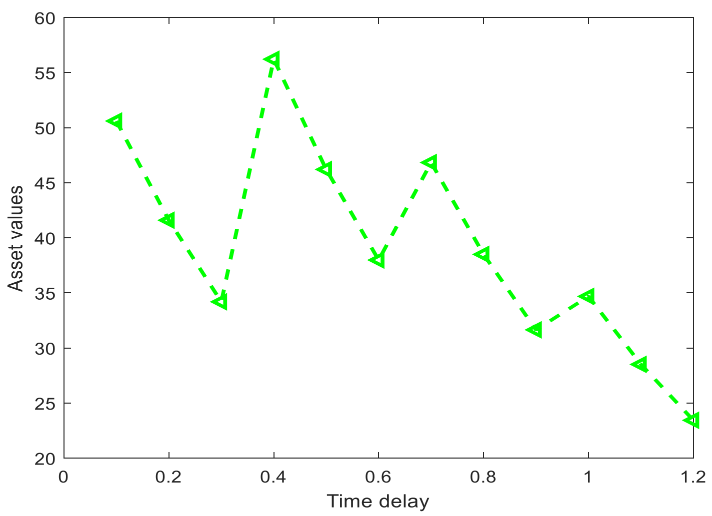

Figure 1: Profile of asset values with variations of time delay when initial price (X_0)

It can be seen in Figure 1, the graphical trend and impact of time delay to accurately assess asset value in time varying investments which is capital intensive. It is obviously informative to an investor whose aim and passion is to maximize profit hence it specifies time interval on the trend lines throughout the trading days.

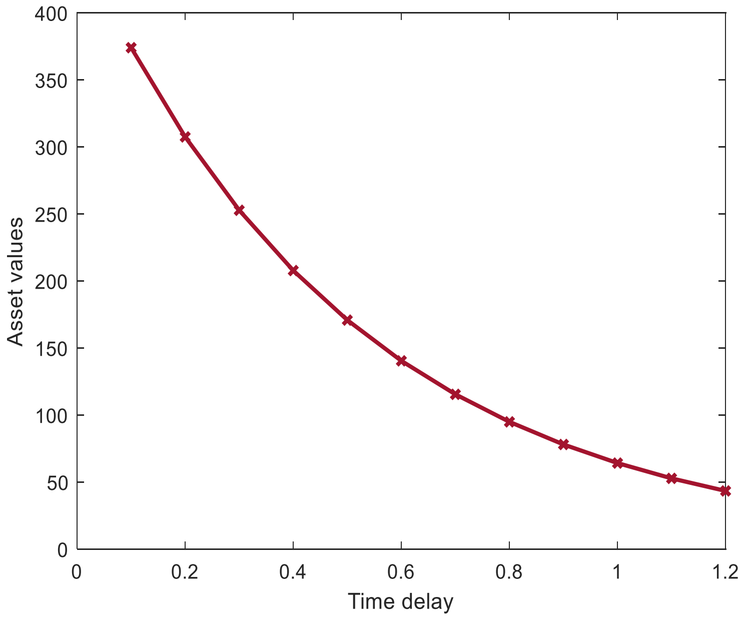

Figure 2: Profile of asset values with variations of time delay when the initial stock is exponential

Figures 2 and 3 describe a growth pattern that indicates dominantly, incremental improvements as time passes. Having an exponential growth in the investment means the trading business is highly indexed with millions of naira over time. This type of situation can interest investor to be more courageous in their economic investment, hence it grows exponentially.

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Table 4: The impact of time delay in the assessment of additive Asset values through the

$$X_1(t)=\frac{1}{e^t}\exp\left\{(t-\tau)\left((\theta_1+\theta_2)^2-\frac{1}{2}\sigma^2\right)t+\sigma dz(t)\right\}$$

solution below

where $dz=1, t=2$ and $S_0=\frac{1}{e^t}$

τ	S_0	σ	$(\theta_1+\theta_2)$	$X_1(t)$	$(\theta_1+\theta_2)$	$X_1(t)$
0.	0.135	0.	1.0000	6.8465	2.0000	611542.2085
0.	0.135	0.	1.0000	5.6279	2.0000	275884.9628
0.	0.135	0.	1.0000	4.6262	2.0000	124459.95
0.	0.135	0.	1.0000	3.8028	2.0000	56147.6022
0.	0.135	0.	1.0000	3.1260	2.0000	25329.8610
0.	0.135	0.	1.0000	2.5695	2.0000	11427.0572
0.	0.135	0.	1.0000	2.1122	2.0000	5155.0869
0.	0.135	0.	1.0000	1.7363	2.0000	2325.6137
0.	0.135	0.	1.0000	1.4272	2.0000	1049.1539
1.	0.135	0.	1.0000	1.1732	2.0000	473.3047
1.	0.135	0.	1.0000	0.9644	2.0000	213.5219
1.	0.135	0.	1.0000	0.7927	2.0000	96.3261

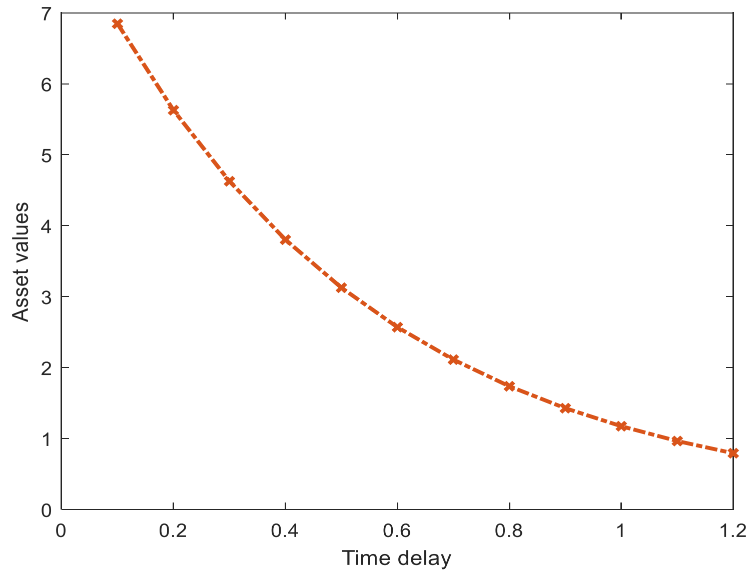

Figure 3 : Profile of Asset values with variations of Time Delay when the initial stock is exponential inverse

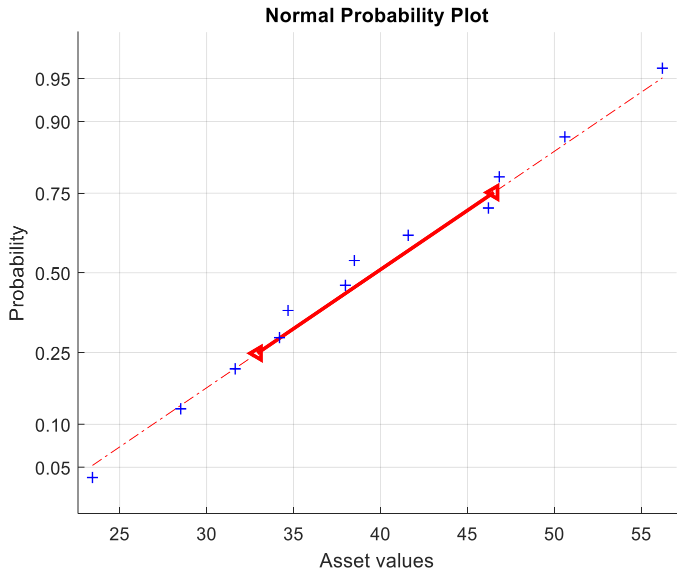

Figure 4 : Normal probability plot on Asset values when the initial stock price is $\left(X_0\right)$

The graphical description of Figure 4 shows that the two asset values through additive effects of Table 1 come from the identical distribution. They are statistically significant and correlated. The plot portrays a significant sure event which is highly beneficial to investor over in a time varying investments and with this scenario; profit margin is unavoidable and decisions can be properly taken to enrich decent management of the business.

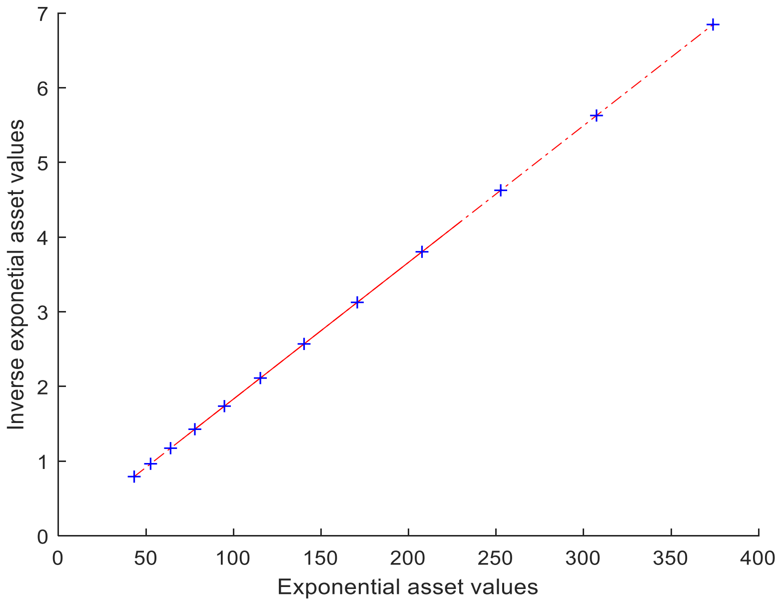

Figure 5 : Quantile-Quantile (QQ) plot assessment on two different Asset values when the initial stock prices is both exponential and exponential inverse

It can be seen in Figure 5 that the QQ plot indicates that the two exponential and inverse exponential asset values with additive effects of Tables 2 and 3 comes from a common distributions. They are statistically significant, correlated and have lots of financial benefits hence it waves around the normal distribution.

4 Conclusion

In this study, the stochastic analysis of variations of initial stock prices was successfully analysed through additive effects with the influence of time delay parameter. The asset values were obtained which all follows exponential trend of business over time. The normality probability and QQ plots all were statistically significant to address the issues in financial market for an assessment of asset pricing for economic investments.

Though, the present paper considered SDDE with additive effects to realistically assess asset values, future study should incorporate multiplicative effects as well and to be solved as stochastic systems.

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