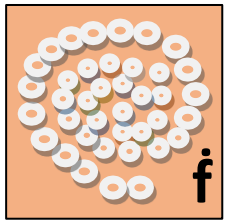

# International Journal of Mathematical Analysis and Modelling

**(Formerly Journal of the Nigerian Society for Mathematical Biology)**

**Volume 9 Issue 1 (June), 2026**

**ISSN (Print): 2682 - 5694**
**ISSN (Online): 2682 – 5708**
**www.tnsmb.org**

# Estimating the fractal dimension of all share index of the Nigerian stock market (ASINSM) from 1999 to 2013

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**Abstract**

In this study, All Share Index data of the Nigerian stock market was estimated using the fractal dimension as an estimator for a period of fifteen years (January 1999–December 2013). The Hurst parameter $H \in [0, 1]$ obtained is used as a dimensionless estimate. The linear relationship between the Hurst parameter and the fractal dimension, $D$, is given by $D = 2 - H$. The study evaluates whether the Nigerian stock market exhibits persistent, random, or anti-persistent behaviour within the period under review. The result of the fractal dimension, $D = 1.54$, obtained showed that the market exhibited weak anti-persistent behaviour and a slight departure from a purely random walk during the particular period. Also, the yearly estimated values of $D$ showed varying behavioural patterns: persistent, anti-persistent, and random behaviour across different economic periods. The result will help policymakers gain a better understanding of market dynamics in order to improve market transparency and stability.

*Keywords and phrases*: fractal dimension; Hurst exponent; persistence; volatility; Nigerian stock market.

## 1 Introduction

The financial system is a system created to transfer funds from surplus economic units to deficit economic units in order to produce goods and services and to make investment so as to facilitate the growth of the economy and improve the standard of living of its citizens [1]. It is generally known that financial system plays a catalytic role in the process of economic development. Through the process of growth, financial system offers wide ranges of portfolio options for savers and issuable instruments for investors, a function often referred to as financial intermediation [2].

The Nigerian financial system comprises of various institutions, markets and operations that are in the business of providing financial services. These institutions can be broadly categorized into money and capital markets. Money market is a market in which short term financial instrument are traded, the capital market deals with long term transactions. The capital market is a major channel for mobilizing long-term funds. The main institutions are the Securities and Exchange Commission (SEC) which is the apex regulatory body, the Nigerian Stock Exchange (NSE), the issues houses, stock broking firms and the registrars [3].

Stock market, a subset of capital market, is the fulcrum of economic, financial and business development globally [4, 5]. The Stock Exchange Market has been one of the most popular investments in the recent past due to its high returns. The Market is an integral part of the global economy to the extent that any fluctuation in this market influences personal and corporate financial lives and the economic health of a country [6]. According to [7] market embrace both the new issues primary market and secondary market.

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The Nigerian stock exchange was establishment in 1961. It was formally called Lagos Stock Exchange, its goal is to promoted private capital investment for growth and development in order to develop the capital markets. The Nigeria's capital market enjoyed a decade of unprecedented growth in the past but was affected by the global financial meltdown in 2008 [8, 9, 10]. This global economic meltdown affected the Nigerian economy adversely in such a way that it made the capital market less attractive to long-term investors and very risky to invest [11]. In the last recent years, interactions between capital market and macroeconomic variables on how to repositioned the market have been an issue among financial economists and practitioners [12, 13].

Apart from the global financial meltdown, Nigeria recently experienced economic recession as a consequence of the 2014–2016 global oil price downturn which also affected the capital market [2]. These made the Nigerian stock market to be full of uncertainty [14]. Because of market uncertainty, trade analyst are in search of good mathematical tools/model that will be able to predict the nature of future market to a good considerable extent. One of such model is fractal dimension. Some researchers have made some research on global stock market with the model. Some examples include: [14, 15, 16, 17, 18, 19, 20, 21, 22], these researchers not much work has been made on Nigerian stock market. It is based on this issue that this paper want to use Fractal Dimension to estimate All Share Index Of The Nigerian Stock Market in other to be able to advice investor.

## 2 Mathematical formulation

From Table 1, we present the analysis of the ASINSM data as follows. Let

$$y_r = x_r = (\text{ASINSM})_r, \quad r = 1, 2, \ldots, n, \tag{1}$$

where $x_r$ $(x_1, \ldots, x_n)$ represent $n$ consecutive terms of $x$ with mean

$$\bar{x} = \frac{1}{n}\sum_{r=1}^{n} x_r, \tag{2}$$

$$W_r = (x_r - \bar{x}) \quad \text{where } r = 1, 2, \ldots, n. \tag{3}$$

$$(W_r)^2 = (x_r - \bar{x})^2 \quad \text{where } r = 1, 2, \ldots, n. \tag{4}$$

$$\sum (W_r)^2 = \sum_{r=1}^{n} (W_r)^2 \tag{5}$$

$$\text{The Variance } (V) = \frac{\sum_{r=1}^{n} (W_r)^2}{n} \tag{6}$$

The standard deviation, $S_d$ of $x$ is estimated as

$$S_d = \sqrt{\frac{\sum_{r=1}^{n} (x_r - \bar{x})^2}{n}} \tag{7}$$

Therefore

$$S_d = \sqrt{\frac{\sum_{r=1}^{n} (W_r)^2}{n}} \tag{8}$$

## 3 METHOD OF SOLUTION

The range:

$$R_n = \max(Y_1, \ldots, Y_n) - \min(Y_1, \ldots, Y_n) \tag{9}$$

### 3.1 Self-similar Process

Self similarity of a process $X = (X - \infty < t < \infty)$ can be defined through its distribution. Assume that $(X_{a,t})$ and $a^k(X_t)$ are identically finite dimensional distribution for all $a > 0$ then $X$ is a self similar with $H$ [23].

### 3.2 Properties of Self-Similar Process

$H$ is a Self-Similar Process whose properties are equivalent to the following:

1. Hurst effect: The $R/S$ is characterised by the power law:

$$\mathbb{E}\left[R(u)/S(u)\right] \sim a_1 u^H \text{ as } u \to \infty \text{ with } \frac{1}{2} < H < 1 \tag{10}$$

2. Slowly decaying variances: The variance of the sample mean are decaying more slowly than the reciprocal of the sample size i.e.

$$\text{VAR}\left[X^{(u)}\right] \sim a_2 u^{2H-2} \text{ as } u \to \infty \text{ with } \frac{1}{2} < H < 1 \tag{11}$$

3. Long-range Dependence (LRD): The autocorrelations decay hyperbolically rather than exponentially in other to show a non-summable autocorrelation function $\sum r(k) = \infty$. This implies that though $r(k)$'s are individually small for large lag, their cumulative effect is important.

4. $L/f$ noise: the spectral density $f(\cdot)$ obeys a power law near the origin, i.e.

$$f(\lambda) \sim a_3 \lambda^{1-2H} \text{ as } \lambda \to 0 \quad \text{with } \frac{1}{2} < H < 1 \tag{12}$$

where

$$f(\lambda) = \sum_k r(k) e^{ik\lambda}.$$

[24]

### 3.3 Rescaled Range R/S Analysis

Then dividing Equation (9) by Equation (8) gives the rescaled range $(R/S)$ value.

The main method for determining the fractal size is the $R/S$ analysis method, which was developed by [25]. According to this method, the Hurst parameter is determined by the following formula:

$$(R/S)_n \propto n^H \tag{13}$$

where $n$ is the number of samples/subsamples, $R$ is the range, i.e. equation (9), and $S$ is the standard deviation, i.e. equation (8).

Equation (13) implies

$$(R/S)_n = cn^H \tag{14}$$

Taking the log of both sides, equation (14) becomes

$$\log(R/S)_n = \log(c) + H\log(n) \tag{15}$$

Where $c$ is a constant. If we assume $c = 1$, then equation (15) becomes

$$\log(R/S)_n = H\log(n) \tag{16}$$

Making $H$ the subject of equation (16) gives

$$H = \frac{\log(R/S)_n}{\log(n)} \tag{17}$$

Let:

$$R/S = a, \quad \log(R/S) = b, \quad \log(n) = c \tag{18}$$

The Hurst exponent may take on values between zero and one. A value of 0.5 indicates a random walk process. For a persistent or trend-reinforcing series, it has a value between 0.5 and 1 ($0.5 < H \leq 1.0$). The more the Hurst exponent approximates 1, the The stronger the Hurst exponent approaches 1, the stronger the system's trend-reinforcing behaviour becomes. Values between 0 and 0.5 ($0 \leq H < 0.5$) indicate anti-persistent or mean-reverting systems [26].

### 3.4 Fractals Dimension(D)

The fractal dimension $D$ has a linear relationship with the Hurst parameter $H$ [27].

This implies

$$D = 2 - H. \tag{19}$$

Table 1: The market nature classification (Robert Hudson, 2004).

| Fractal Dimension | Hurst Exponent | Market Behaviour |
|---|---|---|
| $D < 1.5$ | $H > 0.5$ | Persistent (trending) |
| $D = 1.5$ | $H = 0.5$ | Random walk/Efficient market |
| $D > 1.5$ | $H < 0.5$ | Anti-persistent (mean reverting) |

Table 2: showing stock market data of some selected stocks from January 1999 – December 2013 (central bank of nigerian (CBN) statistical bulletin 2014).

| Yt/Mt | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1999 | 5.494.80 | 5.376.50 | 5.456.20 | 5.315.70 | 5.317.50 | 5.977.90 | 4.964.40 | 4.946.20 | 4.890.80 | 5.032.50 | 5.133.20 | 5.266.40 |
| 2000 | 5.752.90 | 5.955.70 | 5.966.20 | 5.892.80 | 6.095.40 | 6.466.70 | 6.900.70 | 7.394.10 | 7.298.90 | 7.415.30 | 7.164.40 | 8.111.00 |
| 2001 | 8.794.20 | 9.180.50 | 9.159.80 | 9.591.60 | 10.153.80 | 10.937.30 | 10.576.40 | 10.329.00 | 10.274.20 | 11.091.40 | 11.169.60 | 10.963.10 |
| 2002 | 10.650.00 | 10.581.90 | 11.214.40 | 11.399.10 | 11.486.70 | 12.440.70 | 12.458.20 | 12.327.90 | 11.811.60 | 11.451.50 | 11.622.70 | 12.137.70 |
| 2003 | 13.298.80 | 13.668.80 | 13.531.10 | 13.488.00 | 14.086.30 | 14.565.50 | 13.962.00 | 15.426.00 | 16.500.50 | 18.743.50 | 19.319.30 | 20.128.94 |
| 2004 | 22.712.88 | 24.797.43 | 22.896.40 | 25.793.00 | 27.730.80 | 28.887.40 | 27.062.10 | 23.774.30 | 22.739.70 | 23.354.80 | 23.270.50 | 23.844.50 |
| 2005 | 23.078.30 | 21.953.50 | 20.682.40 | 21.961.70 | 21.482.10 | 21.564.80 | 21.911.00 | 22.935.40 | 24.635.90 | 25.873.80 | 24.355.90 | 24.085.80 |
| 2006 | 23.679.40 | 23.843.00 | 23.336.60 | 23.301.20 | 24.745.70 | 26.316.10 | 27.880.50 | 33.096.40 | 32.554.60 | 32.643.70 | 32.632.50 | 33.189.30 |
| 2007 | 36.784.50 | 40.730.70 | 43.456.10 | 47.124.00 | 49.930.20 | 51.330.50 | 53.021.70 | 50.291.10 | 50.229.00 | 50.201.80 | 54.189.90 | 57.990.20 |
| 2008 | 54.189.90 | 265.652.30 | 863.016.50 | 659.440.90 | 158.929.00 | 255.949.00 | 53.110.90 | 147.789.20 | 46.216.10 | 336.325.80 | 633.025.70 | 531.450.78 |
| 2009 | 21.813.76 | 23.377.14 | 19.851.89 | 21.491.11 | 29.700.24 | 26.861.55 | 25.286.61 | 23.009.10 | 22.065.00 | 21.804.69 | 21.010.29 | 20.827.17 |
| 2010 | 22.594.90 | 22.985.00 | 25.966.25 | 26.453.20 | 26.183.21 | 25.384.14 | 25.844.20 | 24.268.20 | 23.050.60 | 25.042.20 | 24.764.70 | 24.770.52 |
| 2011 | 27.356.59 | 26.181.18 | 24.696.95 | 25.041.68 | 25.829.75 | 24.923.34 | 23.826.99 | 21.497.61 | 20.373.00 | 20.934.96 | 20.003.36 | 20.773.98 |
| 2012 | 20.875.83 | 20.123.51 | 20.652.47 | 22.109.76 | 22.170.96 | 21.599.57 | 23.302.22 | 23.750.82 | 26.011.64 | 26.430.92 | 26.494.44 | 28.078.81 |
| 2013 | 31.853.91 | 33.075.14 | 33.536.25 | 33.440.57 | 37.794.75 | 36.168.31 | 37.806.45 | 36.248.53 | 36.585.08 | 37.622.74 | 38.920.85 | 41.329.19 |

Table 4: Showing Standard Error of R/S Estimates of H

| YEAR | N | Range | Mean | |
|---|---|---|---|---|
| | Statistic | Statistic | Statistic | Std. Error |
| 1999–2013 | 15 | 541926.12 | 284107.71 | 41754.86 |

Table 3: Showing $R/S$ Estimates of $H$.

| **YEAR**$_{Ti}$ | $Y_r$ | $W_r$ | $W_r^2$ | $CW_r^2$ |
|---|---|---|---|---|
| 1999 | 63170.30 | -220937.4093 | 48813338843 | 48813338843 |
| 2000 | 80414.10 | -203693.6093 | 41491086483 | 90304425326 |
| 2001 | 122220.90 | -161886.8093 | 26207339036 | 1.16512E+11 |
| 2002 | 139582.40 | -144525.3093 | 20887565038 | 1.37399E+11 |
| 2003 | 186718.74 | -97388.96933 | 9484611348 | 1.46884E+11 |
| 2004 | 296863.81 | 12756.10067 | 162718104.2 | 1.47047E+11 |
| 2005 | 274520.60 | -9587.109333 | 91912665.37 | 1.47139E+11 |
| 2006 | 337219.00 | 53111.29067 | 2820809196 | 1.49959E+11 |
| 2007 | 585279.70 | 301171.9907 | 90704567962 | 2.40664E+11 |
| 2008 | 605096.42 | 320988.7107 | 1.03034E+11 | 3.43698E+11 |
| 2009 | 277098.55 | -7009.159333 | 49128314.56 | 3.43747E+11 |
| 2010 | 297307.12 | 13199.41067 | 174224441.9 | 3.43921E+11 |
| 2011 | 281439.39 | -2668.319333 | 7119928.065 | 3.43928E+11 |
| 2012 | 281600.95 | -2506.759333 | 6283842.355 | 3.43934E+11 |
| 2013 | 433083.66 | 148975.9507 | 22193833877 | 3.66128E+11 |
| TOTAL | 4261615.64 | | | 3.66128E+11 |

Table 5: Descriptive Data Showing the Yearly Data of Stock Market Range and standard deviation

| YEAR | N | Range | Minimum | Maximum | Sum | Mean | | Std. Deviation |
|---|---|---|---|---|---|---|---|---|
| | | | | | | Statistic | Std. Error | Statistic |
| 1999 | 12 | 1087.10 | 4890.80 | 5977.90 | 63170.30 | 5264.19 | 87.84 | 304.30 |
| 2000 | 12 | 2358.10 | 5752.90 | 8111.00 | 80414.10 | 6701.17 | 224.62 | 778.11 |
| 2001 | 12 | 2375.40 | 8794.20 | 11169.60 | 122220.90 | 10185.07 | 238.23 | 825.27 |
| 2002 | 12 | 1876.30 | 10581.90 | 12458.20 | 139582.40 | 11631.87 | 183.78 | 636.63 |
| 2003 | 12 | 6830.14 | 13298.80 | 20128.94 | 186718.74 | 15559.90 | 722.26 | 2501.97 |
| 2004 | 12 | 6174.52 | 22712.88 | 28887.40 | 296863.81 | 24738.65 | 615.29 | 2131.41 |
| 2005 | 12 | 5191.40 | 20682.40 | 25873.80 | 274520.60 | 22876.72 | 451.22 | 1563.05 |
| 2006 | 12 | 9888.10 | 23301.20 | 33189.30 | 337219.00 | 28101.58 | 1260.60 | 4366.83 |
| 2007 | 12 | 21205.70 | 36784.50 | 57990.20 | 585279.70 | 48773.31 | 1710.24 | 5924.44 |
| 2008 | 12 | 34201.60 | 31450.78 | 65652.38 | 605096.42 | 50424.70 | 3350.86 | 11607.72 |
| 2009 | 12 | 9848.35 | 19851.89 | 29700.24 | 277098.55 | 23091.55 | 824.13 | 2854.86 |
| 2010 | 12 | 3858.30 | 22594.90 | 26453.20 | 297307.12 | 24775.59 | 379.31 | 1313.97 |
| 2011 | 12 | 7353.23 | 20003.36 | 27356.59 | 281439.39 | 23453.28 | 745.85 | 2583.70 |
| 2012 | 12 | 7955.30 | 20123.51 | 28078.81 | 281600.95 | 23466.75 | 771.78 | 2673.54 |
| 2013 | 12 | 9475.28 | 31853.91 | 41329.19 | 433083.66 | 36090.30 | 772.89 | 2677.37 |

Table 6: Yearly Estimate of Hurst Parameter ($H$) Showing the Varying Behaviours of the Stock Market

| YEAR | n | log n | R/S | log(R/S) | H | D | Market Behaviour |
|---|---|---|---|---|---|---|---|
| 1999 | 12 | 1.08 | 3.57 | 0.55 | 0.51 | 1.49 | Persistent |
| 2000 | 12 | 1.08 | 3.03 | 0.48 | 0.44 | 1.56 | Anti-persistent |
| 2001 | 12 | 1.08 | 2.88 | 0.46 | 0.43 | 1.57 | Anti-persistent |
| 2002 | 12 | 1.08 | 2.95 | 0.47 | 0.44 | 1.56 | Anti-persistent |
| 2003 | 12 | 1.08 | 2.73 | 0.44 | 0.41 | 1.59 | Anti-persistent |
| 2004 | 12 | 1.08 | 2.90 | 0.46 | 0.43 | 1.57 | Anti-persistent |
| 2005 | 12 | 1.08 | 3.32 | 0.52 | 0.48 | 1.52 | Anti-persistent with slightly Random Walk |
| 2006 | 12 | 1.08 | 2.26 | 0.35 | 0.32 | 1.68 | Anti-persistent |
| 2007 | 12 | 1.08 | 3.58 | 0.55 | 0.51 | 1.49 | Persistent |
| 2008 | 12 | 1.08 | 2.95 | 0.47 | 0.44 | 1.56 | Anti-persistent |
| 2009 | 12 | 1.08 | 3.45 | 0.54 | 0.50 | 1.50 | Random Walk |
| 2010 | 12 | 1.08 | 2.94 | 0.47 | 0.44 | 1.56 | Anti-persistent |
| 2011 | 12 | 1.08 | 2.85 | 0.46 | 0.43 | 1.57 | Anti-persistent |
| 2012 | 12 | 1.08 | 2.98 | 0.47 | 0.44 | 1.56 | Anti-persistent |
| 2013 | 12 | 1.08 | 3.54 | 0.55 | 0.51 | 1.49 | Persistent |

## 4 Results and discussions

The result is presented as follows, in table 3:

The mean (equation 2) of $x$ is $\bar{x} = 284107.71$

The range (equation 9) is $R_n = 541926.12$

The standard deviation (equation 8) is $S_d = 156232.3$ with standard error $= 41754.86$.

The $R/S$ (equation 18) is

$$R/S = 3.47.$$

Furthermore,

$$\log(R/S) = 0.54,$$

$$n = 15,$$

$$\log(n) = 1.176,$$

$$H = 0.46,$$

and

$$D = 1.54 \quad \text{in equation (19).}$$

$D = 1.54$ implies that the nature of the trading activities of the estimated stock market are as follows: The market exhibit weak anti-persistent behaviour which is a mind departure from a purely random work process. In table 6: The result obtained shows varying nature of stock market behaviour. 1999, 2007 and 2013 shows persistent market behaviour; 2005 shows random walk behaviour. 2009 shows anti-persistent and slightly random walk behaviour while other years shows anti-persistant behaviour.

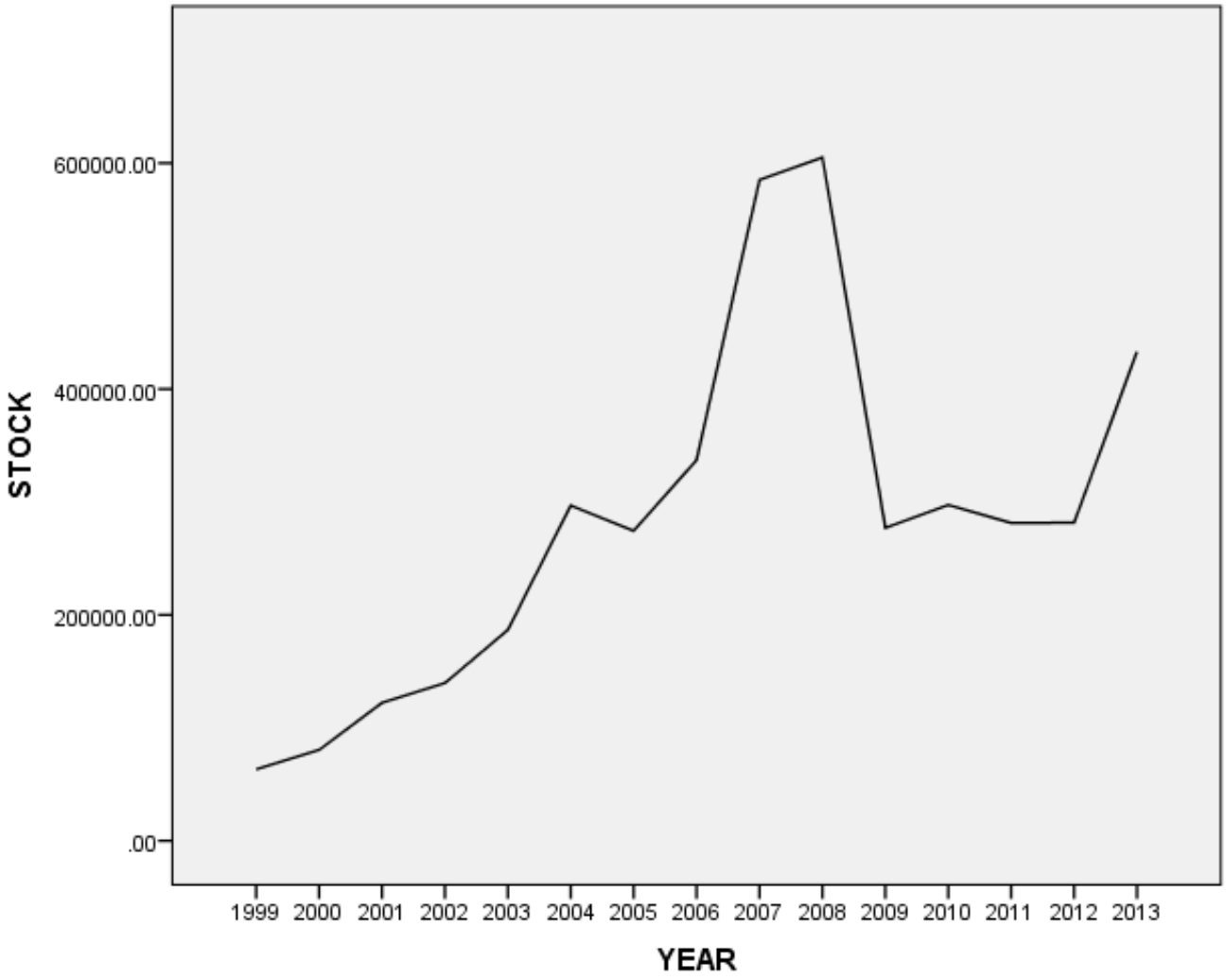


Figure 1: A line graph showing varying nature of trading activities for the period

## 5 Conclusion

In this paper, the all share index of the Nigerian stock market was estimated from 1999 to 2013 using fractal dimension. The R/S statistic was used to find the Hurst parameter $H$, which has a linear relationship with fractal dimension. The study evaluates whether the Nigerian stock market exhibits persistent, random, or anti-persistent behaviour within the period under review. The result of the fractal dimension, $D = 1.54$, obtained showed that the market exhibited weak anti-persistent behaviour and a slight departure from a purely random walk during the particular period. Also, the yearly estimated values of $D$ showed varying patterns of behaviour, namely persistent, anti-persistent, and random behaviour across different economic periods. The result will help policymakers to have a good knowledge of market dynamics in order to improve market transparency and stability.

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