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Analytical shooting technique for fluid model

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Abstract

Solution to boundary value problems emanated from modeling of fluid flow problems like Jeffery-Hamel flow, couette flow and the likes without an exact solution has been the desire of researchers in recent years; with this a semi-analytical scheme was developed to solve some of the differential equations of nth order ($n \geq 2$) emanated from modeling of fluid problems in engineering. This approach was used to solve some of the problems, and results obtained are continuous solutions that are free of discretization errors. Two shooting gradients were used to adjust the shooting angle namely Cubic Newton formula (CNF) and Modified Fourth-Order formula (MFOF), and it was observed that MFOF needs less iteration than CNF to produce an approximate solution with a very low difference from the exact solution where the exact solution is availabl

Keywords and phrases: shooting technique; fluid; engineering; Adomian Decomposition Method; boundary value problem

1 Introduction

Many problems emanating from engineering, most especially fluid dynamics, are being modelled mathematically using differential equations such as mathematical modelling [4, 19]. Differential equations serve as great tool in modeling such problems, such as [10] where the study investigates the optimization of entropy in viscous fluid flow over the surface of a rotating cone. The analysis incorporates the influence of internal heat generation and viscous dissipation on the thermal field. The governing nonlinear system of ordinary differential equations, formulated from the underlying physical model, was addressed numerically through the application of the shooting method. Also [23] carried out research on Criticality of the branch chain and heat explosion of an old 6-constant fluid for generalized viscous shear flow with the aid of differential equations, where relative fluid was assumed to be exothermic. Most of the derived differential equations from modeling are nonlinear in nature, which in most cases, has no direct solution. However, numerous approaches have been developed to solve nonlinear Boundary Value Problems (BVPs) emanating from modeling in engineering such as; shooting techniques coupled with Runge-Kutta method to solve BVPs emanated from magneto-hydrodynamics (MHD) Falker-Skan model over a permeable wall [15]. This is a fluid problem that deals with electromagnetic phenomena where the resulting differential equations were solved with Runge-Kutta Shooting method. Weighted residual collocation method was employed by authors in [4] to generate solution of a couette flow model that was developed in[24]. Two point Taylor iterative linearization via Legendre was used in [17] to solve a model with estimated kinetics that depicts the thermal dispersion patterns of reactive space which is a second order nonlinear BVP at different boundary conditions. Author in [21] applied the collocation method in finding the solution to nonlinear BVP of Flow and heat transfer characteristics of an Eyring–Powell fluid past a linearly stretching surface. Similarity transformation procedure was also used in [20] to transform coupled partial differential equation into nonlinear coupled ordinary differential equation (ODE) generated from the governing Casson flow problem, and

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Chebyshev collocation method was used to solve the resulting linearized ODE. Among other methods that have been used to solve ODE that were developed from mathematical modeling are the Homotopy analysis method [8] and the Keller box method [3].

The contribution of this research lies in providing an analytical approach for solving boundary value problems (BVPs) arising from the mathematical modeling of fluid dynamics, while also minimizing discretization errors through the application of higher-order root-finding formulas as shooting gradient methods within the shooting technique framework

2 Methodology

The focus of this study is to implement shooting technique with semi-analytical method which is Adomian Decomposition Method (ADM)

2.1 Semi-Analytical Shooting Technique (SAST)

The shooting technique commonly employed are developed for second order ODE [5, 14, 21], one of the purpose of this work is to modified the technique to suit any nth order differential equation $n \geq 2$. Consider the nth order boundary value problem which can be written as

$$p^{(n)}(x) = k\left(x, p, p'p'', \cdots, p^{(n-1)}\right), \quad a \leq x \leq b, \tag{1}$$

having the boundary conditions (BC)

$$p(a) = \alpha_1,\, p'(a) = \alpha_2,\, \cdots, p^{(n-2)}(a) = \alpha_{n-2},\, p(b) = \phi. \tag{2}$$

To get the approximate solution of (1) with its boundary conditions (BC's) (2), it requires reducing (1) to its appropriate IVP subjected to initial conditions and introducing the parameter v, which will serve as the shooting gradient (SG)

$$\begin{gathered} p^{(n)}(x) = k\left(x, p, p'p'', \cdots, p^{(n-1)}\right), \quad a \leq x \leq b, \\ p(a) = \alpha_1,\, p'(a) = \alpha_2,\, \cdots, p^{(n-2)}(a) = \alpha_{n-2},\, p^{n-1}(a) = v_i. \end{gathered} \tag{3}$$

Equation (3) is an approximation of (1) subject to the BC in (2), this equation (3) is then solved by the process of ADM repeatedly until $\lim\limits_{i\to\infty} p(b, v_i) = p(b) \approx \phi$ where v_i is called shooting gradient (SG) and $i = 0, 1, 2, \cdots$. The initial SG is given in terms of slope as in [14, 21]

$$v_0 = \frac{\phi - \alpha_1}{b - a}, \tag{4}$$

Solving (3) with the initial SG (4), using ADM gives an analytical solution in form of polynomial. If $p(b, v_0)$ is not sufficiently close to ϕ which in most cases it is expected, the approximation will be adjusted by choosing another SG $v_1, v_2, \cdots$, until $p(b, v_i)$ is sufficiently close to ϕ where the subsequent gradient will be determined by shooting gradient procedure.

2.2 Shooting Gradient Procedure

These are the formulae used to adjust the SG when using shooting technique in solving BVP, this formulas are derived from root finding formulas. In literature, Newton's and Secant methods are the most used SG formulae [5, 21, 25].

2.2.1 Cubic Newton's Formula (CNF)

This formula was firstly used in [16] for second order BVP, and is given by

$$v_i = v_{i-1} - \left(\frac{p(b, v_{i-1}) - \phi}{g(b, v_{i-1})} - \left(\frac{p(b, v_{i-1}) - \phi}{g(b, v_{i-1})} \right)^2 \left(\frac{r(b, v_{i-1})}{2g(b, v_{i-1})} \right) \right), \; i = 1, 2, 3, \cdots \tag{5}$$

where $g = \frac{\partial p}{\partial v}$ and $r = \frac{\partial^2 p}{\partial v^2}$ in (5).

The solution for $p(b, v_{i-1})$ can always be obtained from the reduced IVP, while $g(b, v_{i-1})$ and $r(b, v_{i-1})$ requires derivation of supplementary equation from (3) more details of the derivation is in [15]. Using the approach in the work of [16], the supplementary equations are

$$\begin{gathered} g^{(n)}(x, v) = k_p g + k_{p'} g' + k_{p''} g'' + \cdots + k_{p^{(n-1)}} g^{(n-1)}, \\ g(a) = 0, \; g'(a) = 0, \; \cdots, g^{(n-1)}(a) = 1. \end{gathered} \tag{6}$$

$$\begin{aligned} r^{(n)}(x, v) = k_p r + g \left(k_{pp} g + k_{pp'} g' + \cdots + k_{pp^{(n-1)}} g^{(n-1)} \right) \\ + k_{p'} r' + g' \left(k_{pp'} g + k_{p'p'} g' + \cdots + k_{p'p^{(n-1)}} g^{(n-1)} \right) + \cdots + \\ k_{p^{(n-1)}} r^{(n-1)} + g^{(n-1)} \left(k_{pp^{(n-1)}} g + k_{p^{(n-1)}p'} g' + \cdots + k_{p^{(n-1)}p^{(n-1)}} g^{(n-1)} \right) \\ r(a) = 0, \; r'(a) = 0, \cdots, r^{(n-1)}(a) = 0. \end{aligned} \tag{7}$$

2.2.2 Modified Fourth-Order Formula (MFOF)

Another formula to be used as SG is derived from expansion of Taylor series to $4th$ order, that is Let

$$\begin{aligned} p(x_n + h) =& p(x_n) + p'(x_n)(x_{n+1} - x_n) + \frac{1}{2!} p''(x_n)(x_{n+1} - x_n)^2 \\ &+ \frac{1}{3!} p'''(x_n)(x_{n+1} - x_n)^3 + \cdots \end{aligned} \tag{8}$$

setting $p(x_n + h) = 0$, then (10) will be equivalent to

$$p(x_n) + p'(x_n)(x_{n+1} - x_n) + \frac{1}{2} p''(x_n)(x_{n+1} - x_n)^2 + \frac{1}{6} p'''(x_n)(x_{n+1} - x_n)^3 = 0, \tag{9}$$

$$(x_{n+1} - x_n) \left(p'(x_n) + \frac{1}{2} p''(x_n)(x_{n+1} - x_n) + \frac{1}{6} p'''(x_n)(x_{n+1} - x_n)^2 \right) = -p(x_n), \tag{10}$$

$$x_{n+1} = x_n - \frac{6p(x_n)}{6p'(x_n) + 3p''(x_n)(x_{n+1} - x_n) + p'''(x_n)(x_{n+1} - x_n)^2}. \tag{11}$$

From the first order Taylor series

$$x_{n+1} - x_n = -\frac{p(x_n)}{p'(x_n)}, \tag{12}$$

replacing $(x_{n+1} - x_n)$ in (11) with its expression from (12) and simplify further to have

$$x_{n+1} = x_n - \frac{6p(x_n)p'(x_n)^2}{6p'(x_n)^3 - 3p(x_n)p'(x_n)p''(x_n) + p(x_n)^2 p'''(x_n)}. \tag{13}$$

So if $v_i = x_{n+1}$ and let $g = p'$, $r = p''$ and $u = p'''$ in (13),then equation (13) becomes

$$v_i = v_{i-1} - \left(\frac{6\,(p(b, v_{i-1} - \phi))\,(h(b, v_{i-1}))^2}{6\,(g(b, v_{i-1}))^2 - 3\,(p(b, v_{i-1}) - \phi)\,(g(b, v_{i-1}))\,(r(b, v_{i-1})) + (p(b, v_{i-1} - \phi))^2\,(u(b, v_{i-1}))} \right), \tag{14}$$

this is called Modified Fourth-Order formula (MFOF) for adjusting SG where $i = 1, 2, 3, \cdots$. Equation (3) will be used to formulate $u(b, v_{i-1})$ as

$$\frac{\partial^3 p^n(x,t)}{\partial t^3} = \frac{\partial^3 k}{\partial t^3}\left(x, p(x,t), p'(x,t), \cdots p^{(n-1)}(x,t)\right), \tag{15}$$

where x and t are independent

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\left(\frac{\partial k}{\partial t}\right)\right) = \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\left(\frac{\partial k}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial k}{\partial p}\cdot\frac{\partial p}{\partial t} + \frac{\partial k}{\partial p'}\cdot\frac{\partial p'}{\partial t} + \cdots + \frac{\partial k}{\partial p^{(n-1)}}\cdot\frac{\partial p^{(n-1)}}{\partial t}\right)\right). \tag{16}$$

Because x and t are independent, then $\frac{\partial x}{\partial x}$ and (16) becomes

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\left(\frac{\partial k}{\partial t}\right)\right) = \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\left(\frac{\partial k}{\partial p}\cdot\frac{\partial p}{\partial t} + \frac{\partial k}{\partial p'}\cdot\frac{\partial p'}{\partial t} + \frac{\partial k}{\partial p''}\cdot\frac{\partial p''}{\partial t} + \cdots + \frac{\partial k}{\partial p^{(n-1)}}\cdot\frac{\partial p^{(n-1)}}{\partial t}\right)\right). \tag{17}$$

Equation (16) can be simplify further with the use of chain rule and let $g = \frac{\partial p}{\partial v}$, $g = \frac{\partial^2 p}{\partial v^2}$ and $g = \frac{\partial^3 p}{\partial v^3}$ to have

$$\begin{aligned} u^{(n)}(x,v) &= k_p u + 2r\left(k_{pp}g + k_{pp'}g' + k_{pp''}g'' + \cdots + k_{pp^{(n-1)}}g^{(n-1)}\right) \\ &\quad + k_{p'}u' + 2r'\left(k_{p'p}g + k_{p'p'}g' + k_{p'p''}g'' + \cdots + k_{p'p^{(n-1)}}g^{(n-1)}\right) \\ &\quad + k_{p''}u'' + 2r''\left(k_{p''p}g + k_{p''p'}g' + k_{p''p''}g'' + \cdots + k_{p''p^{(n-1)}}g^{(n-1)}\right) + \\ &\quad \vdots \\ &+ k_{p^{(n-1)}}u^{(n-1)} + 2r^{(n-1)}\left(k_{p^{(n-1)}p}g + k_{p^{(n-1)}p'}g' + k_{p^{(n-1)}p''}g'' + \cdots + k_{p^{(n-1)}p^{(n-1)}}g^{(n-1)}\right) \\ &\quad u(a) = 0,\ u'(a) = 0,\ \cdots, u^{(n-1)}(0) = 0. \end{aligned} \tag{18}$$

2.3 Concept of Adomian Decomposition Method for IVP

Considering a nonlinear initial value problem (IVP) having the

$$L(p(x)) + R(p(x)) + N(p(x)) = m(x), \tag{19}$$

here, L denotes the linear operator corresponding to the highest-order derivative, R represents the linear operator of lower order terms, and N stands for the nonlinear operator. [9, 11] which will require to be decomposed by Adomian polynomial [2, 11], while $m(x)$ is the source term. Taking the L^{-1} of both side of (19)

$$p(x) = \Upsilon_0 + L^{-1}(m(x)) - L^{-1}(Rp(x)) + L^{-1}(Np(x)) = m(x), \tag{20}$$

where L^{-1} is multiple definite integral of order $n[0tox]$, $Np(x)$ is the nonlinear. Extensive information on the ADM procedure is available in [2, 7, 9, 11, 18]. Thus, solution to the given problem can be written in an infinite series as

$$p(x) = \sum_{n=0}^{\infty} p_n(x). \tag{21}$$

3 Numerical Application

3.1 Example 1:

Consider the MHD Jeffery-Hamel flow [1] given as

$$p'''(x) = -\beta Re p(x)p'(x) - (4 - Ha)\left(\frac{\beta}{2}\right)^2 p'(x), \tag{22}$$
$$p(0) = 1,\ p'(0) = 0,\ p(1) = 0.$$

Where Re is the Reynolds number, Ha is Hartmann number and β is the angle of inclination between the two walls at which the fluid flows. MHD Jeffery-Hamel flow is a kind of fluid dynamics issue in which a magnetic field is applied while an electrically conducting fluid flows between two non-parallel surfaces. [6] Equation (23) is reduced to the IVP is

$$p'''(x) = -\beta Re\, p(x)p'(x) - (4 - Ha)\left(\frac{\beta}{2}\right)^2 p'(x), \tag{23}$$
$$p(0) = 1,\ p'(0) = 0,\ p''(0) = v_i.$$

Using (6) and (7) to generate the supplementary equations associated with Cubic Newton's formula to have

$$g'''(x) = -\beta Re\, g(x)p'(x) - \beta Re\, p(x)g'(x) - (4 - Ha)\left(\frac{\beta}{2}\right)^2 g'(x), \tag{24}$$
$$g(0) = 1,\ g'(0) = 0,\ g''(0) = 1,$$

and

$$r'''(x) = -\beta Re\, r(x)p'(x) - 2\beta Re\, g(x)g'(x) - \beta Re\, p(x)r'(x) - (4 - Ha)\left(\frac{\beta}{2}\right)^2 r'(x), \tag{25}$$
$$r(0) = 1,\ r'(0) = 0,\ r''(0) = 0,$$

respectively while the MFOF make use of additional supplement equation to (24) and (25) which can be generated with (18)

$$\begin{aligned} u'''(x) = &-\beta Re\, r(x)p'(x) - 2\beta Re\, r(x)g'(x) - 2\beta Re\, g(x)r'(x) \\ &-\beta Re\, p(x)u'(x) - (4 - Ha)\left(\frac{\beta}{2}\right)^2 u'(x), \end{aligned} \tag{26}$$
$$u(0) = 1,\ u'(0) = 0,\ u''(0) = 0,$$

Equation (23)-(26) are solve simultaneously using ADM repeatedly till $\lim_{i\to\infty} p(b, v_i) = p(b) \approx \phi$

3.2 Example 2

Consider

$$\frac{d^2T}{dy^2} + \eta e^{\lambda T}, \tag{27}$$
$$T(0) = 0,\ T(1) = 1.$$

This is a model equation of thermal effect on Couette flow [12], describing fluid motion between two parallel plates, where η is the heating parameter, λ is the fluid viscosity, T is the temperature of the fluid and y is the perpendicular distance between the two parallel plates. This problem was solved by

[12] using perturbation method and Hermite–Padé approximation technique. Solving (27) using the proposed method, the reduced IVP of (27) is

$$\frac{d^2T}{dy^2} = -\eta e^{\lambda T}, \qquad T(0) = 0,\ T'(0) = v_i. \tag{28}$$

The supplementary equations to be solved alongside with (28) are

$$\frac{d^2g}{dy^2} = -g\lambda\eta e^{\lambda T}, \qquad g(0) = 0,\ g'(0) = 1, \tag{29}$$

$$\frac{d^2r}{dy^2} = -r\lambda\eta e^{\lambda T} - g^2\lambda^2\eta e^{\lambda T}, \qquad r(0) = 0,\ r'(0) = 0, \tag{30}$$

and

$$\frac{d^2u}{dy^2} = -u\lambda\eta e^{\lambda T} - 2gr\lambda^2\eta e^{\lambda T}, \qquad u(0) = 0,\ u'(0) = 0, \tag{31}$$

generated with (6), (7) and (18) respectively.

4 Results and Discussion

The results obtained when using SAST with ADM were presented in Table 1 to 9 when using CNF and MFOF to adjust the SG.

For example 1 different solution was presented by varying Re, Ha and β parameters. Table 1 presents a comparison between the exact solution and the approximate solution of the skin friction $p''(0)$ when parameters at $Re = 10$ and $\beta = 5^0$ are fixed. It was observed that as the Hartmann number (Ha) increases down the table the skin friction also increases. Table 2 also has similar observation with table only that $\beta = -5^0$. Table 3 shows how skin friction $p'\prime(0)$ can be affected when Reyonlds number was varies and $Ha = 0$ and $\beta = 5^0$ are fixed. It was observed that as there is increase in Reyonld number, the skin friction $p'\prime(0)$ decreases down the table for both methods used in adjusting the SG. But Table 4 shows the affected of variation Reyonlds number and when $Ha = 0$ and $\beta = -5^0$ are fixed on skin friction $p'\prime(0)$. It was observed that increase in Reynolds number leads to decreases in skin friction. Generally, the approximate solutions obtained was also closed to exact solution for both CNF and MFOF only that MFOF converges faster than CNF.

Example 2 has no exact solution it was compared with solution obtained with shooting method via Runge-Kutta Method (RK) at fourth iterate.

Table 1: Approximate Solution for Example 1 at $Re = 10$ and $\beta = 5^0$.

Ha	Exact Solution [1]	CNF at v_3	Absolute Error for CNF at v_3	MFOF at v_2	Absolute Error for MFOF at v_2
0	-2.25194860298	-2.25192790491	0.00002069807	-2.25192790483	0.00002069815
200	-1.98460616460	-1.98460617738	0.00000001278	-1.98460617737	0.00000001277
600	-1.55460599206	-1.55478671513	0.00018072307	-1.55478671513	0.00018072307

Table 2: Approximate Solution for Example 1 at $Re = 10$ and $\beta = -5^0$.

Ha	Exact Solution [1]	CNF at v_3	Absolute Error for CNF at v_3	MFOF at v_2	Absolute Error for MFOF at v_2
0	-1.78454684058	-1.78453514248	0.00001169810	-1.784535142486	0.0.00001169809
200	-1.57612988885	-1.57607504866	0.00005484019	-1.576075048664	0.00000001277
600	-1.24053445388	-1.24063818711	0.00010373323	-1.240638187105	0.00010373323

Table 3: Approximate Solution for Example 1 at $Ha = 0$ and $\beta = 5^0$.

Re	Exact Solution [1]	CNF at v_3	Absolute Error for CNF at v_3	MFOF at v_2	Absolute Error for MFOF at v_2
10	-2.25194860298	-2.25192790491	0.00002069807	-2.25192790483	0.00002069815
20	-2.52719223269	-2.25192790491	0.00002069807	-2.52680592101	0.00038631168
40	-3.16971218754	-3.16179337578	0.00791881176	-3.16172130205	0.00799088549

Example 2 has no exact solution it was compared with solution obtained with shooting method via Runge-Kutta Method (RK) at fourth iterate. Table 5- 9 show the approximate solution of the new methods and RK at different value of η and λ. Figure 1 and 2 show the effect of the heating parameter over temperature of the fluid when using CNF and MFOF respectively. Figure 3 and 4 show the effect of the fluid viscosity λ over temperature of the fluid when using CNF and MFOF respectively. It was observed that the increases in the fluid viscosity λ leads to increase in temperature of the fluid, but at lower fixed plate the temperature is at minimum. It also observed that the new methods have similar result with RK, and MFOF is even closer to the boundary condition than RK and also has less iteration than other method used.

Table 5: Computed Approximate Solution for Example 2 when $\eta = 0.5$ and $\lambda = 1$.

x	RK	CNF at v_3	MFOF at v_3
0	0.00000000000000	0.00000000000000	0.00000000000000
0.2	0.26690965164730	0.26703149502070	0.26703149502242
0.4	0.50761709596949	0.50785739624295	0.50785739624633
0.6	0.71505049728830	0.71539779454574	0.71539779455065
0.8	0.88161750928471	0.88200894316011	0.88200894316645
1.0	0.99999999999896	0.99999999999273	1.00000000000000

Table 4: Approximate Solution for Example 1 at $Ha = 0$ and $\beta = -5^0$.

Re	Exact Solution [1]	CNF at v_3	Absolute Error for CNF at v_3	MFOF at v_2	Absolute Error for MFOF at v_2
10	-1.78454684058	-1.78452491763	0.00002192295	-1.78444957315	0.00009726743
20	-1.58815353699	-1.58797519752	0.00017833947	-1.58784685642	0.00030668057
40	-1.25899393509	-1.25758016557	0.00141376952	-1.25752705539	0.00146687970

Table 6: Computed Approximate Solution for Example 2 when $\eta = 1$ and $\lambda = 1$.

x	RK	CNF at v_3	MFOF at v_3
0	0.00000000000000	0.00000000000000	0.00000000000000
0.2	0.35001452330256	0.35531505537760	0.35531572057486
0.4	0.64304223745372	0.65332401918971	0.65332530962443
0.6	0.86004499889808	0.87442443882383	0.87442625375054
0.8	0.98302534796918	0.99862850894299	0.99863066775139
1.0	0.99999999556607	0.99999782609439	1.00000000000030

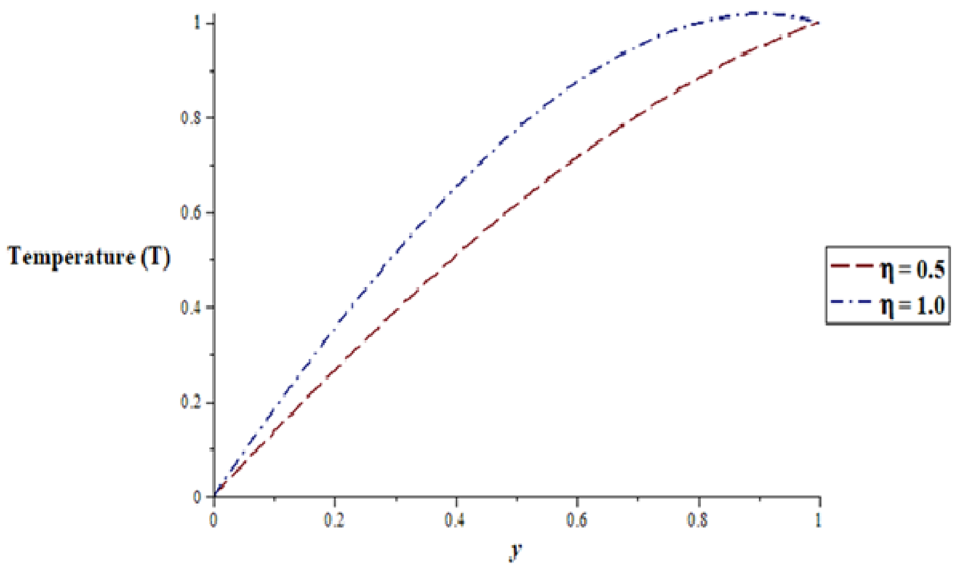

Figure 1: Effect of heating parameter over temperature when using CNF as SG.

Table 7: Approximate Solution for Example 2 when $\eta = 0.1$ and $\lambda = 0.1$.

x	RK	CNF at v_3	MFOF at v_3
0	0.00000000000000	-0.0000000007400	0.0000000002300
0.2	0.20833691417523	0.20833787958100	0.20833787972200
0.4	0.41258961805011	0.41259152636500	0.41259152638800
0.6	0.61267383897997	0.61267650985000	0.61267650980000
0.8	0.80850531912882	0.80850794827500	0.80850794827000
1.0	0.99999999999996	1.0000000002700	0.99999999991300

Table 8: Approximate Solution for Example 2 when $\eta = 0.1$ and $\lambda = 1.5$

x	RK	CNF at v_3	MFOF at v_3
0	0.00000000000000	0.00000000000000	0.00000000000000
0.2	0.21593094592640	0.21592052478095	0.21592052478111
0.4	0.42628849781021	0.42626774572161	0.42626774572190
0.6	0.62901070201894	0.62898082805153	0.62898082805196
0.8	0.82139473971016	0.82136275490126	0.82136275490184
1.0	0.99999999987205	0.99999999999929	1.00000000000000

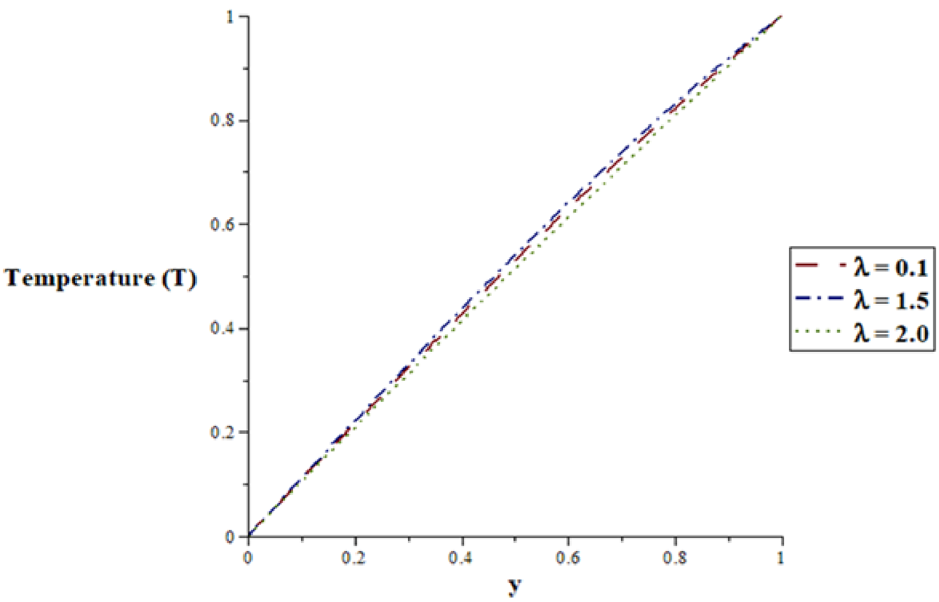

Figure 3: Effect of fluid viscosity over temperature when using CNF as SG.

Table 9: Approximate Solution for Example 2 when $\eta = 0.1$ and $\lambda = 2.0$

x	RK	CNF at v_3	MFOF at v_3
0	0.00000000000000	0.00000000000000	0.00000000000000
0.2	0.22092925033276	0.22086589952372	0.22086589957502
0.4	0.43554363733365	0.43541795544017	0.43541795554202
0.6	0.64047421252145	0.64029231478795	0.64029231493836
0.8	0.83085066497519	0.83065093566148	0.83065093585600
1.0	0.99999999999216	0.99999999977221	1.00000000000260

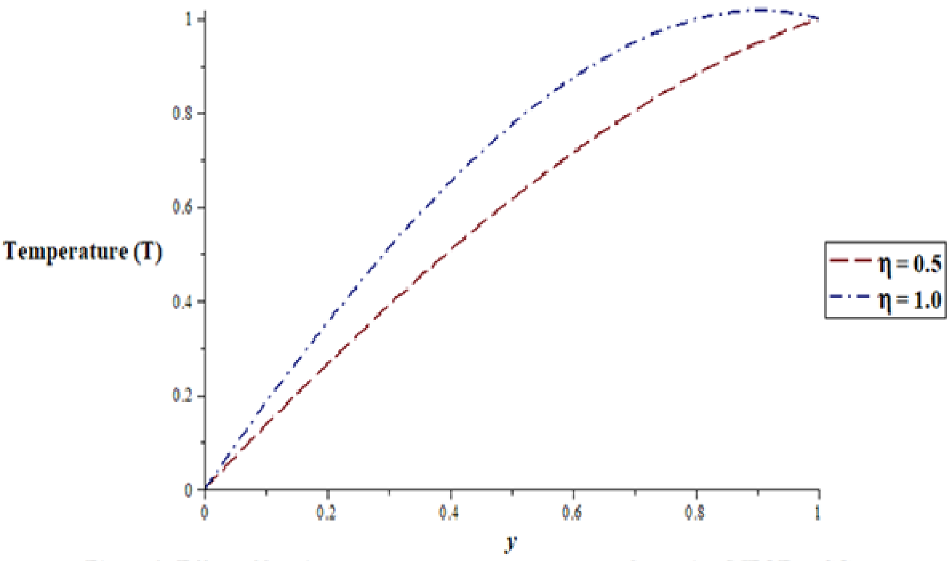

Figure 2: Effect of heating parameter over temperature when using MFOF as SG.

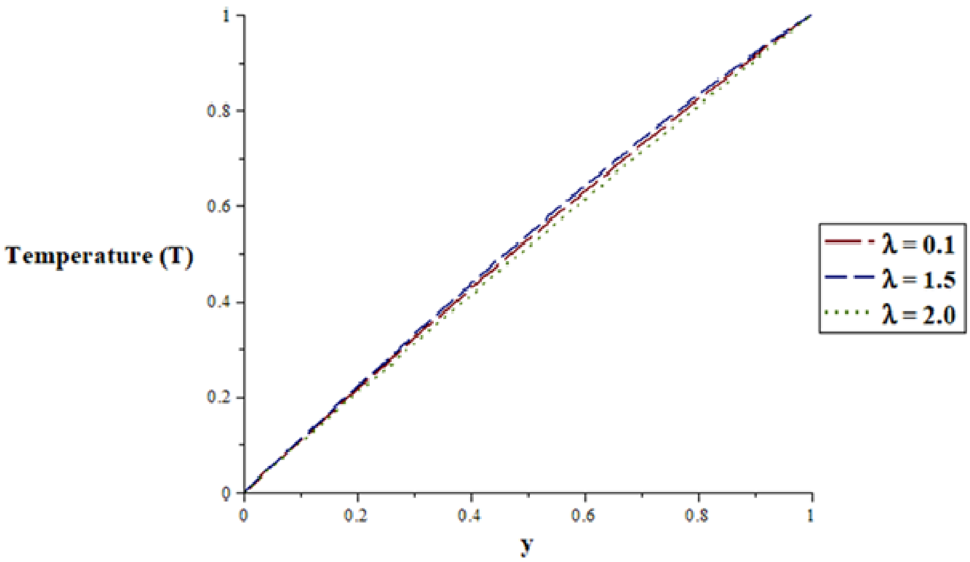

Figure 4: Effect of fluid viscosity over temperature when using MFOF as SG.

5 Conclusion

Semi-Analytical Shooting Techniques was used to solved the resultant differential equation of nth order ($n \geq 2$) of fluid problems, and two different examples was carried out and two method was used to adjust the shooting gradient. It was observed generally that the results obtained are form of polynomial which can be evaluated at any point with the boundary condition. It was also observed the MFOF converges faster than other method been used to adjust the SG.

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